Dense matter in neutron stars: progress, challenges, and implications

Sanjay Reddy, Institute for Nuclear Theory, University of Washington, Seattle.

Precision Many-Body Physics, Paris, June 12-14, 2023

- Introduction to dense matter in neutron stars
- Neutron star structure and the equation of state.
- Transport in a solid and superfluid crust.
- Neutrino transport in warm dense matter.

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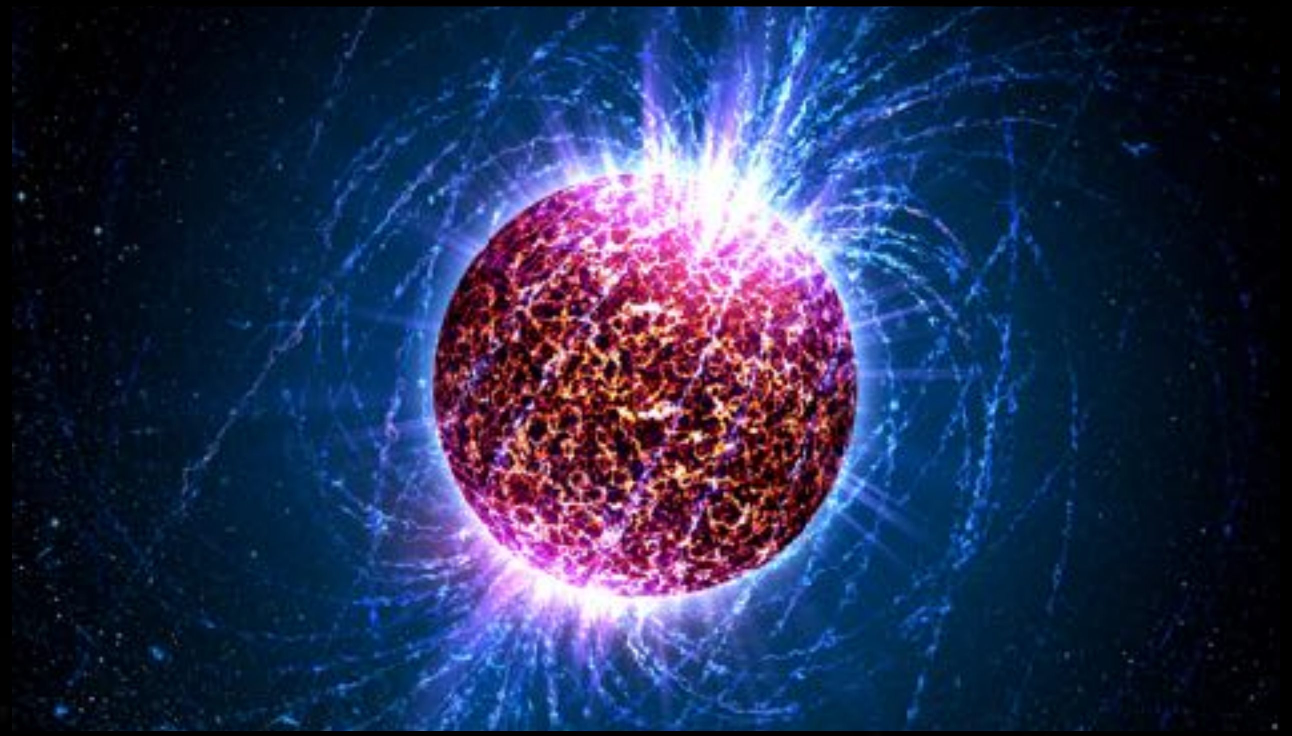
Neutron Stars and Big Questions



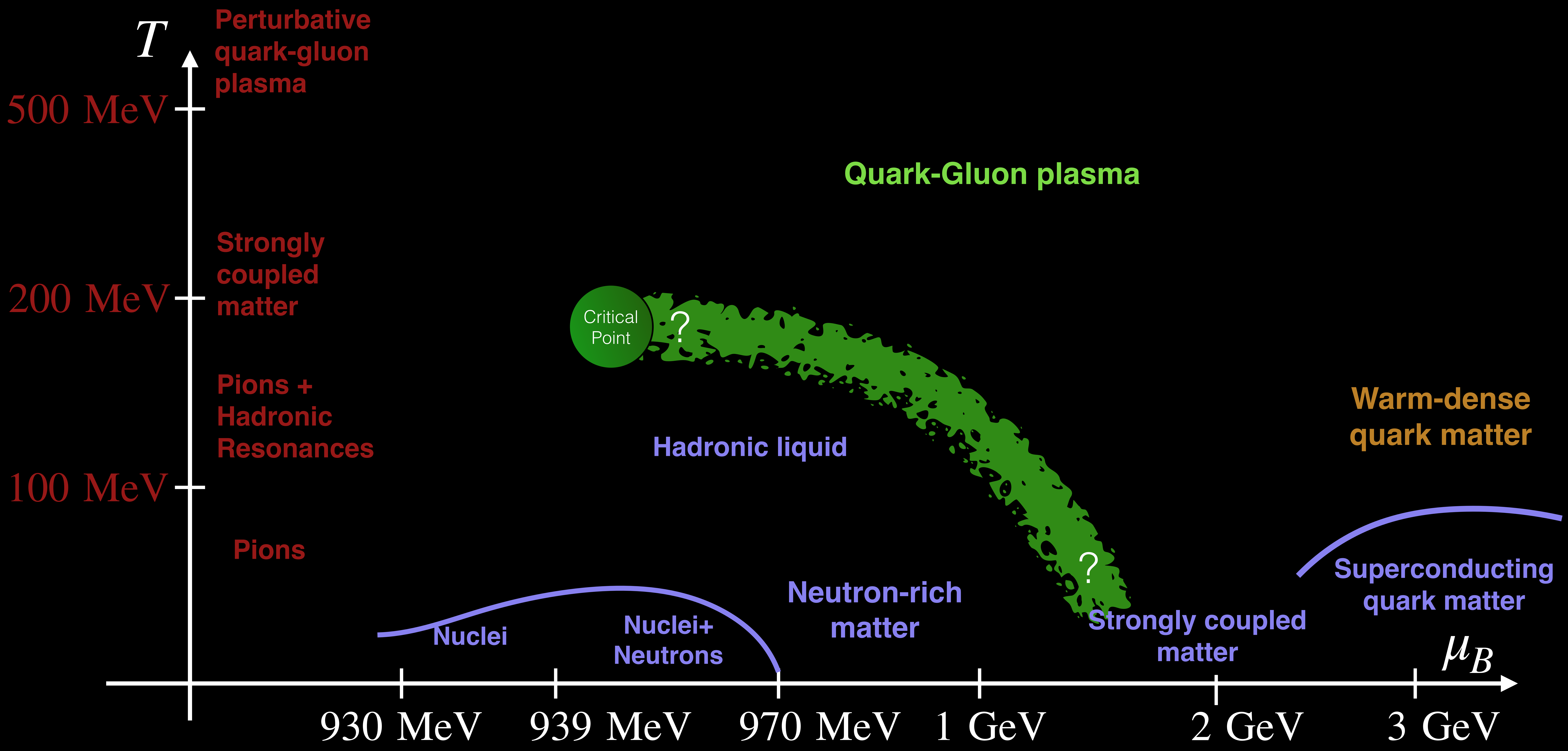
Nature of matter at extreme density?

Origin of cosmic explosions?

Synthesis of heavy elements?

Nature of dark matter?

Measuring and interpreting neutron star properties has far reaching implications.



T
Perturbative quark-gluon plasma
500 MeV
Quark-Gluon plasma
Strongly coupled matter
200 MeV
Critical Point
?
Pions + Hadronic Resonances
Hadronic liquid
Warm-dense quark matter
100 MeV
Pions
?
Superconducting quark matter
Neutron-rich matter
Nuclei+ Neutrons
Nuclei
Strongly coupled matter
μ_B
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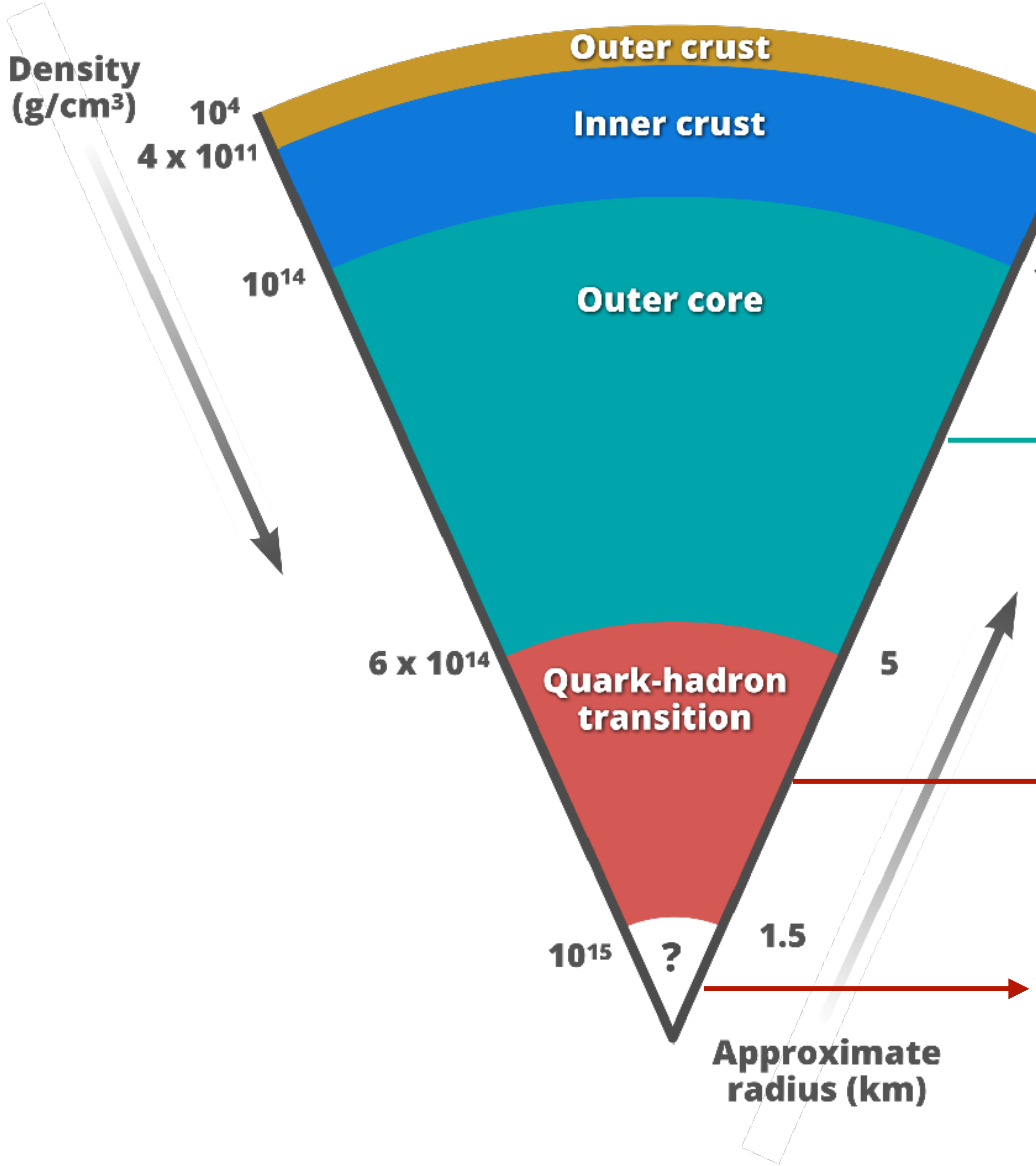
Inside Neutron Stars



Nuclei and relativistic electrons.

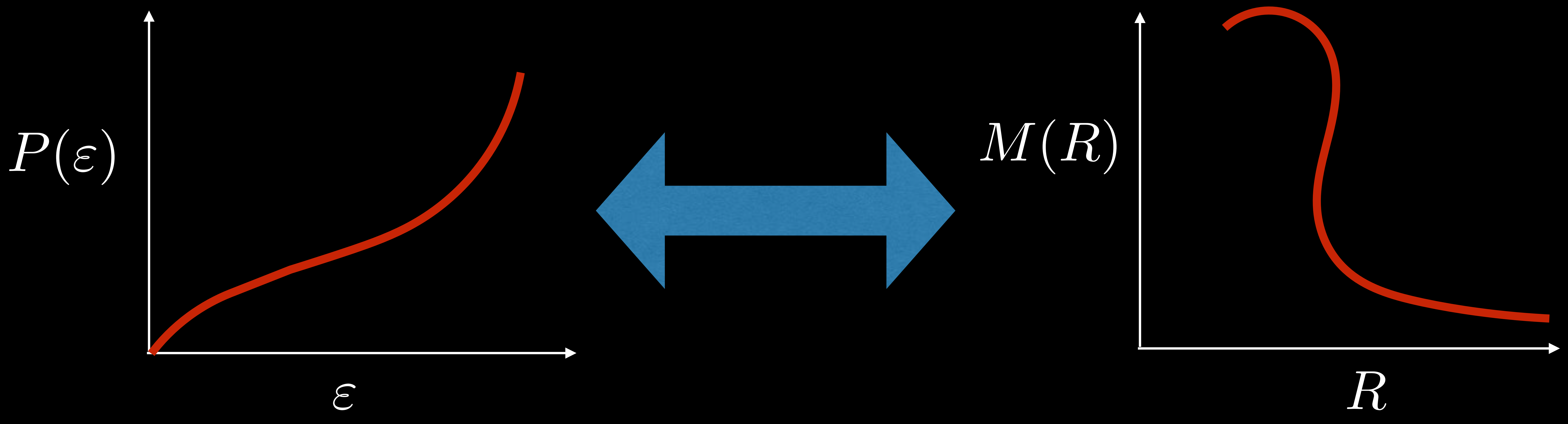
Neutron-rich nuclei, relativistic electrons, superfluid neutrons

Neutrons (~ 90%), protons, relativistic electrons, muons. Description in terms of baryons remains useful. Superfluid neutrons & superconducting protons.

Complex strongly interacting relativistic matter. Description not simple in terms of either baryons or quarks. Quarkyonic Matter?

Ordered quark matter? color superconductor?

Part I: Equation of State and Neutron Star Structure



$$P(\varepsilon) + \mathrm{Gen.Rel.} = M(R)$$

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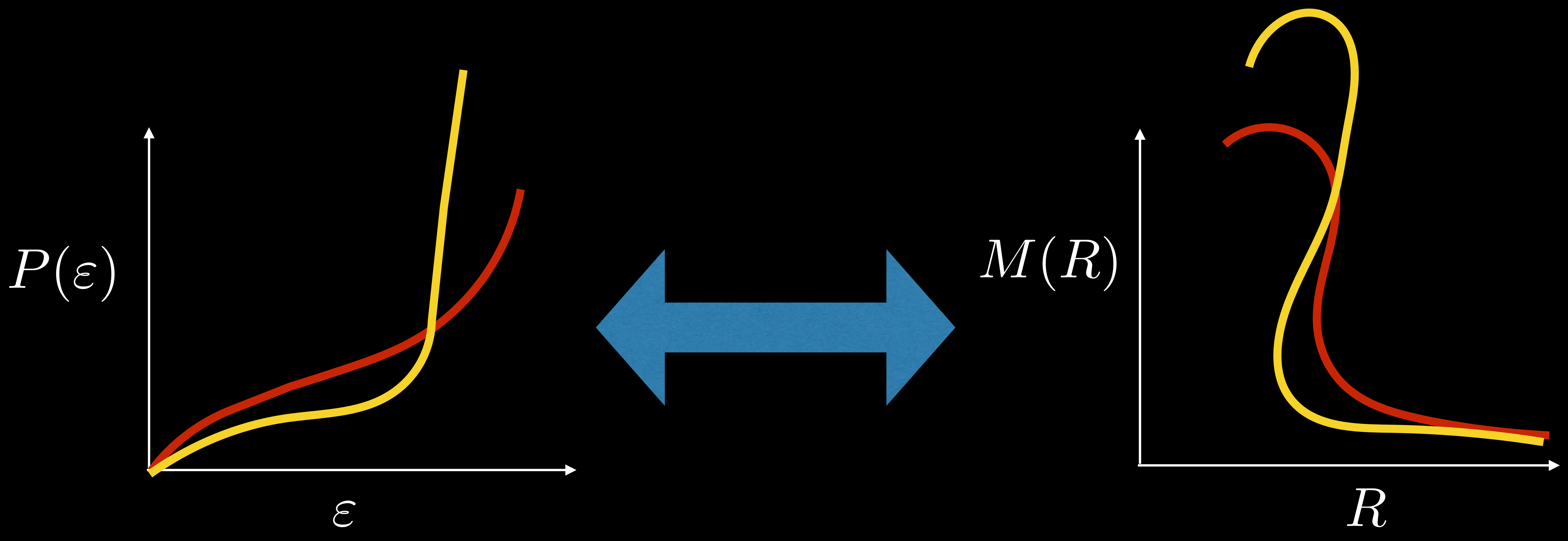

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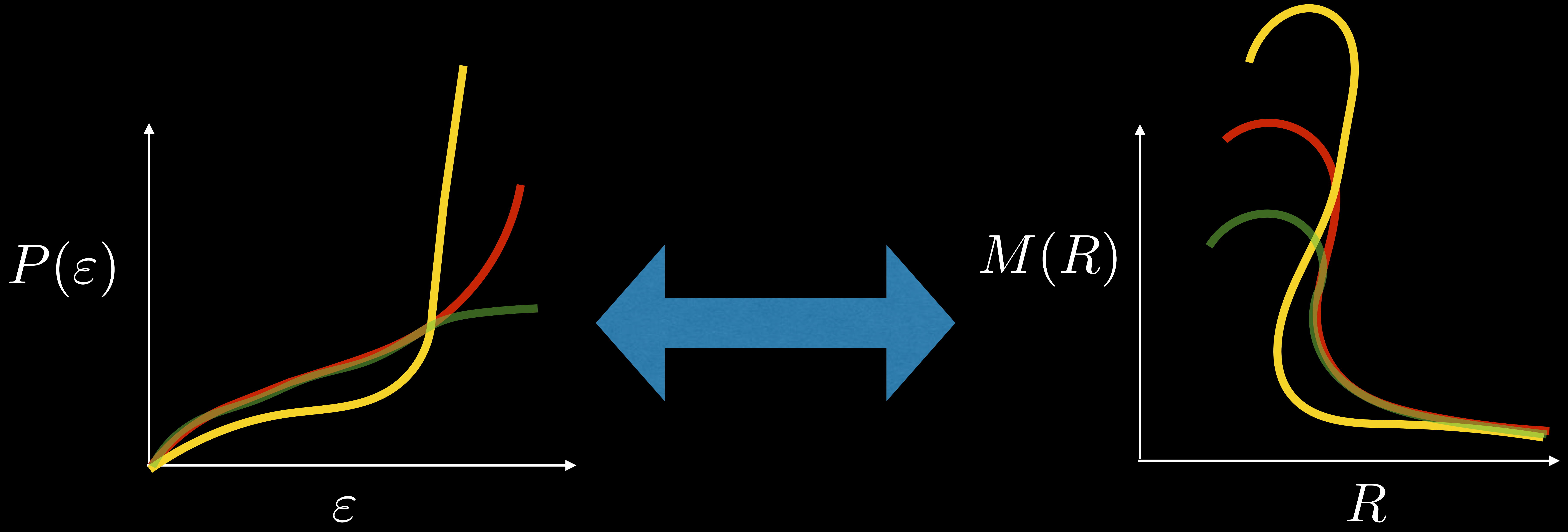

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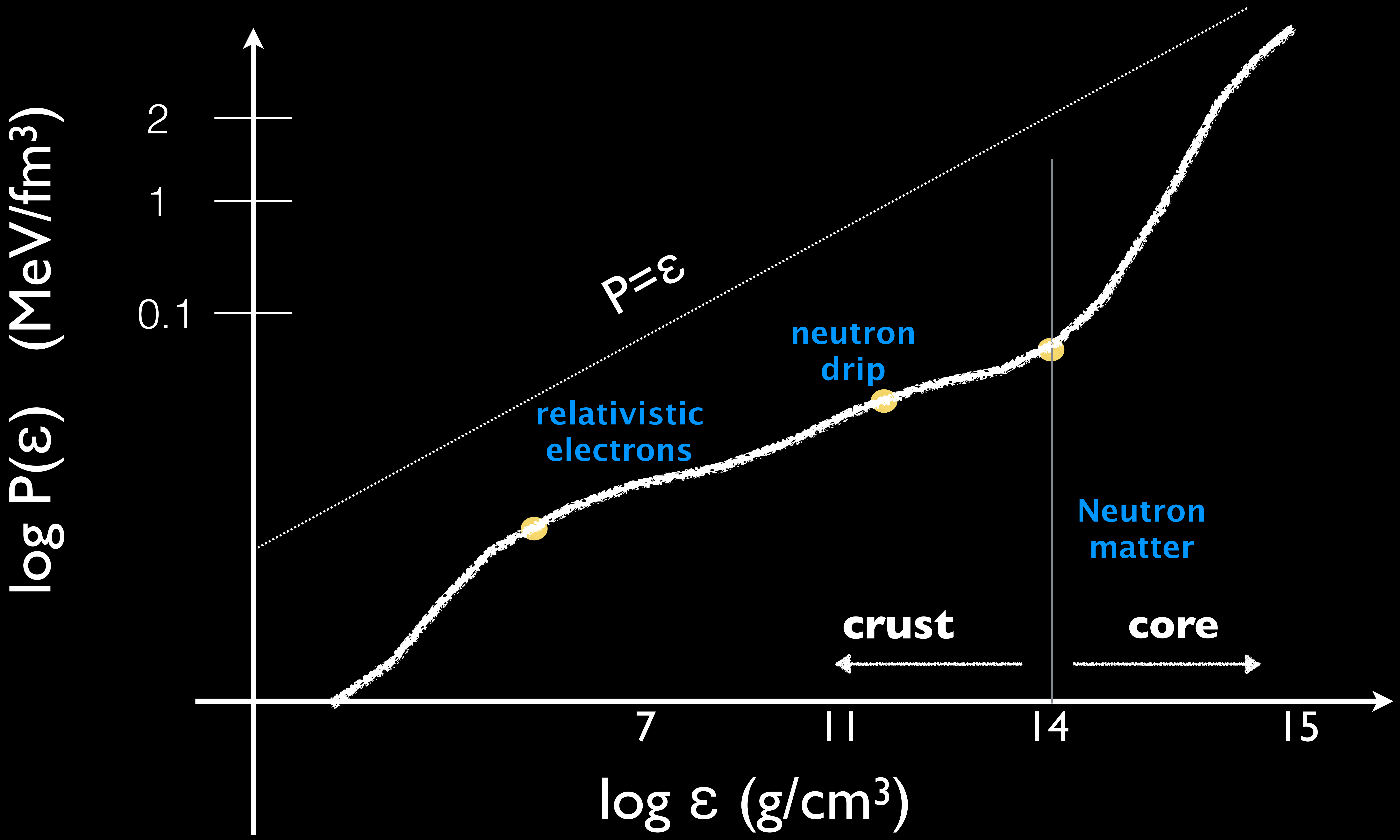
Pressure v/s Energy Density (EoS)



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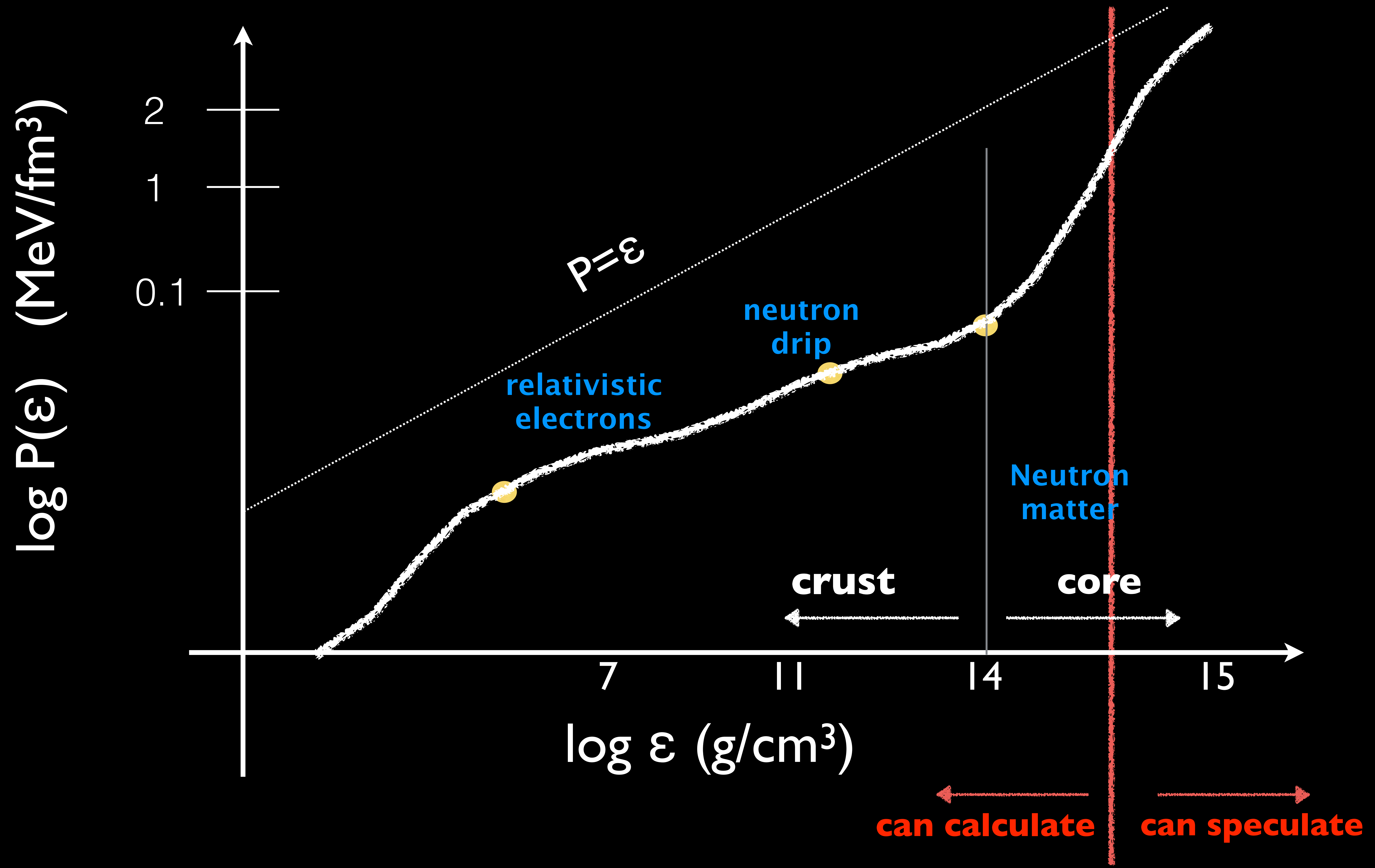

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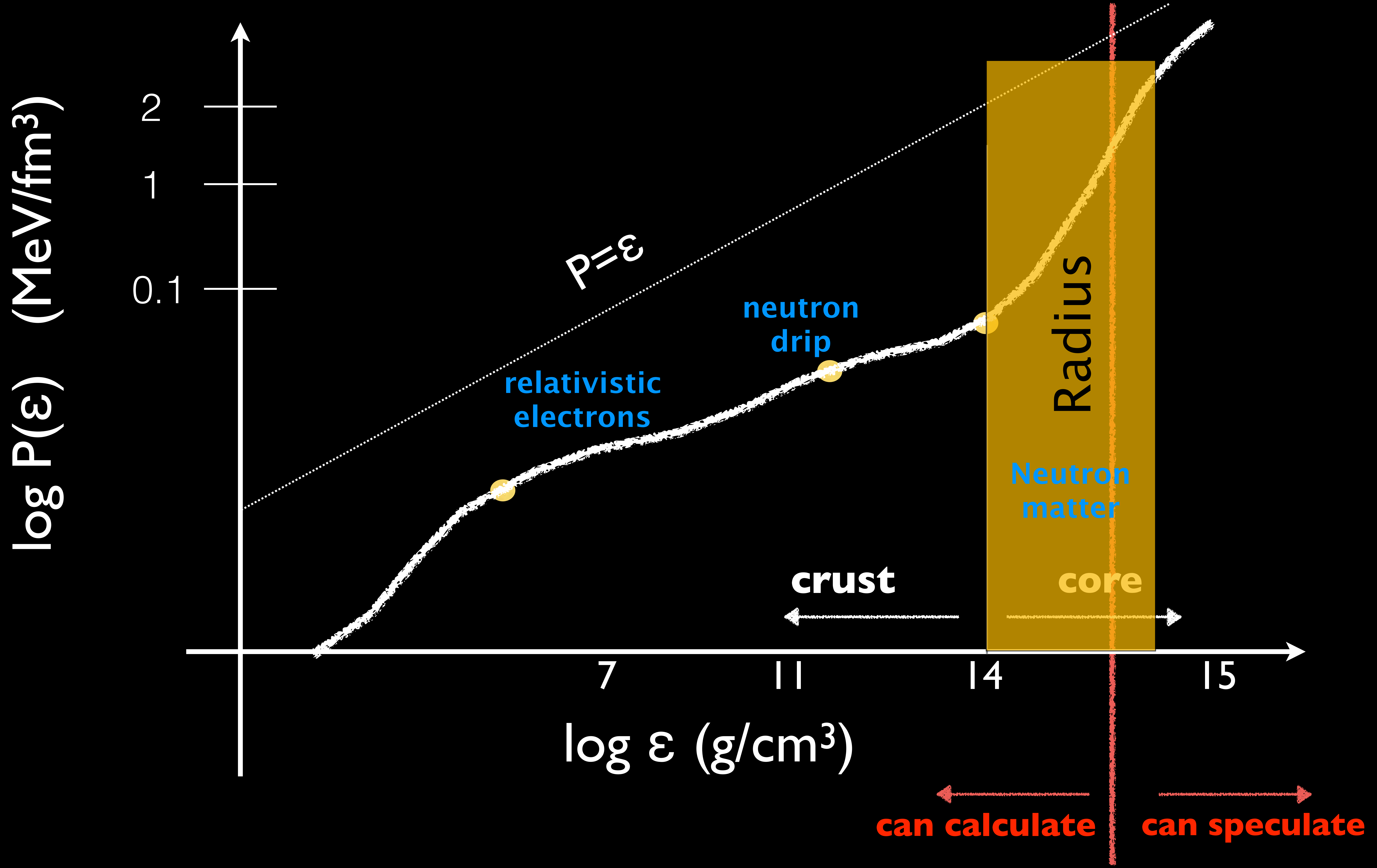

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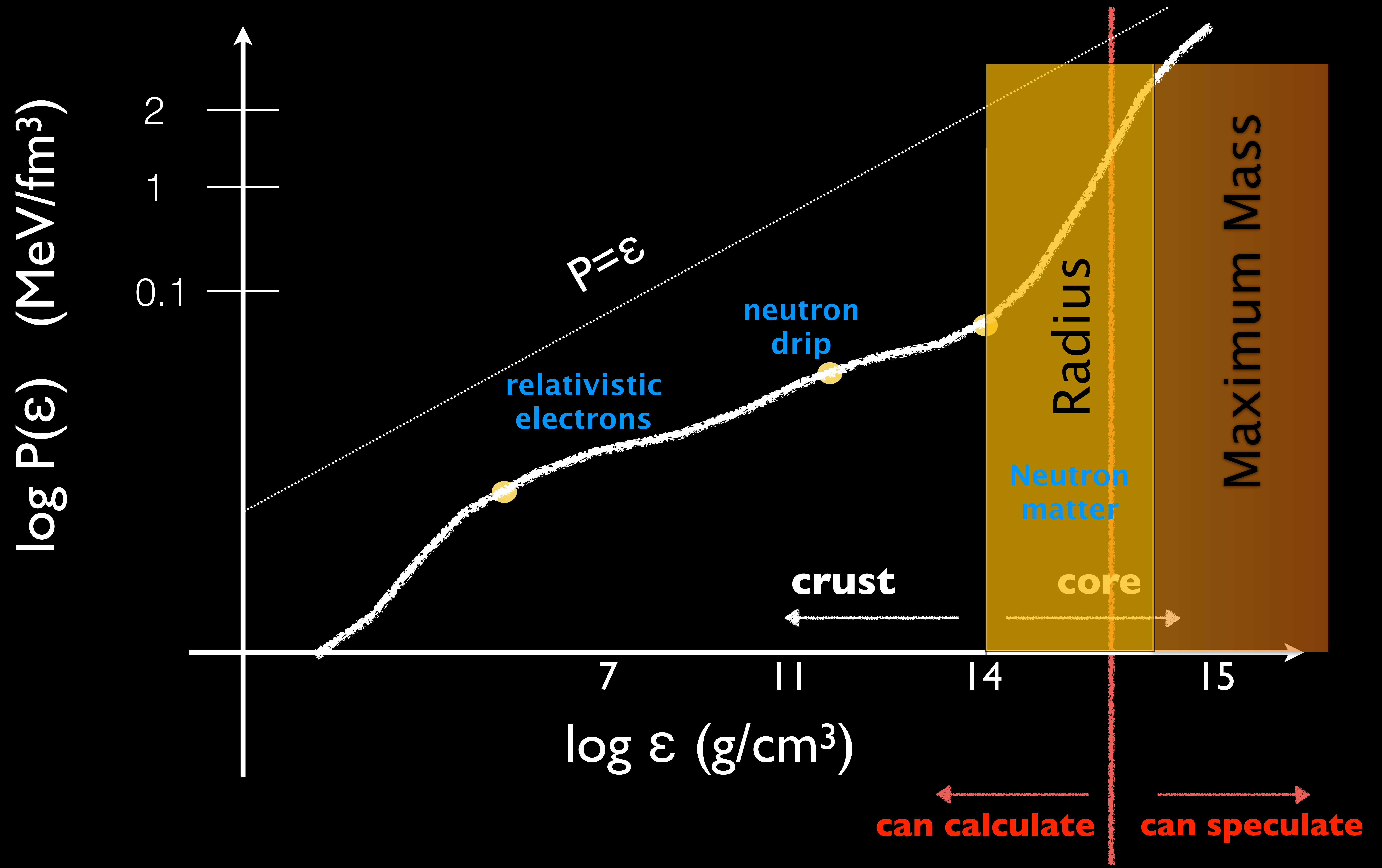
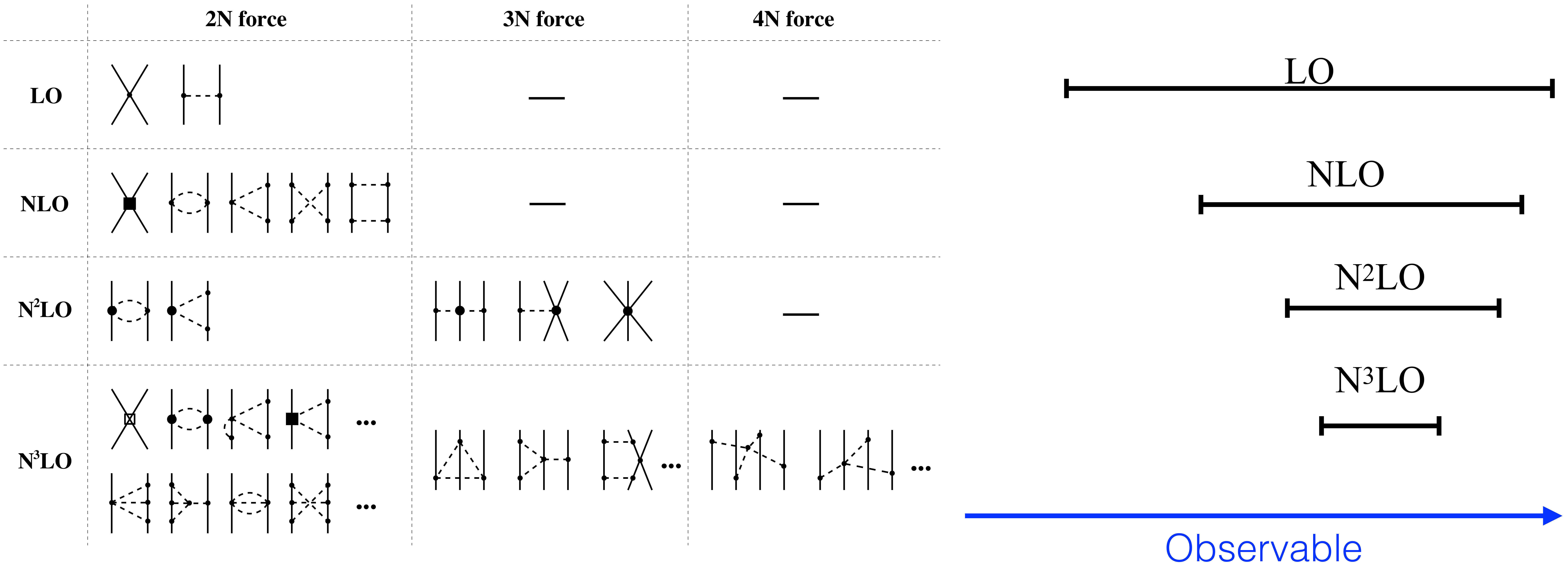


Nuclear Forces from Effective Field Theory (EFT)

EFT Hamiltonians organizes operators in powers of the momentum: $\dfrac{Q}{\Lambda_B}$



Allows for error estimation*. Provides guidance for the structure of three and many-body forces.

Beane, Bedaque, Epelbaum, Kaplan, Machliedt, Meisner, Phillips, Savage, **van Kolck**, **Weinberg,** Wise ..

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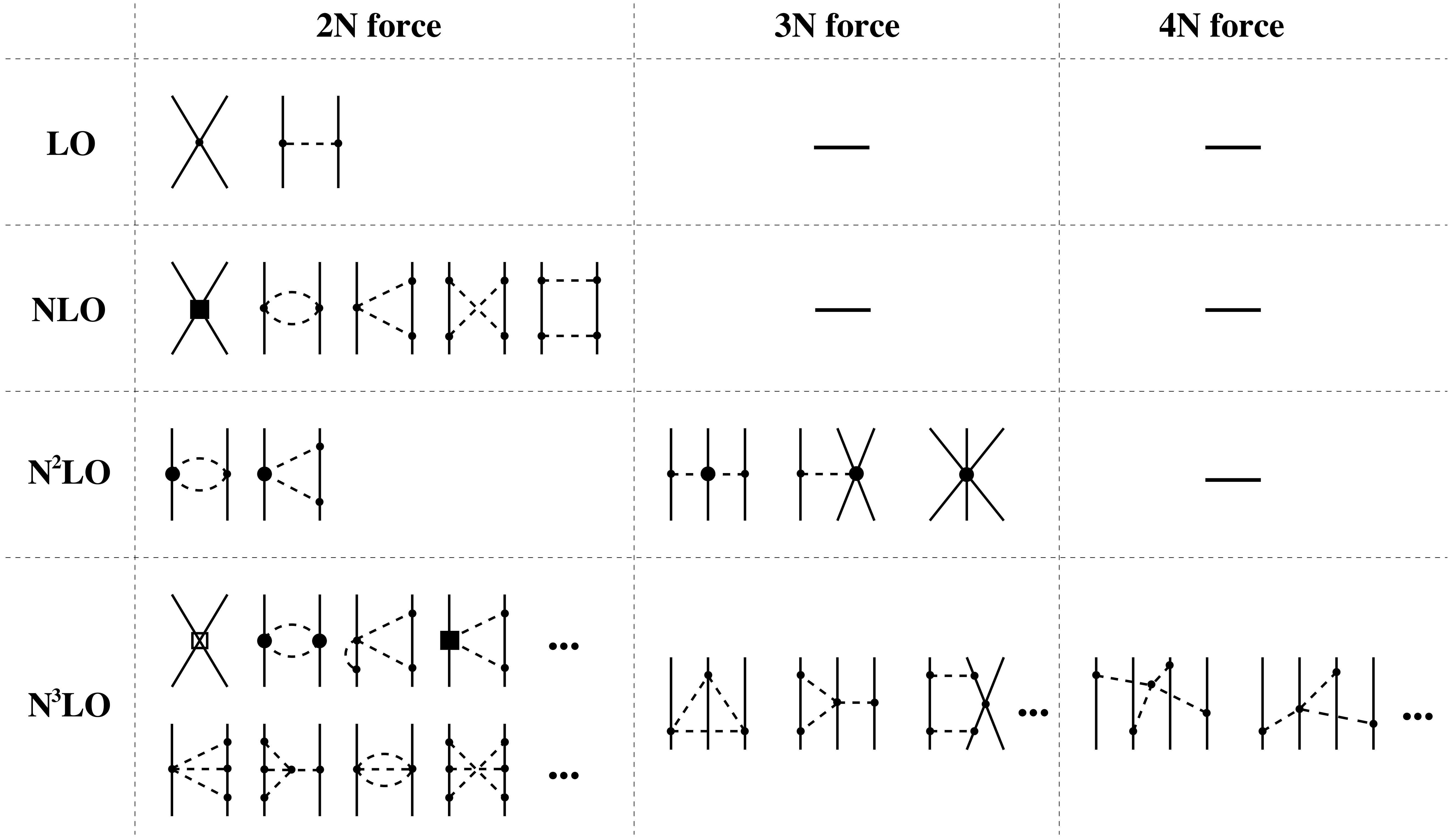

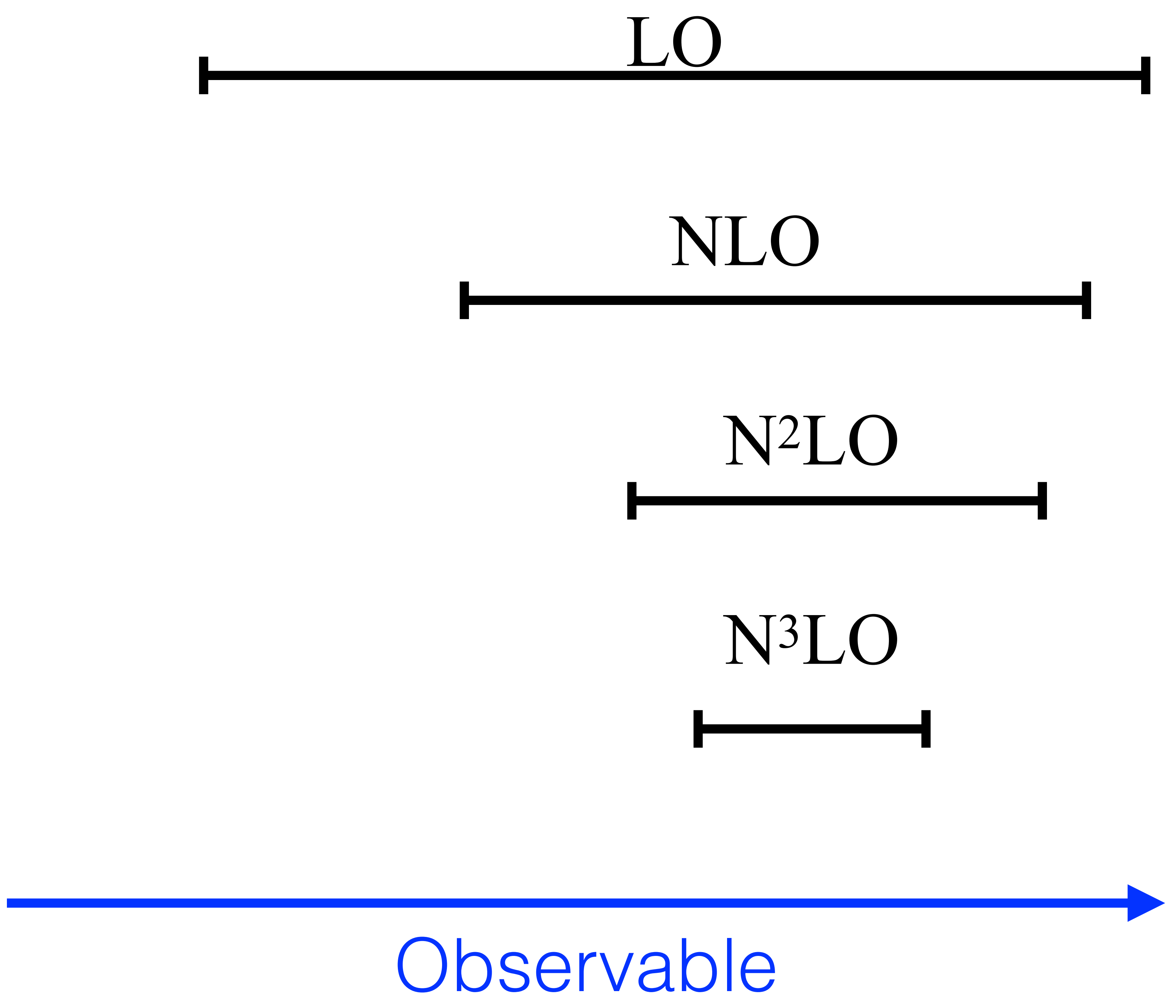

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Equation of State of Dense Nuclear Matter

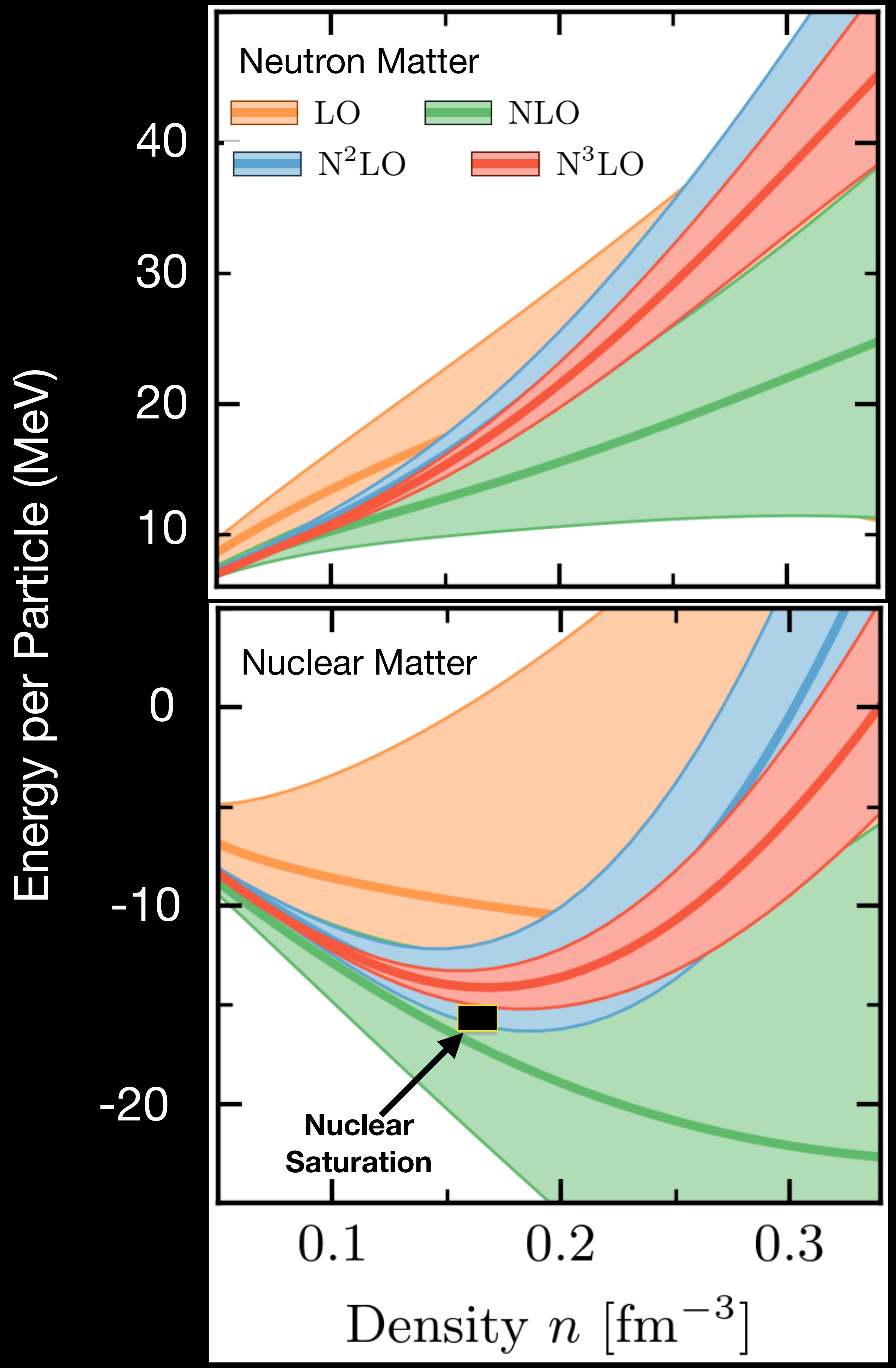
Quantum many-body calculations of neutron matter and nuclear matter using EFT-inspired potentials show convergence up to about twice nuclear saturation density.

Many-body perturbation theory, coupled-cluster, and Quantum Monte Carlo methods have been employed to calculate the energy of dense neutron matter.

Hebeler and Schwenk (2009), Gandolfi, Carlson, Reddy (2010), Gezerlis et al. (2013), Tews, Kruger, Hebeler, Schwenk (2013), Holt Kaiser, Weise (2013), Hagen et al. (2013), Roggero, Mukherjee, Pederiva (2014), Wlazlowski, Holt, Moroz, Bulgac, Roche (2014), Tews et al. (2018), Drischler et al., (2020).

Drischler et al. used Bayesian methods to systematically estimate the EFT truncation errors in neutron and nuclear matter.

Drischler, Furnstahl, Melendez, Phillips, (2020).



Drischler et al., (2020).

Equation of State of Neutron Star Matter

Many-body perturbation theory and Bayesian analysis of the EFT truncation errors predict:

$$P_{\mathrm{NSM}}(n_B = 0.16\ \mathrm{fm}^{-3}) = 3.0 \pm 0.2\ \mathrm{MeV/fm}^3$$

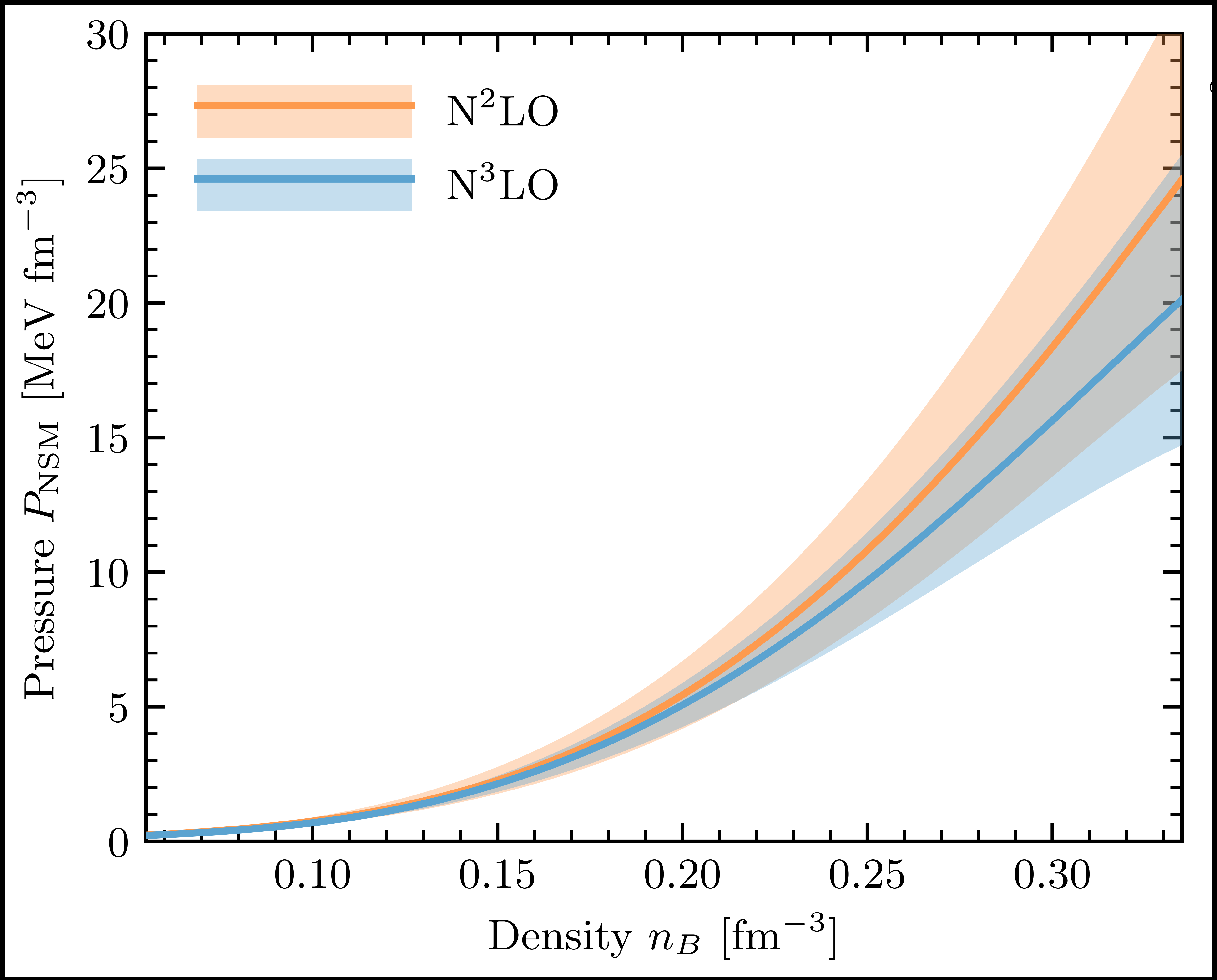
$$P_{\mathrm{NSM}}(n_B = 0.34\ \mathrm{fm}^{-3}) = 20.0 \pm 5\ \mathrm{MeV/fm}^3$$


Christian Drischler


Sophia Han


Tianqi Zhao

Important caveat: Errors due to cut-off and regularization schemes are not well understood. Perturbative methods fail for EFT potentials with large cut-off.



Drischler, Han, Lattimer, Prakash, Reddy, Zhao (2020)

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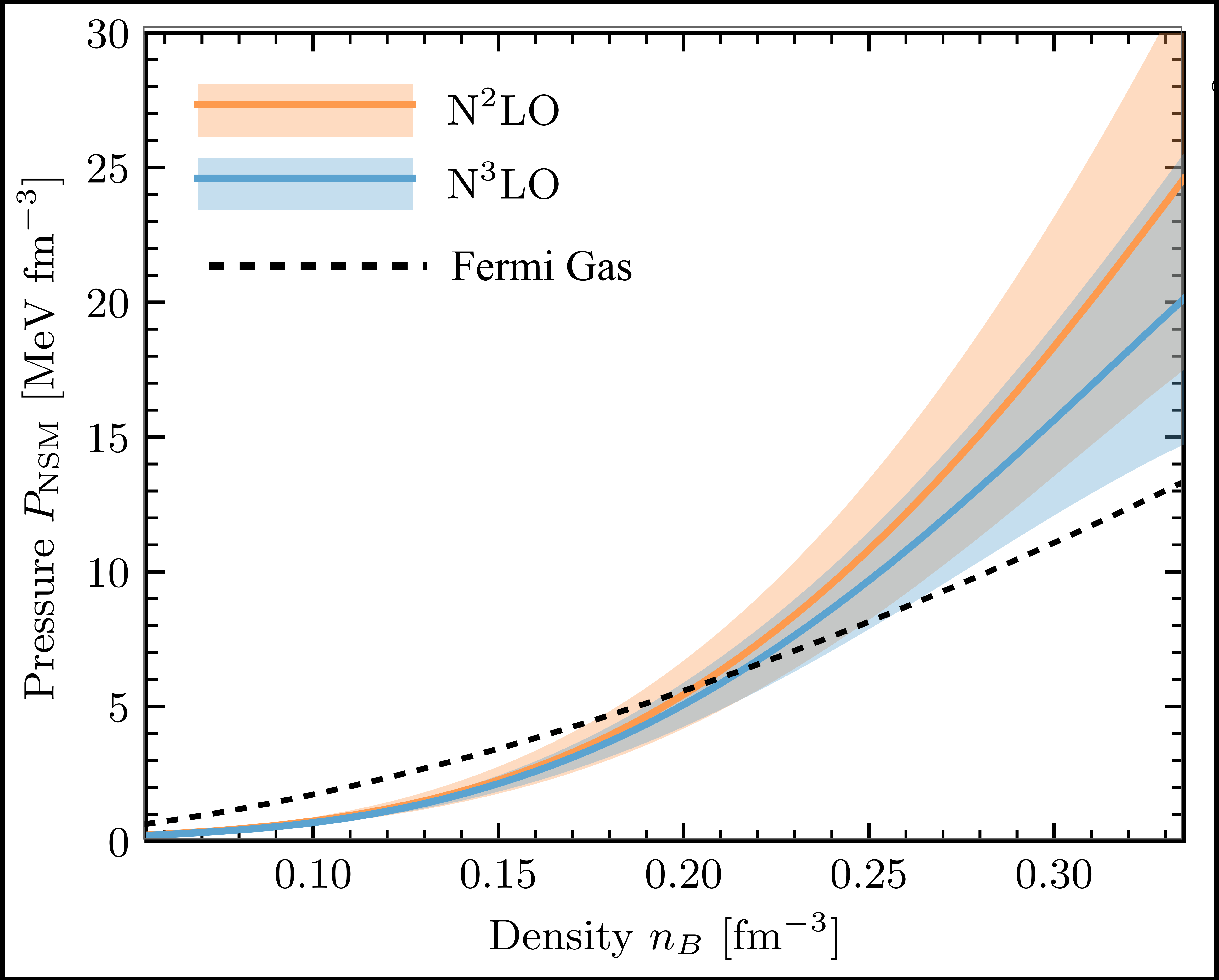

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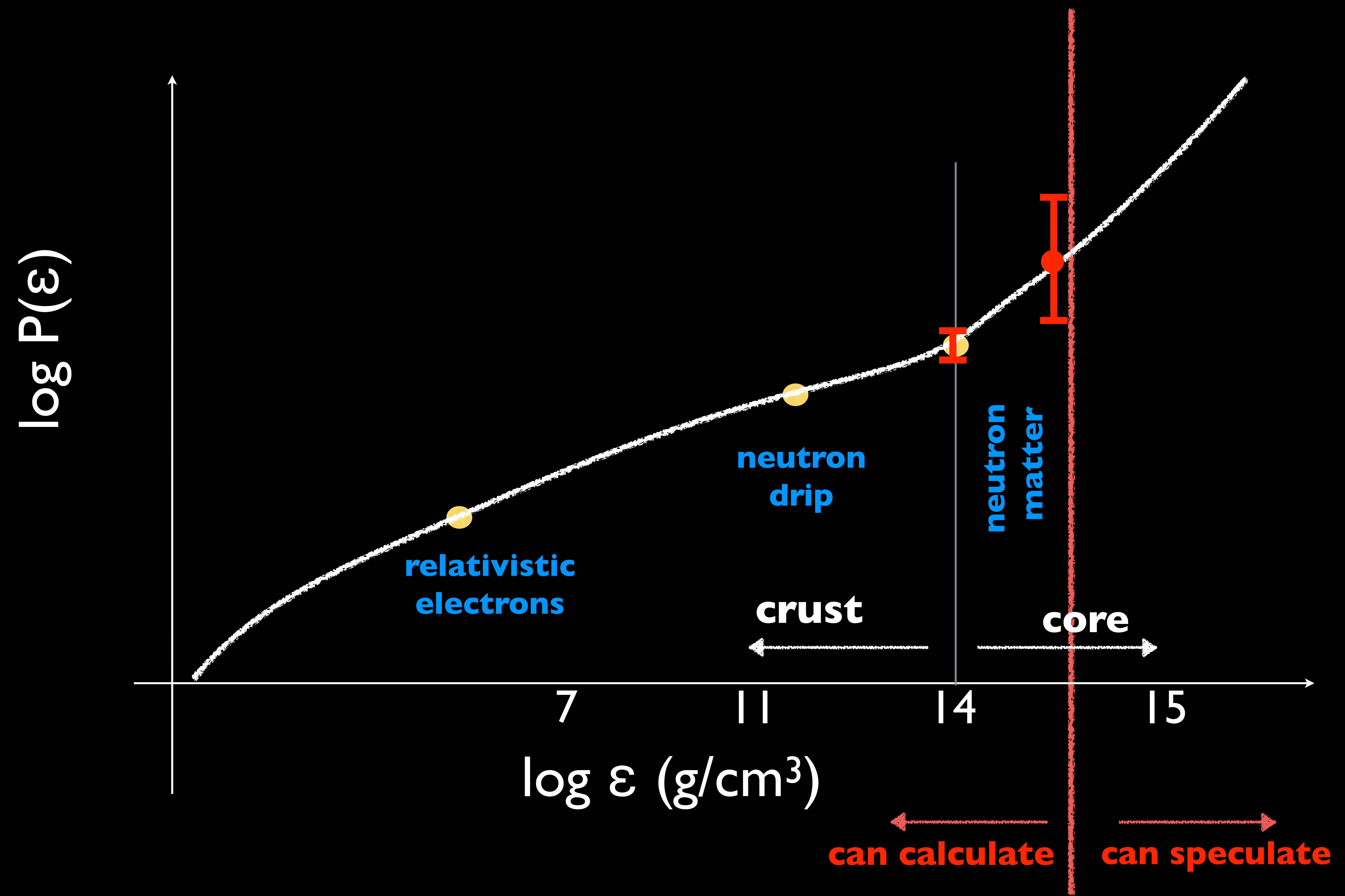
General Constraints on the Equation of State



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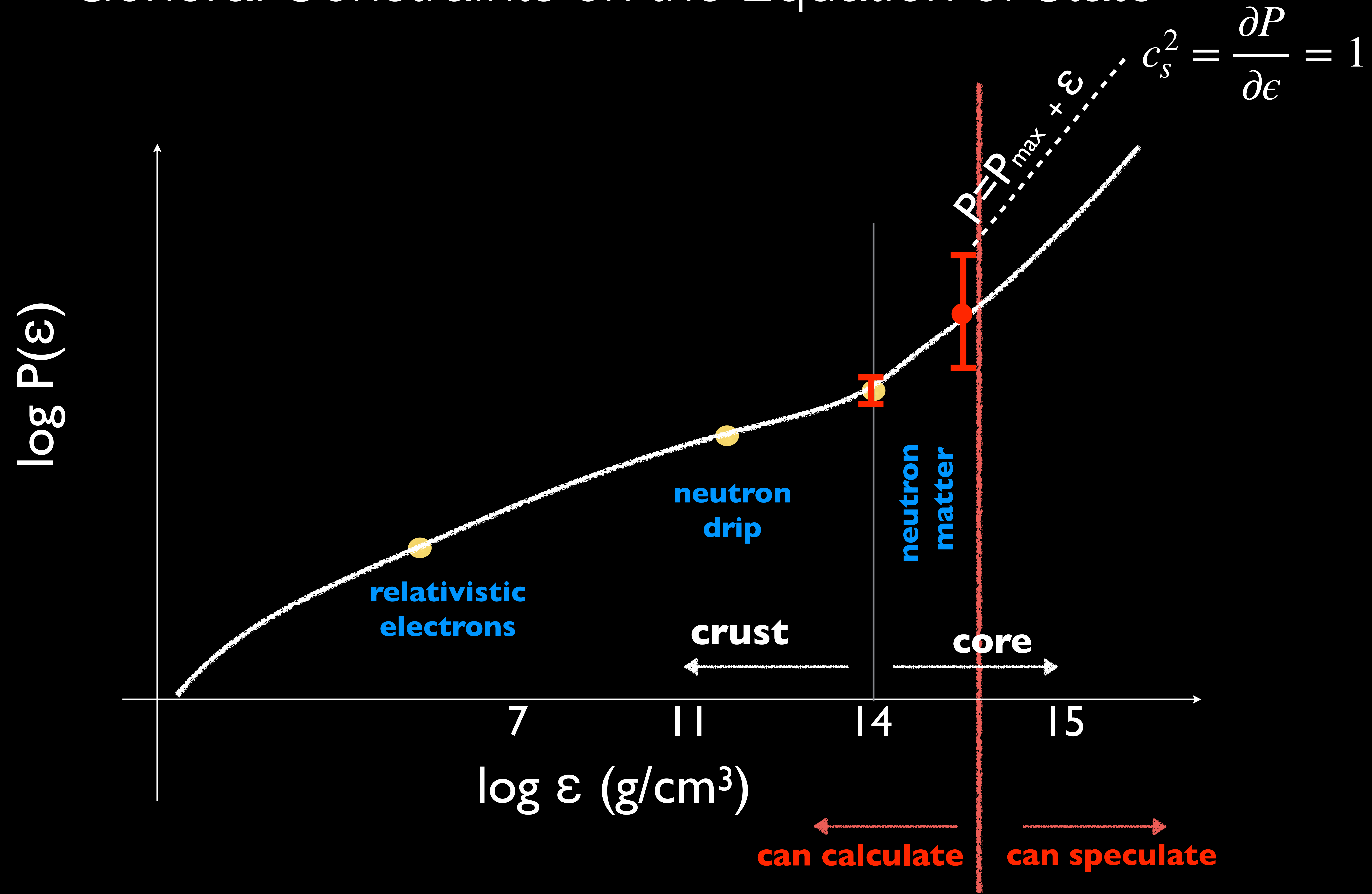

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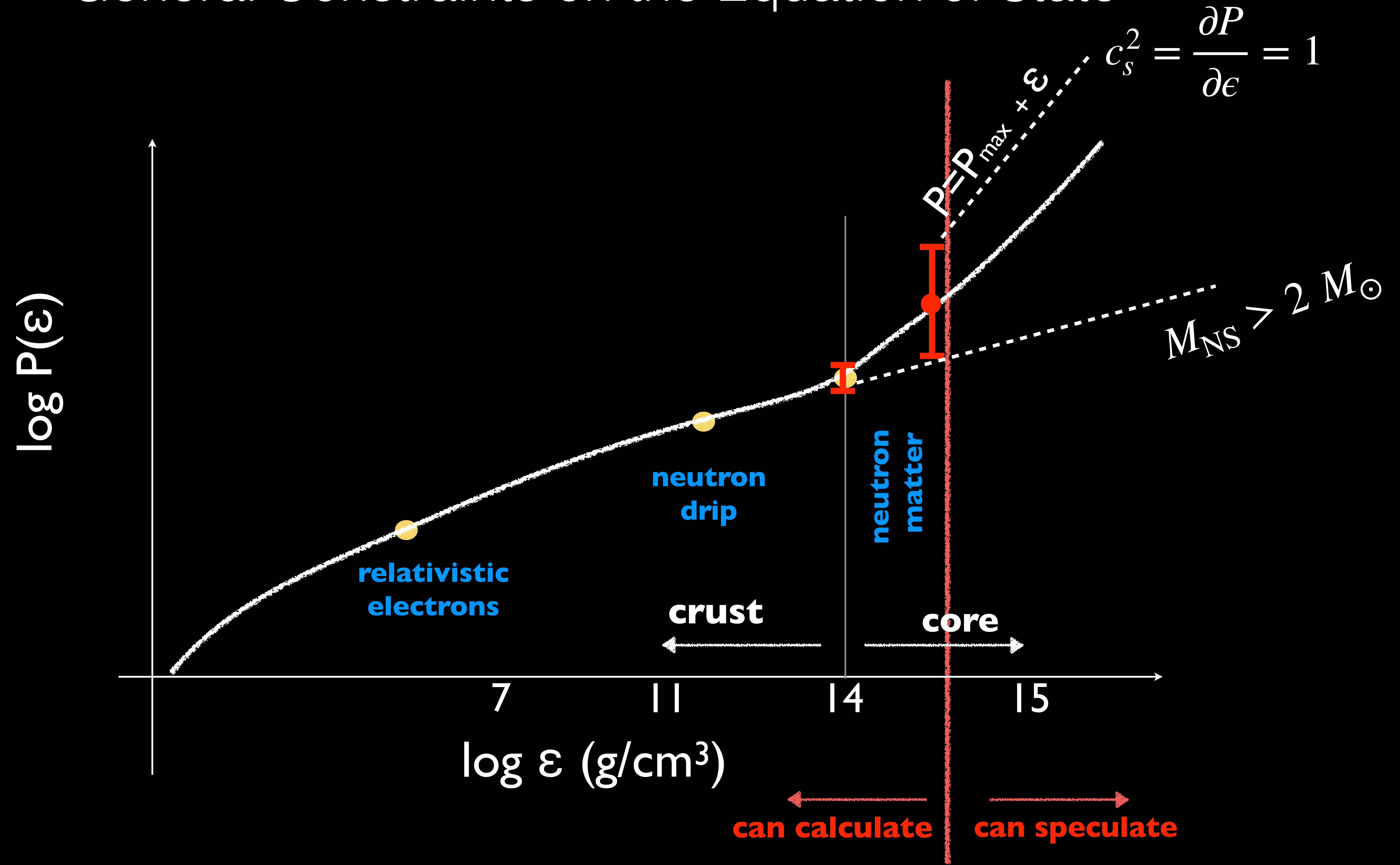


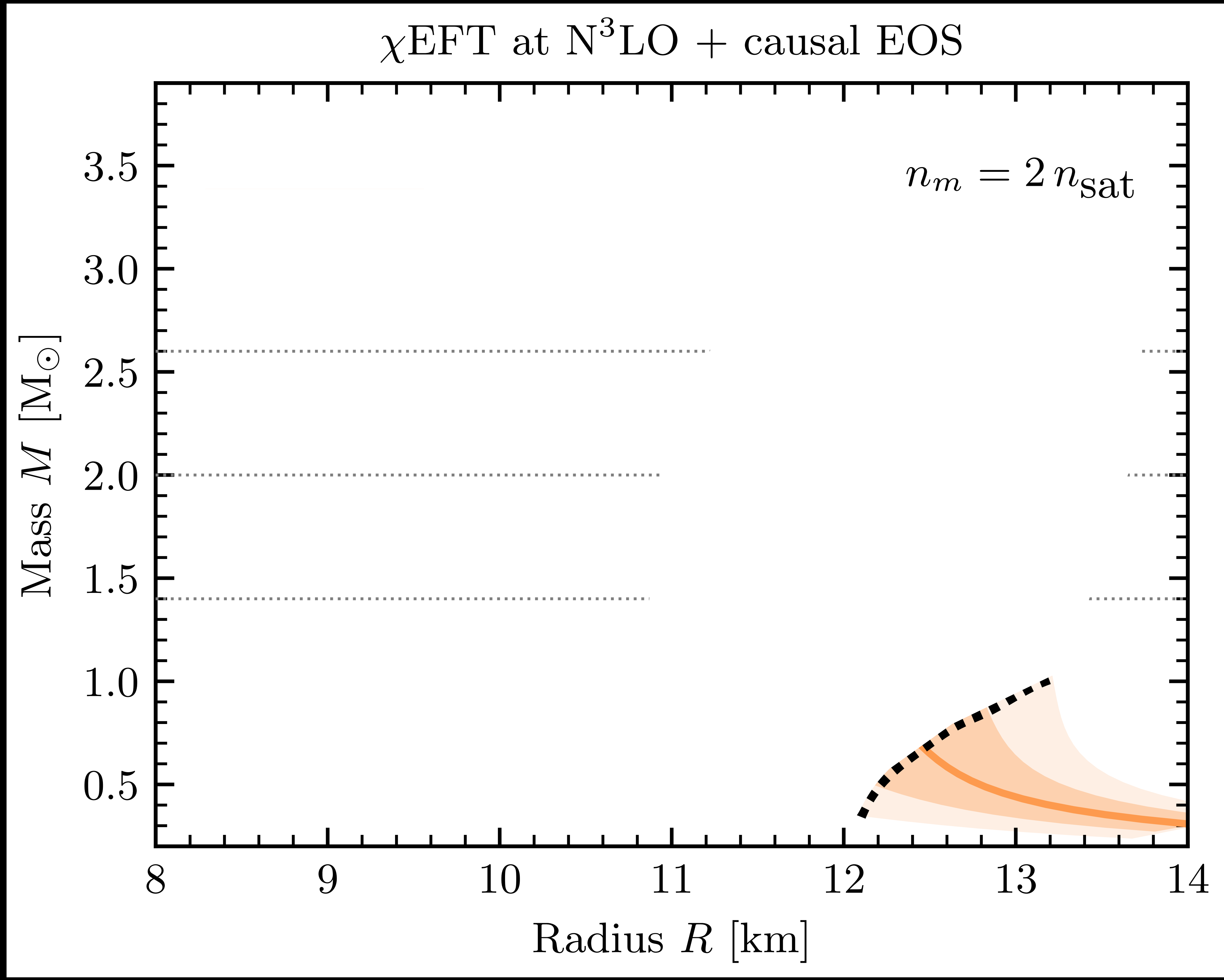
Bounds on Neutron Star Radii

EFT predictions for the EOS can be combined with extremal high-density EOS (with $c_s^2 = 1$) to derive robust bounds on the radius of a NS of any mass.

The lower limit on the NS maximum mass obtained from observations strengthen these bounds:

- $M_{\rm max} > 2.0\ M_\odot$, 9.2 km < $R_{1.4}$ < 13.2 km
- $M_{\rm max} > 2.6\ M_\odot$, 11.2 km < $R_{1.4}$ < 13.2 km

If $R_{1.4}$ is small (<11.5 km) or large (>12.5 km), it would imply a very large speed of sound in the cores of massive neutron stars.



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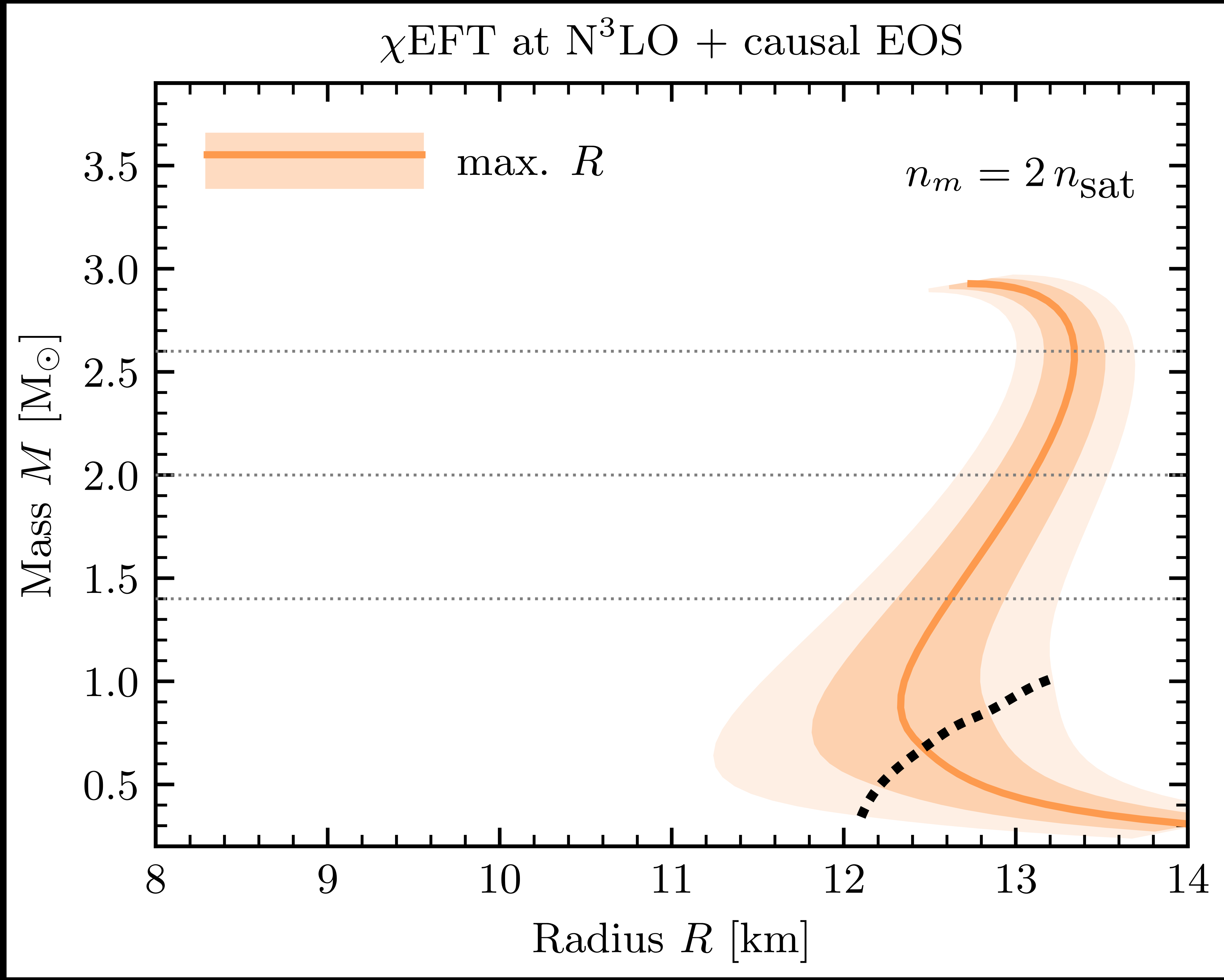

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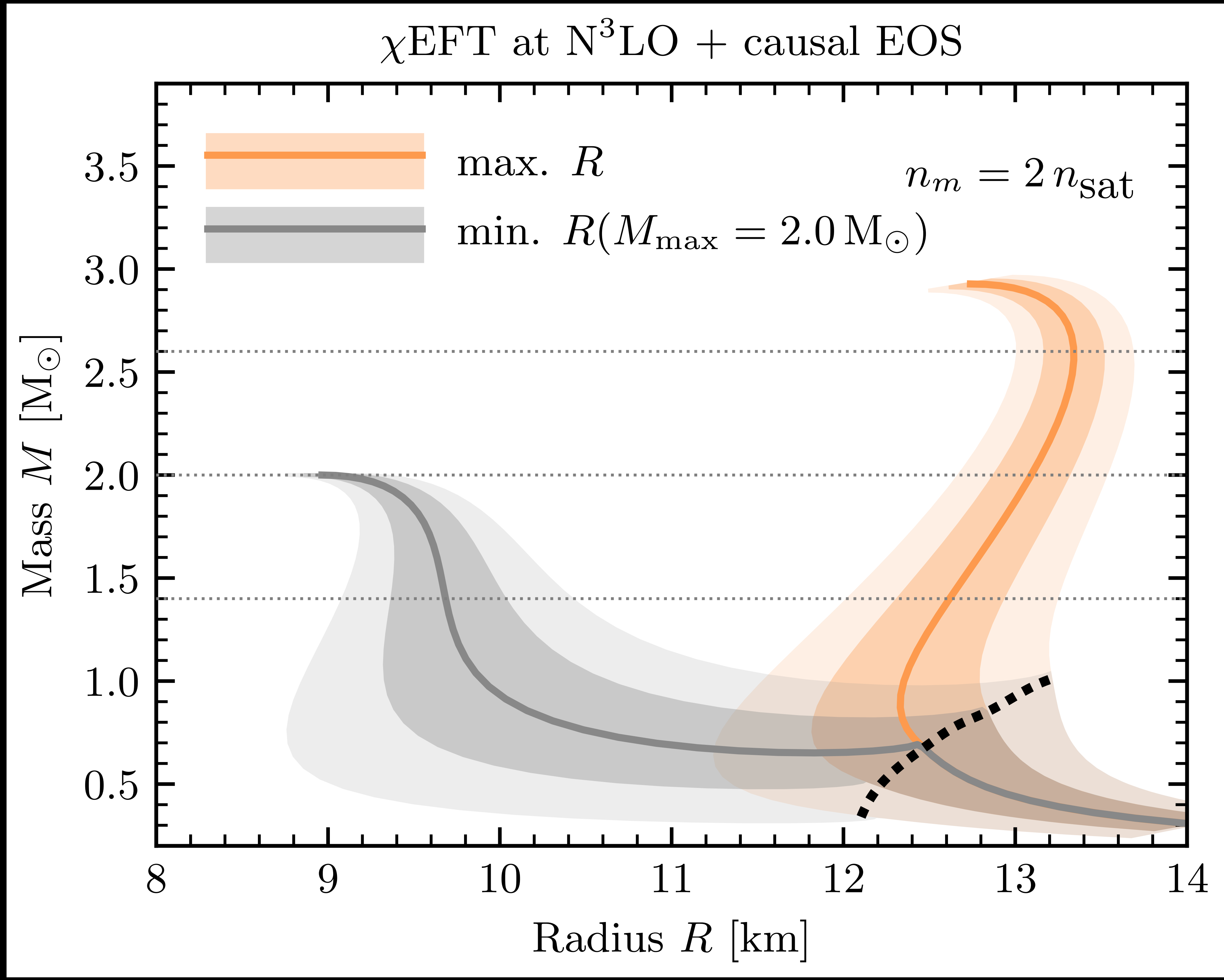

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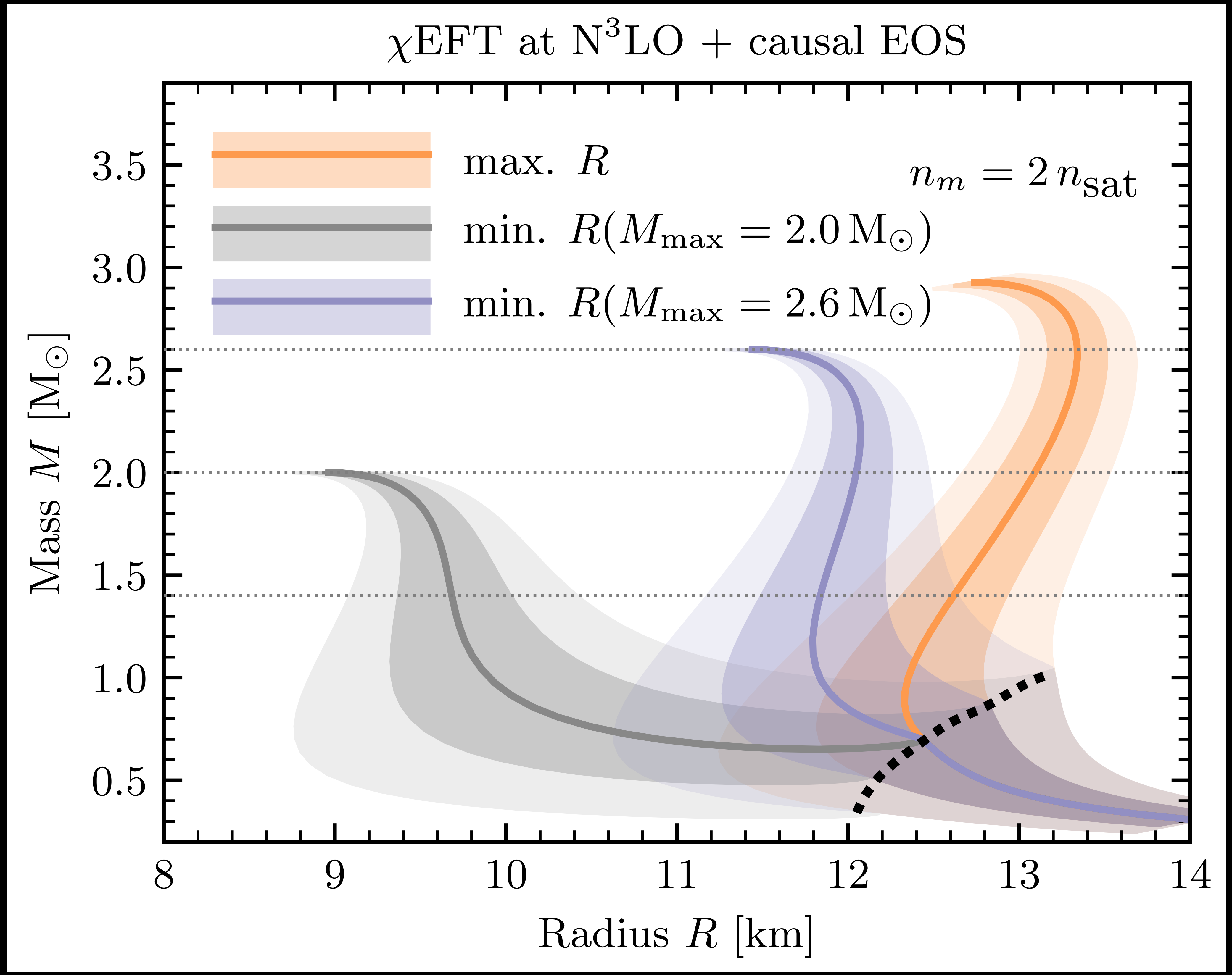

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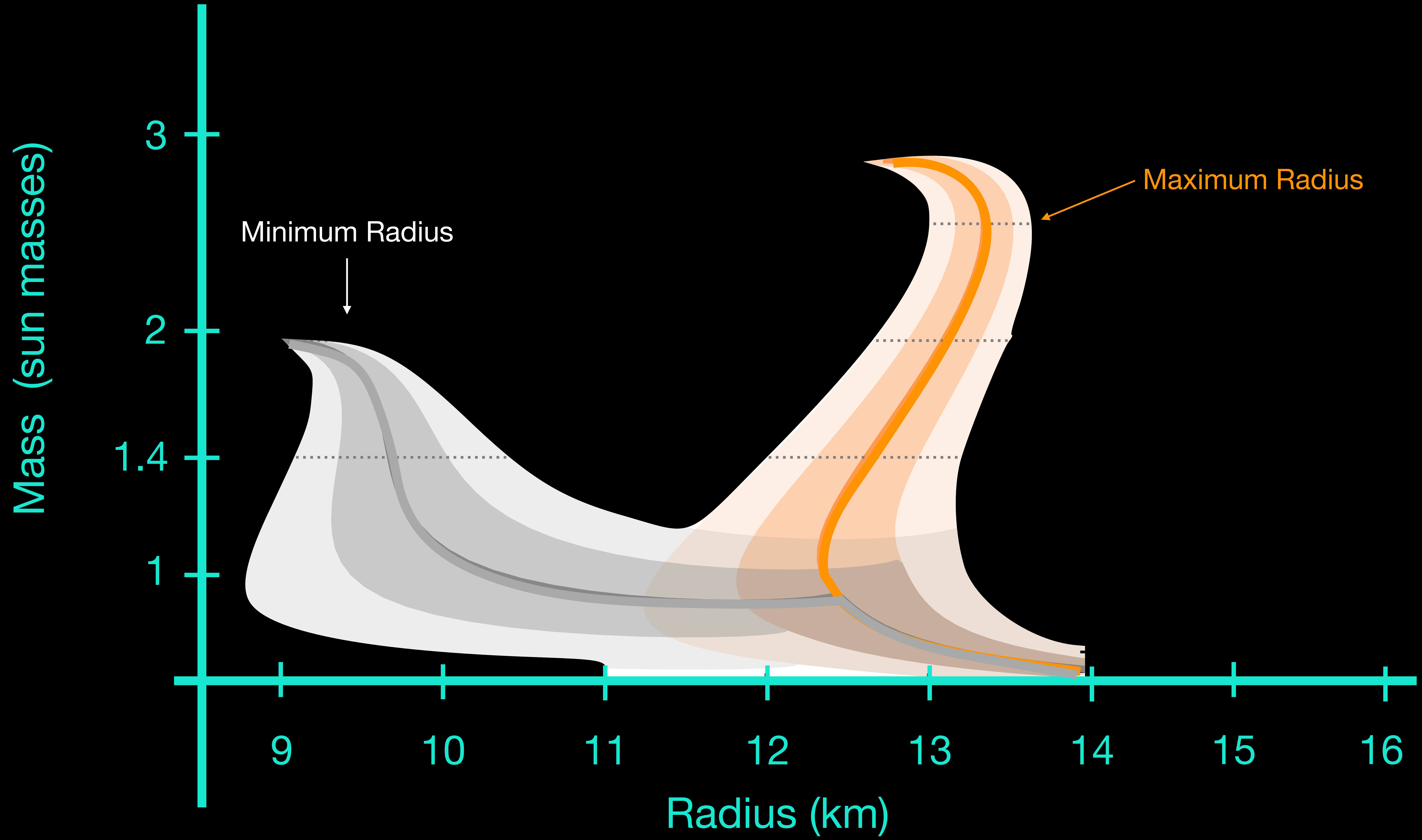

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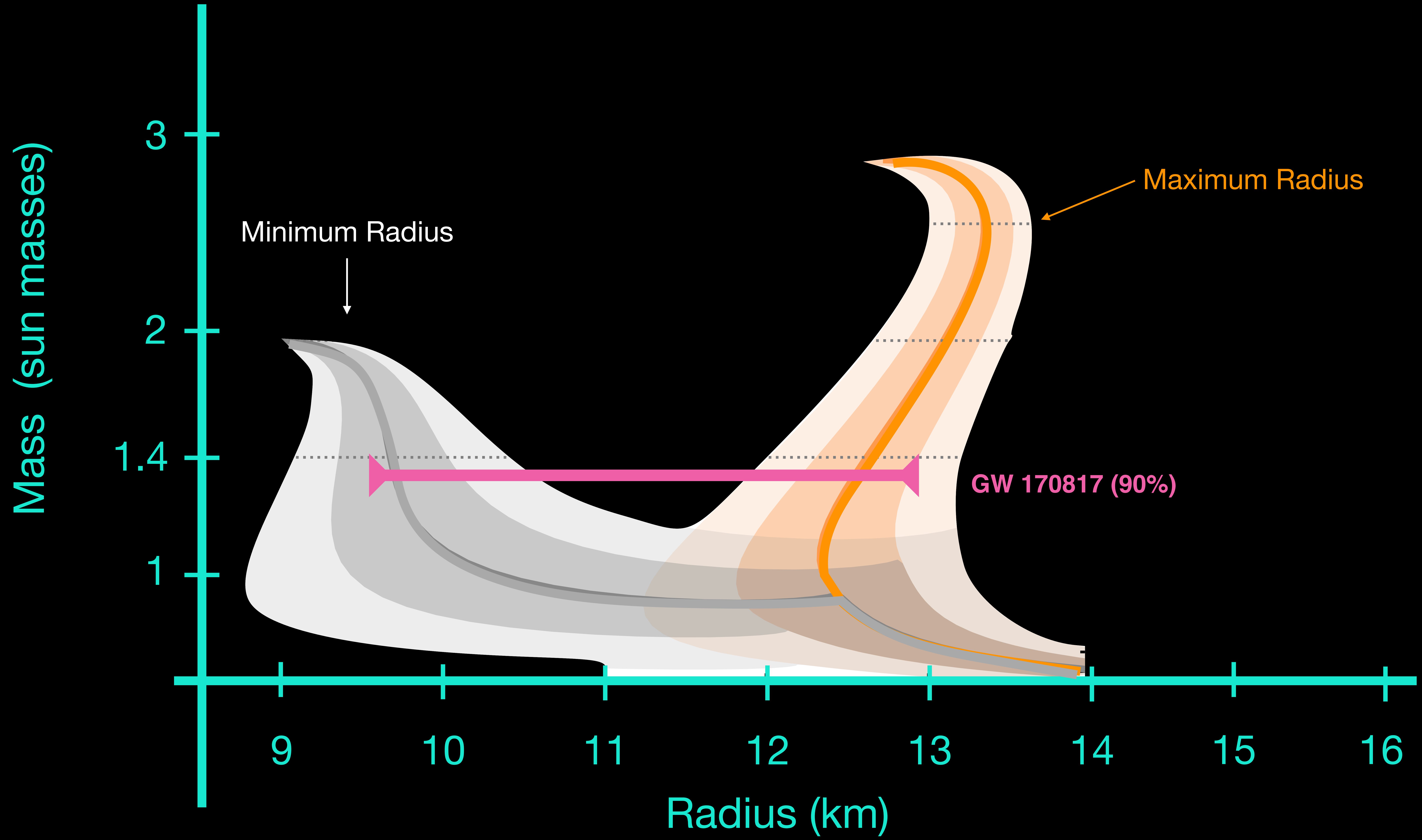

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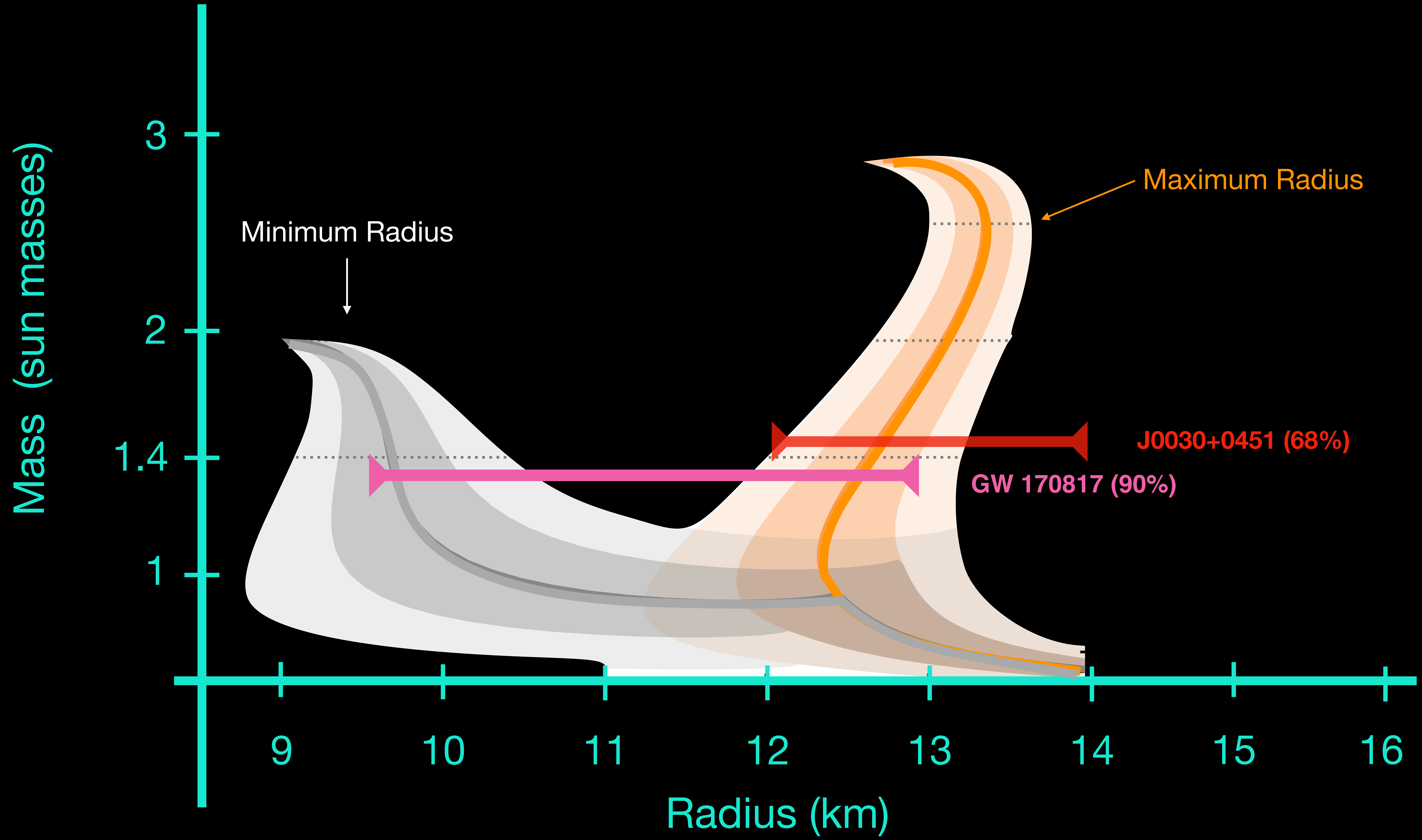

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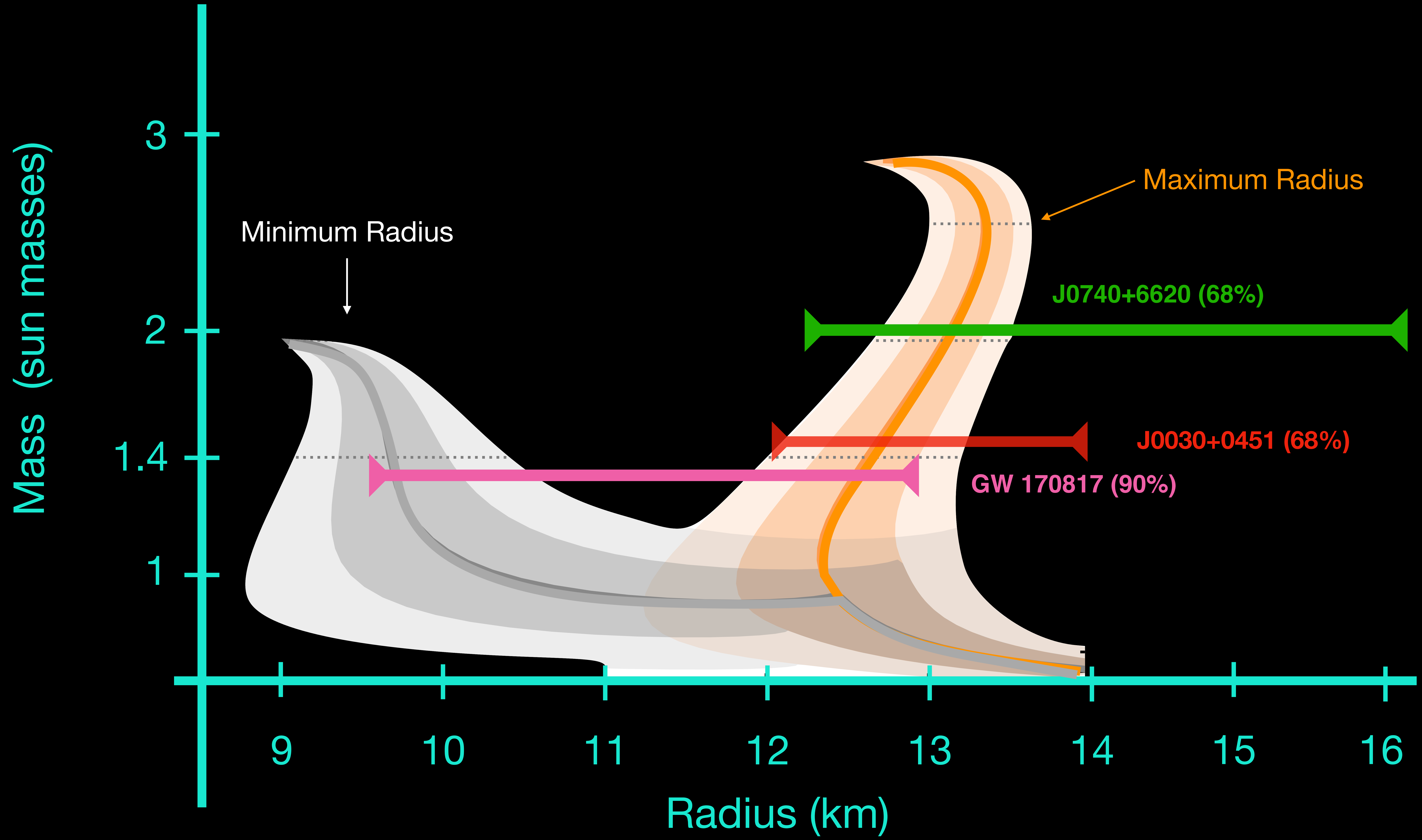

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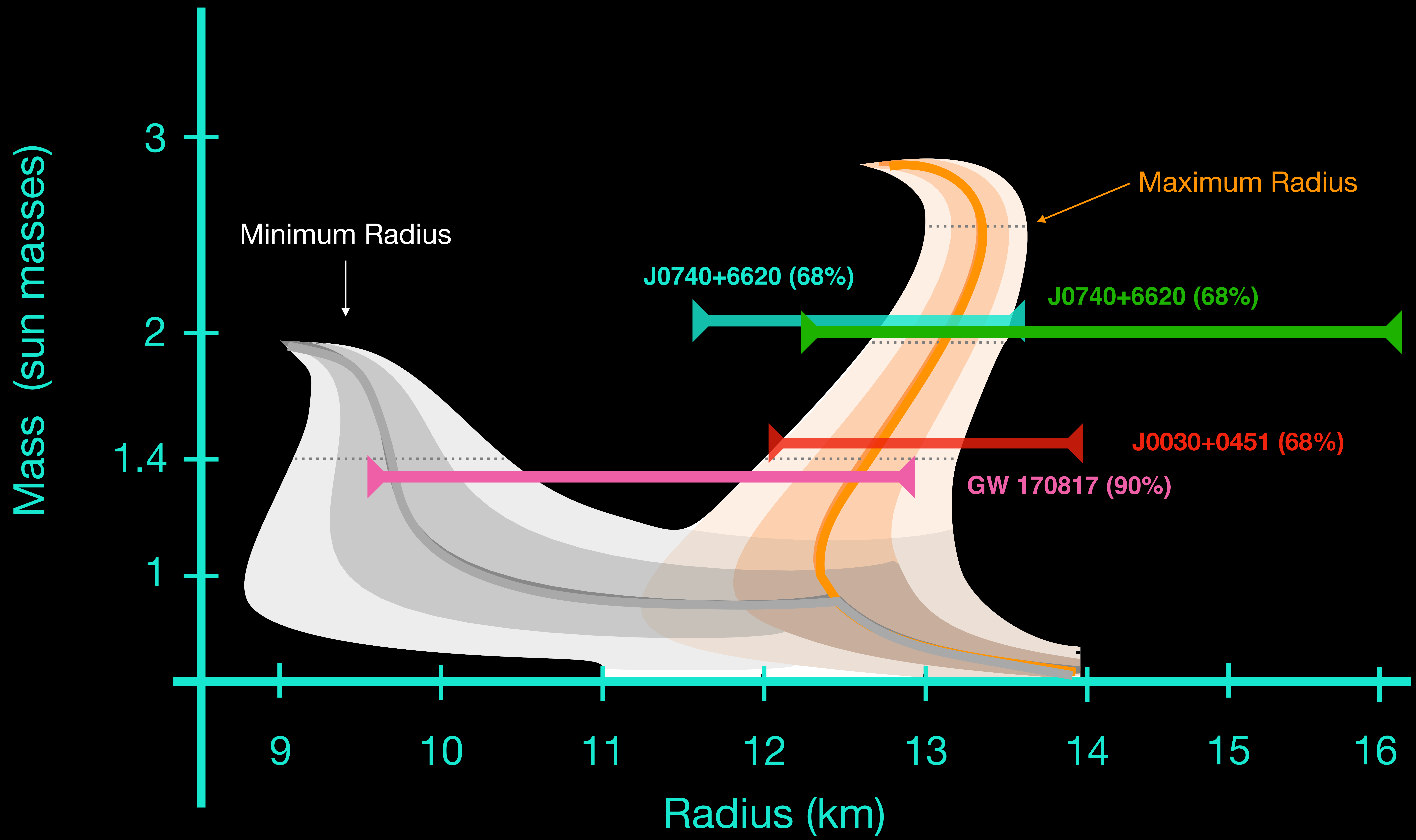

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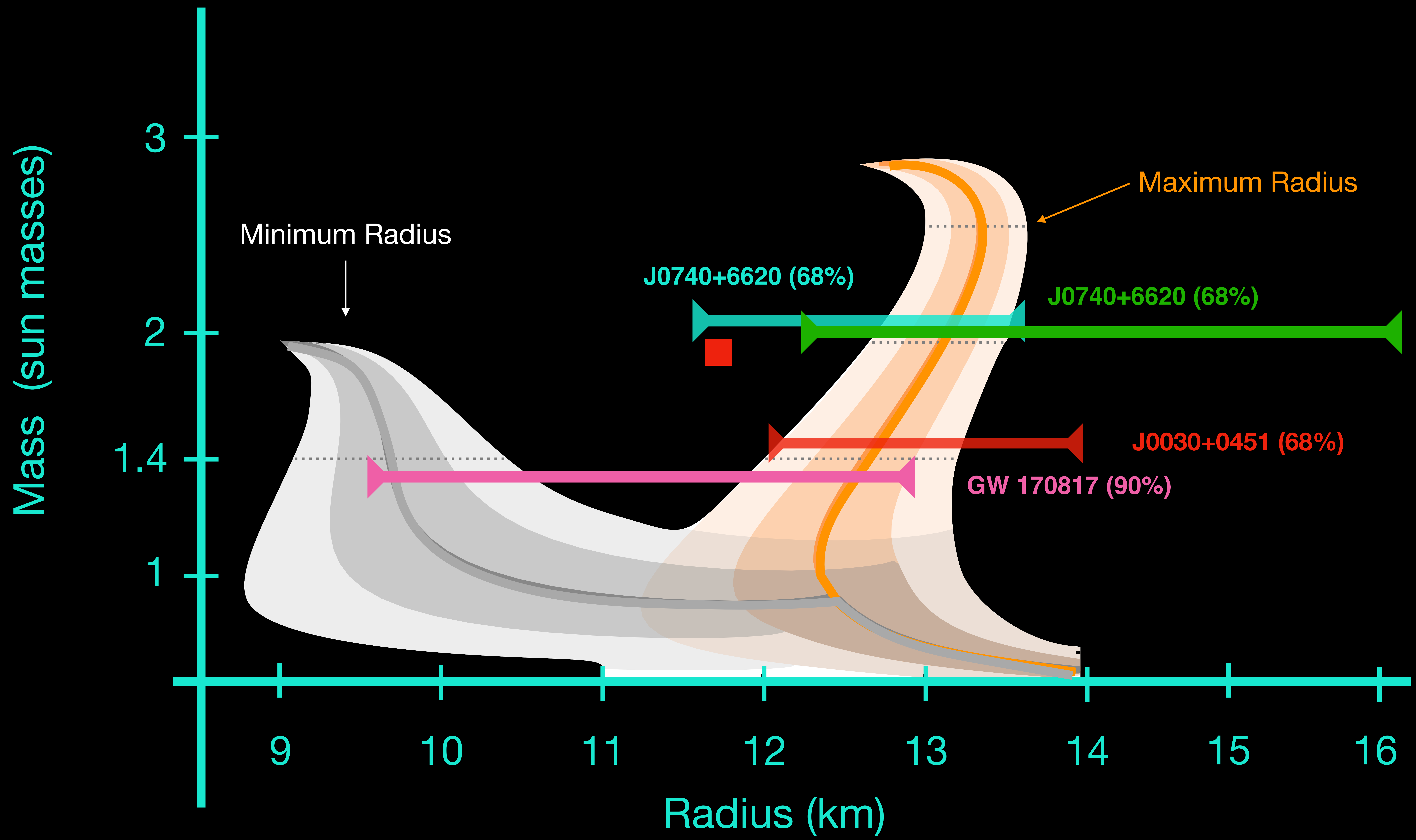

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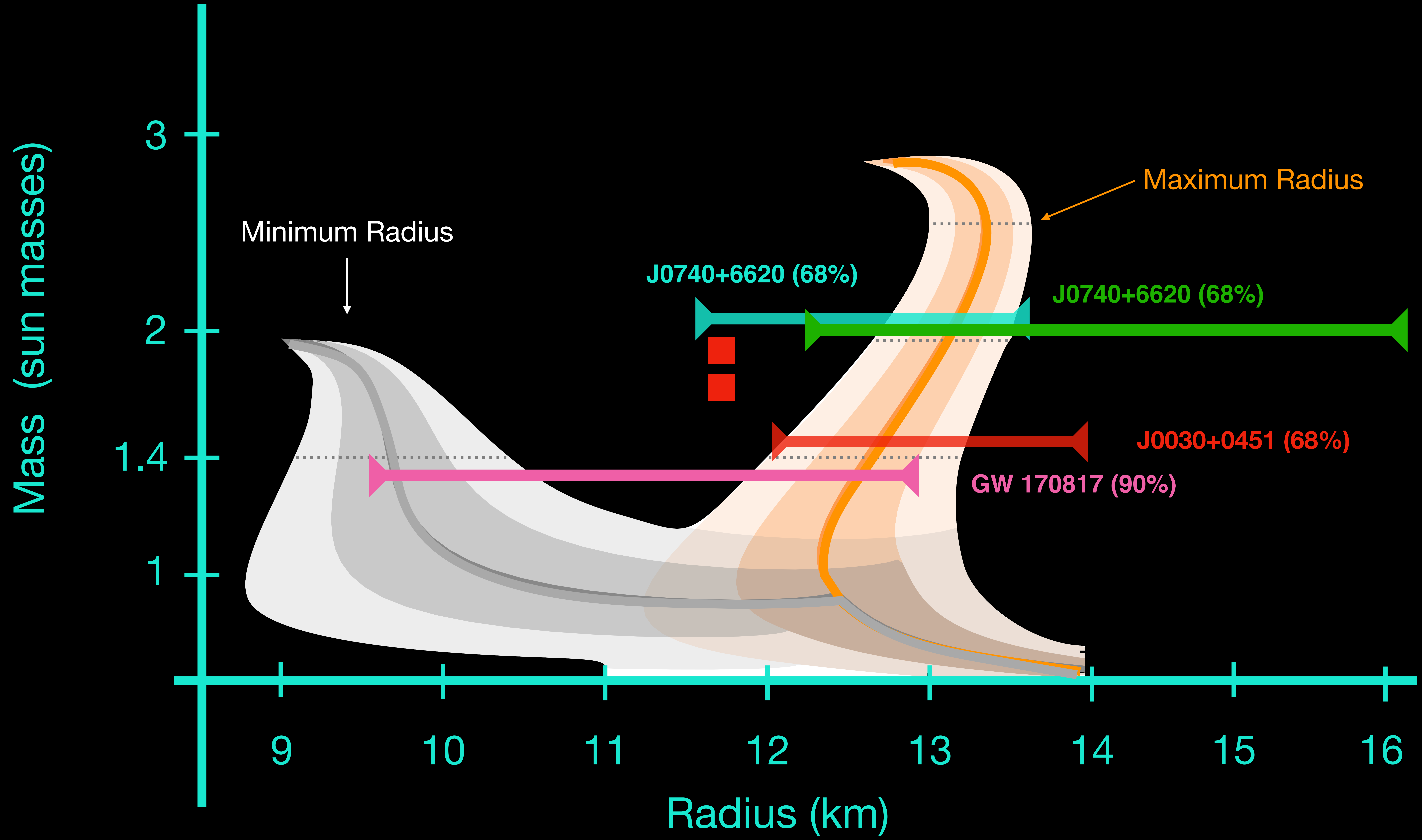

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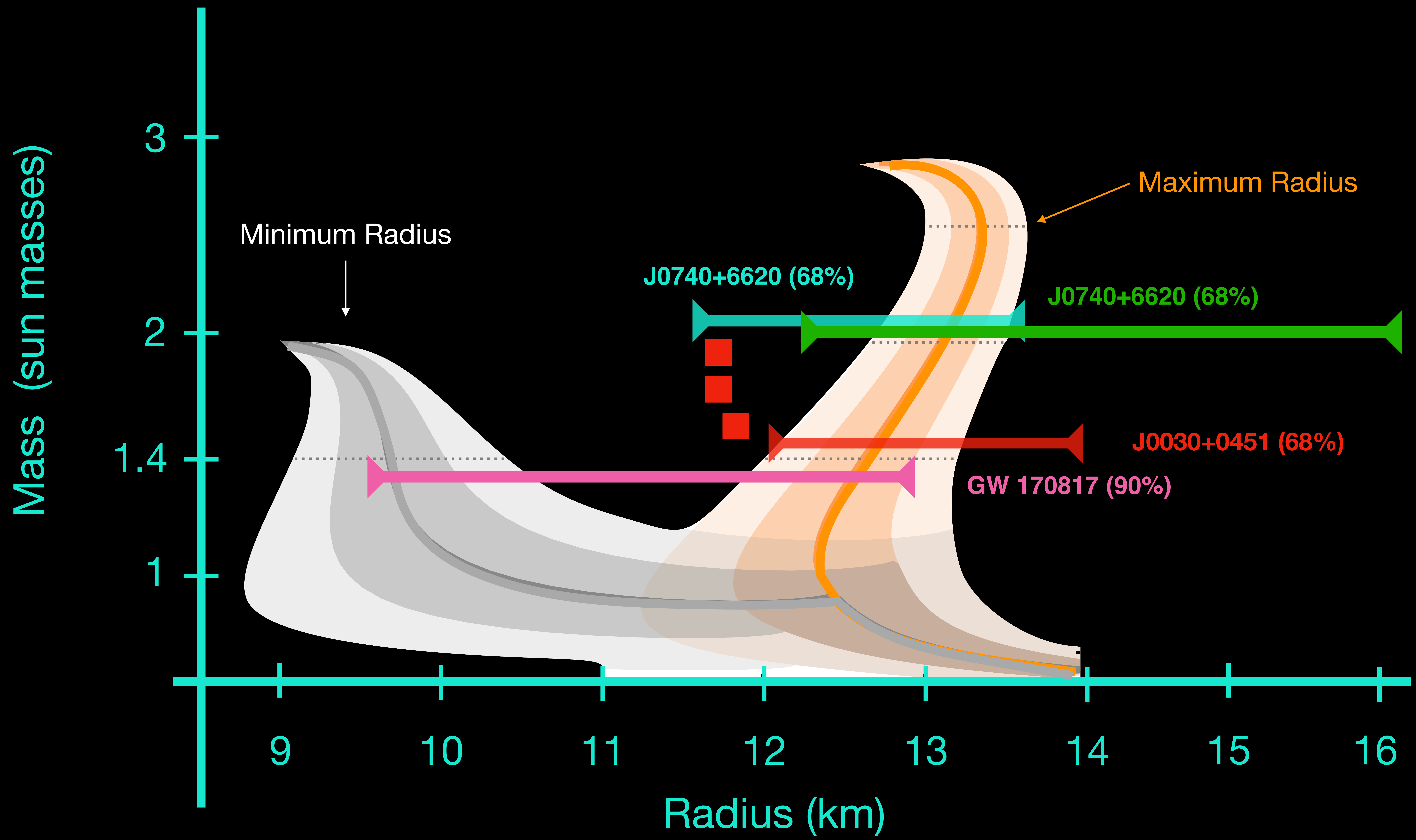


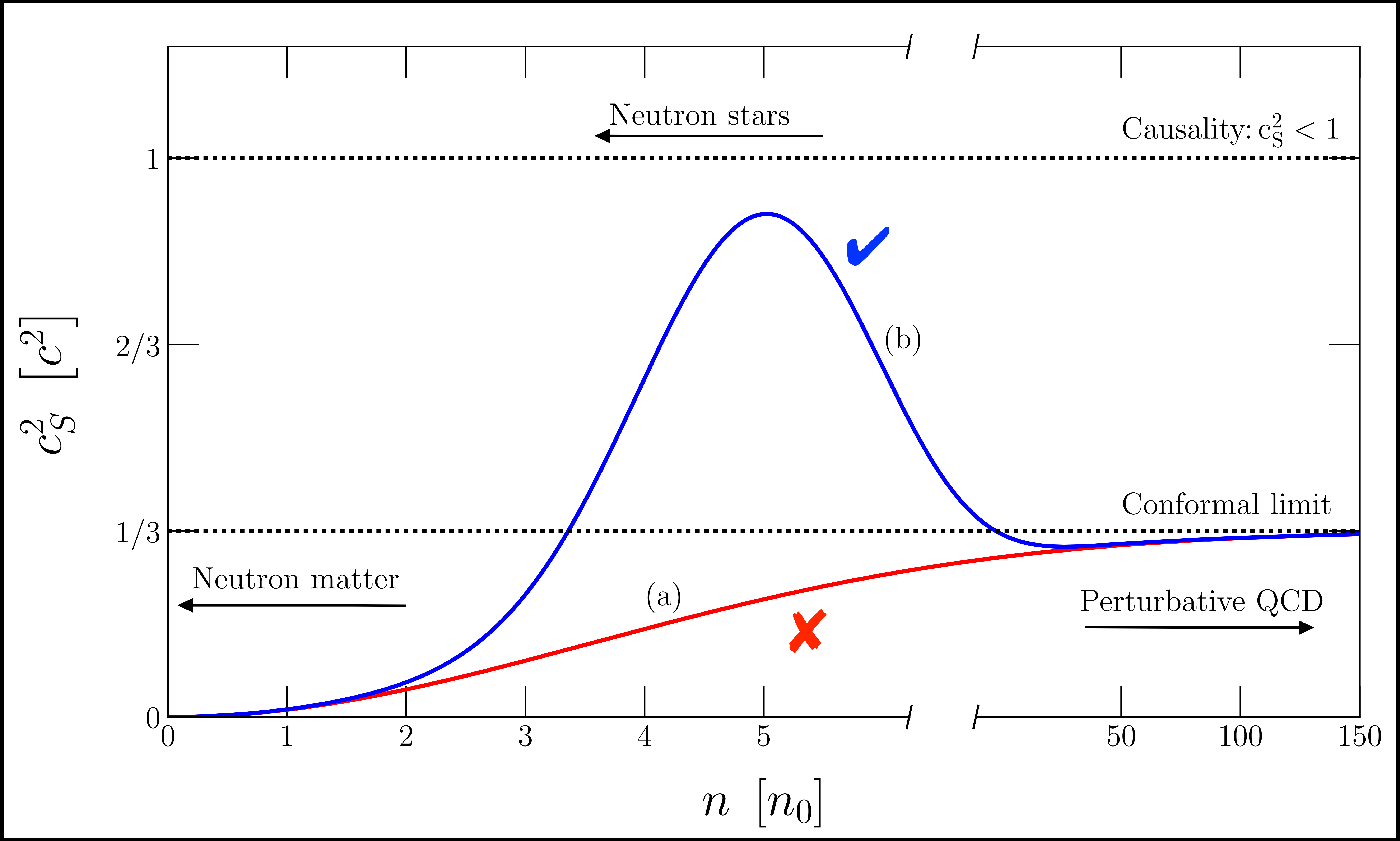
Speed of Sound in Dense Matter

$$c_s^2 = \frac{\partial P}{\partial \epsilon}$$

Large maximum mass and observed radii, combined with neutron matter calculations, suggest a rapid increase in pressure in the core.

This implies a large and non-monotonic sound speed in dense QCD matter.

Suggests the existence of a strongly interacting phase of relativistic matter in the inner core.



Tews, Carlson, Gandolfi and Reddy (2018)
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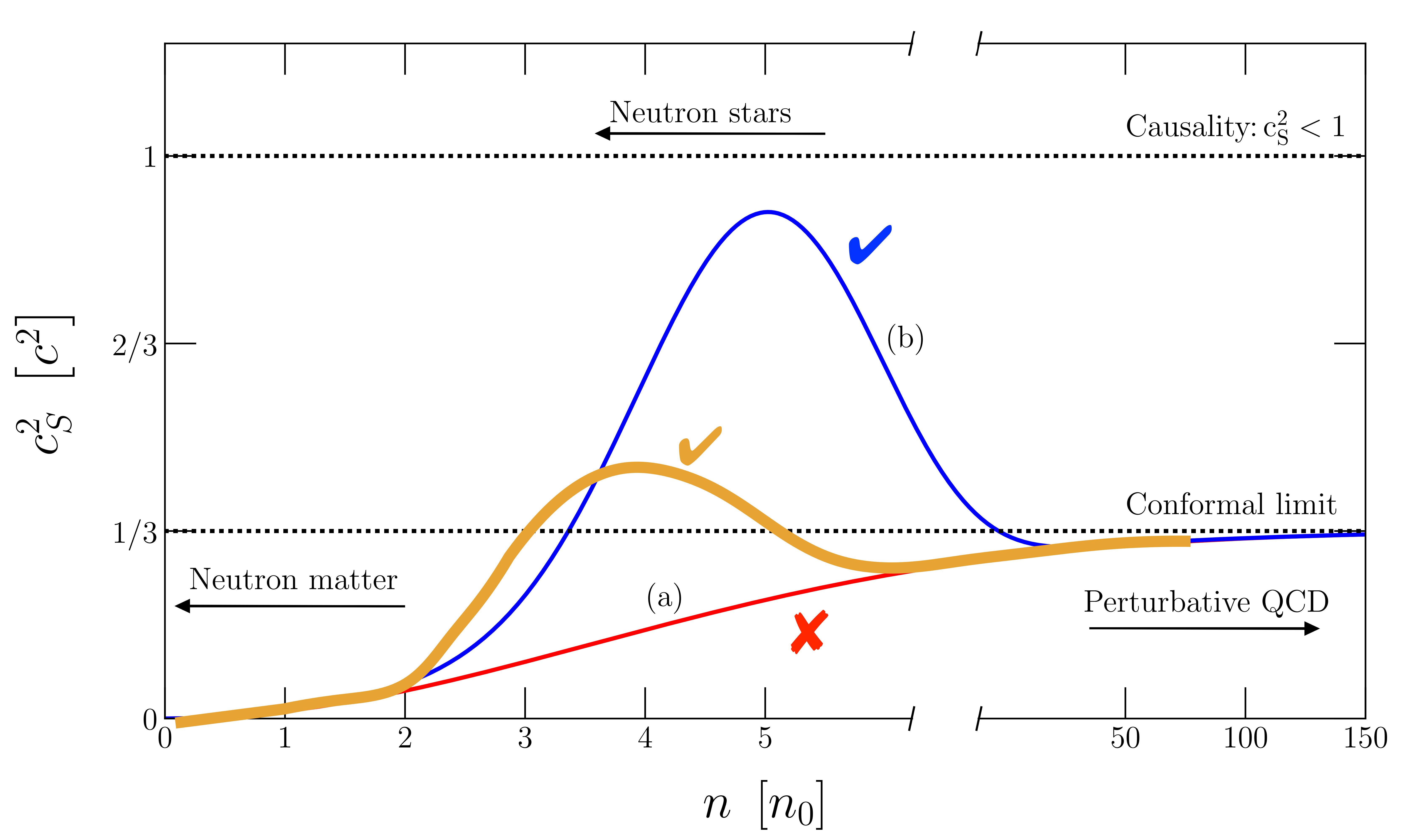

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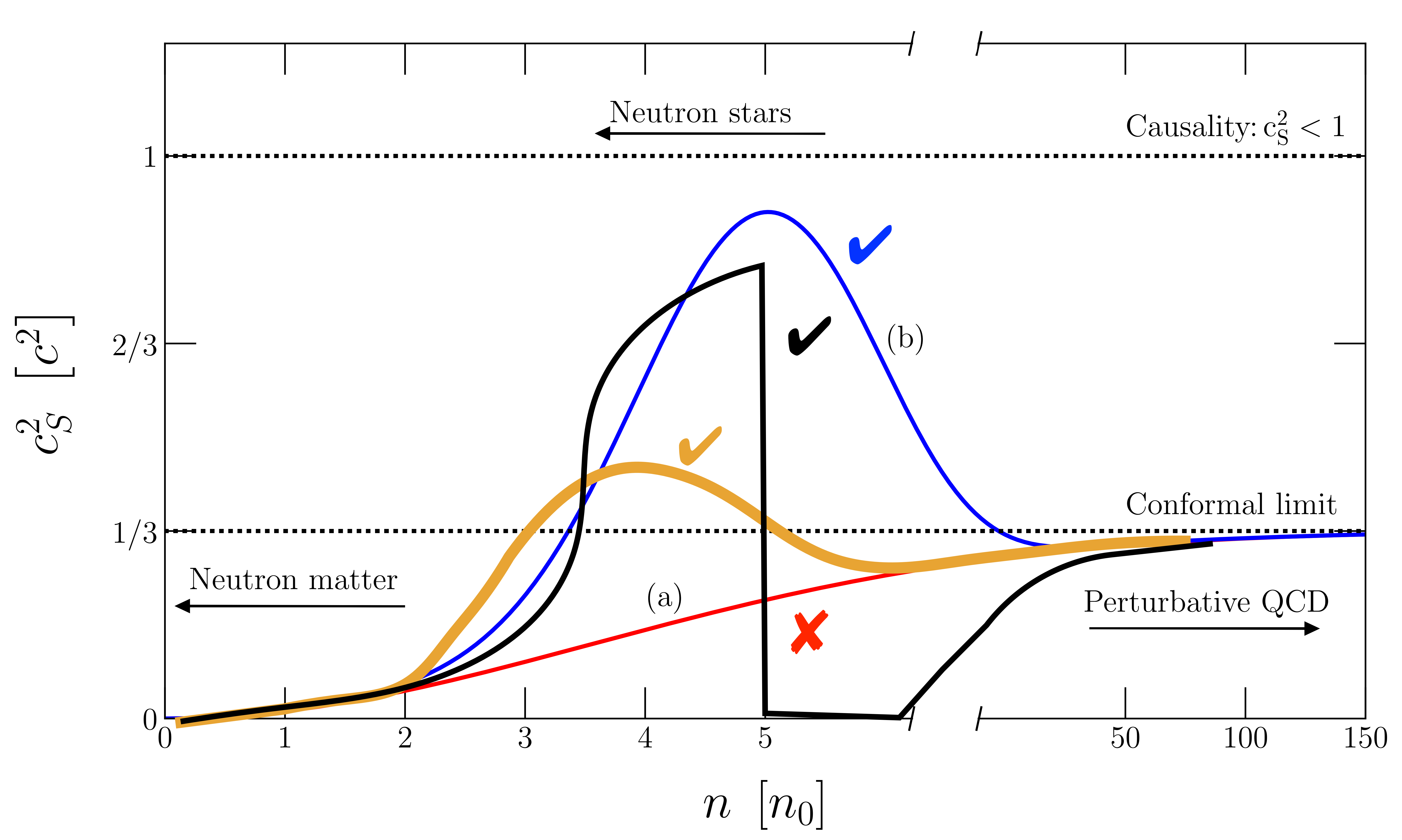

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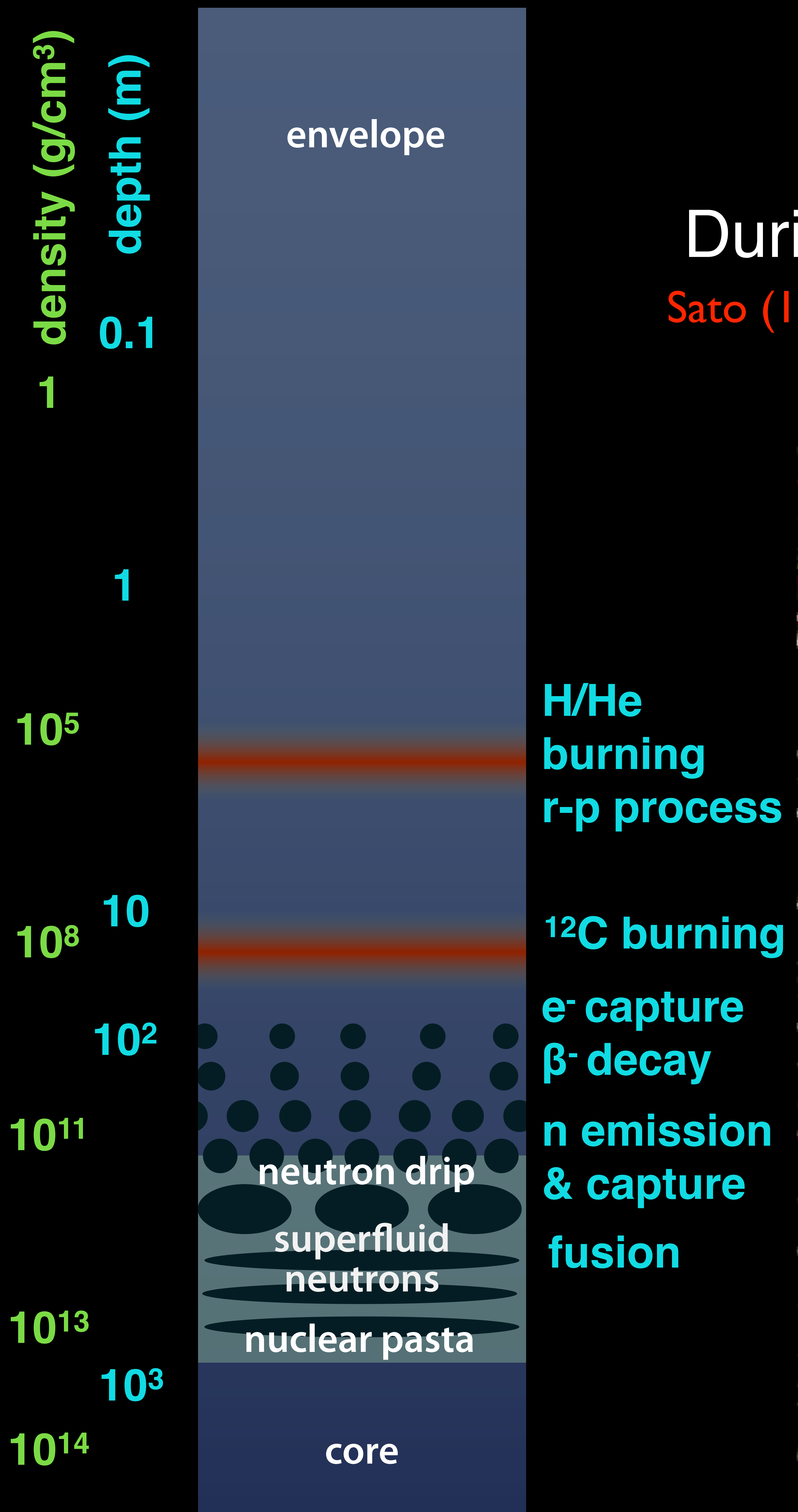
Part II: Crust

Evidence for a solid and superfluid state of matter.

Deep Crustal Heating

During accretion nuclear reactions release: ~ 2-4 MeV / nucleon

Sato (1974), Haensel & Zdunik (1990), Brown, Bildsten Rutledge (1998) Gupta et al (2007,2011).



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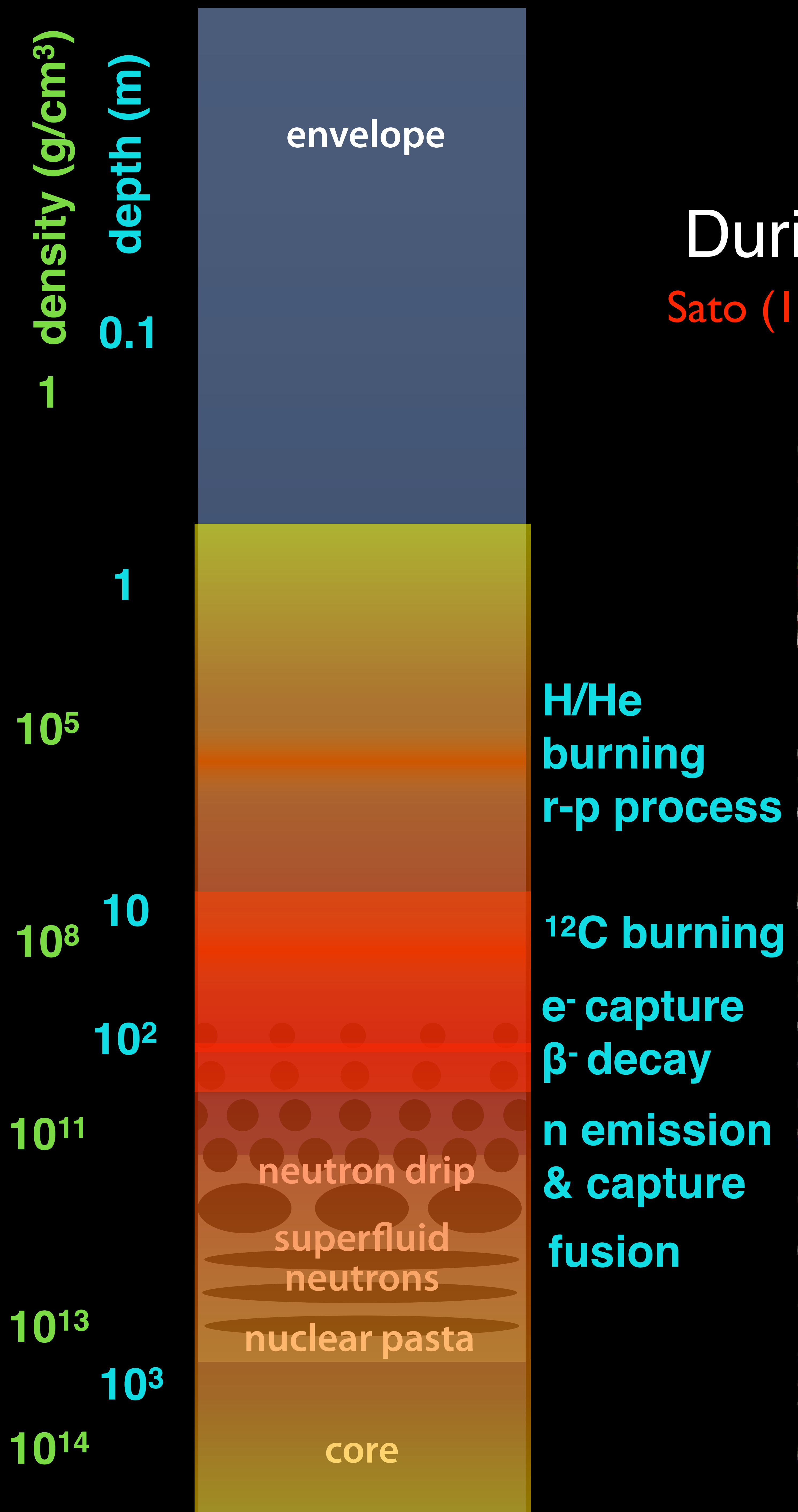

Cooling Post Accretion

- Relaxation observed in 7 sources to date.

- All known Quasi-persistent sources show cooling after accretion

- Cools on a time scale of ~1000 days.

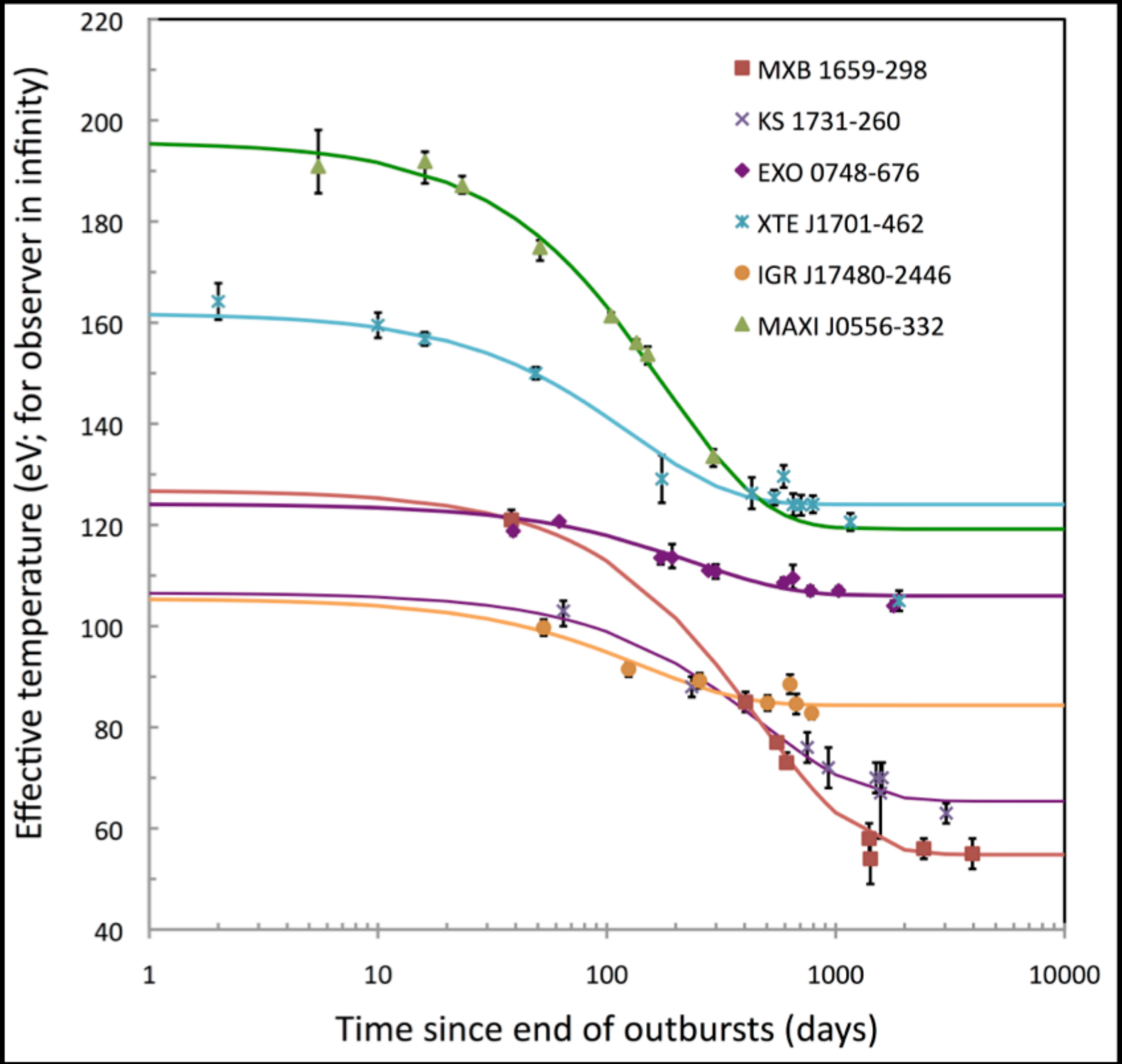

Figure from Rudy Wijnands (2013)

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Crustal Specific Heat

Crust Thickness

$$\tau_{\rm cool} \simeq \frac{C_V}{\kappa} \Delta R^2$$

Thermal Conductivity

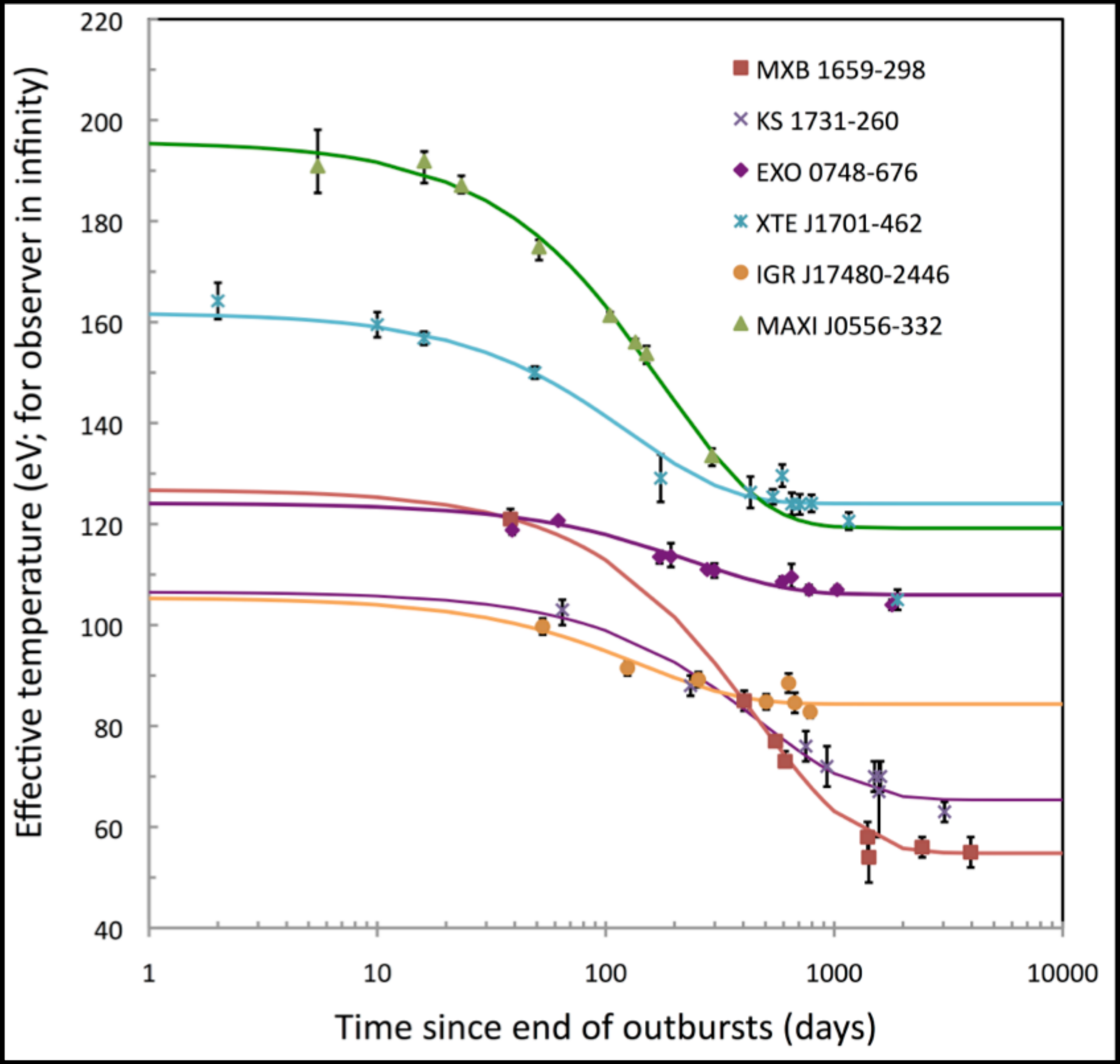
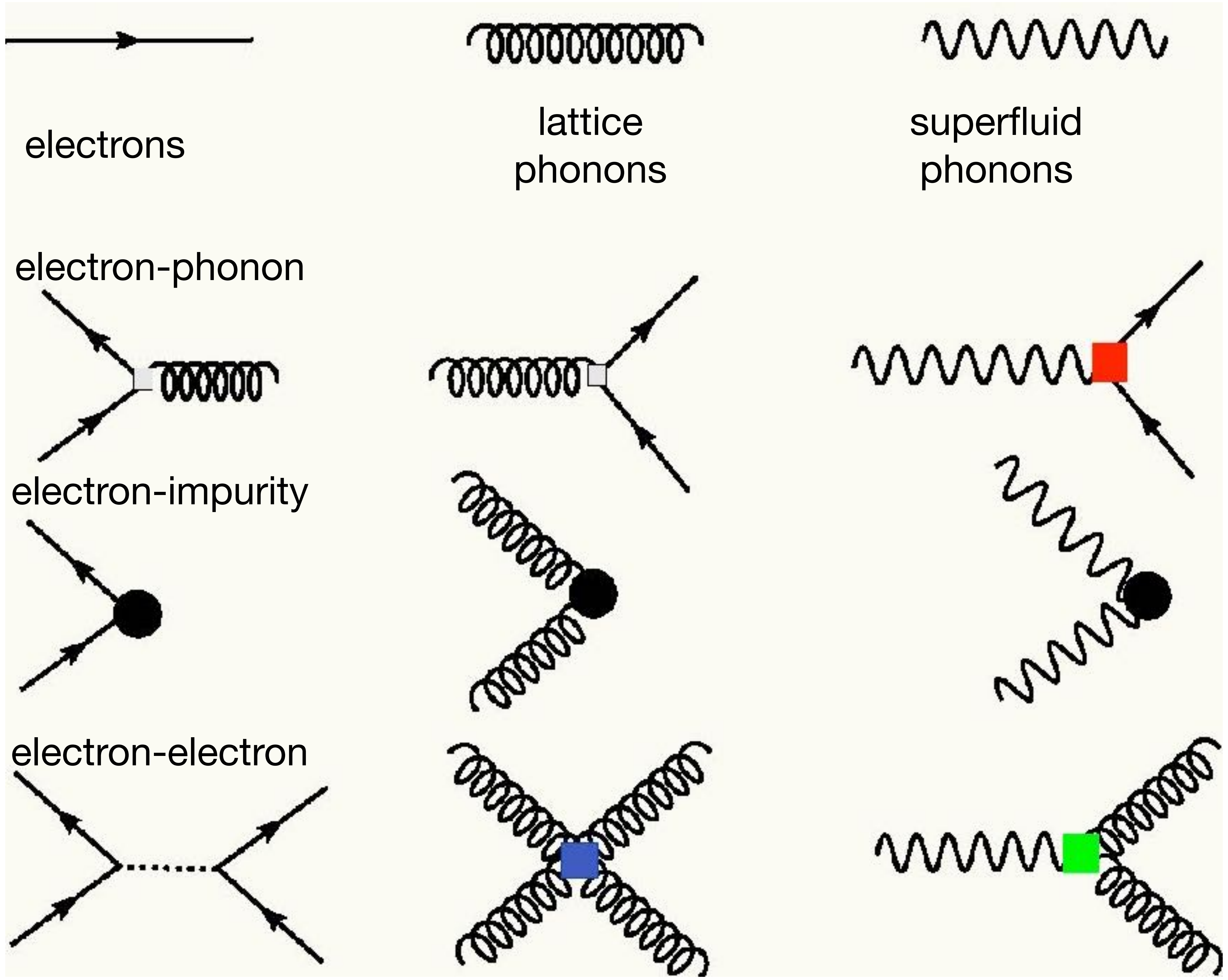
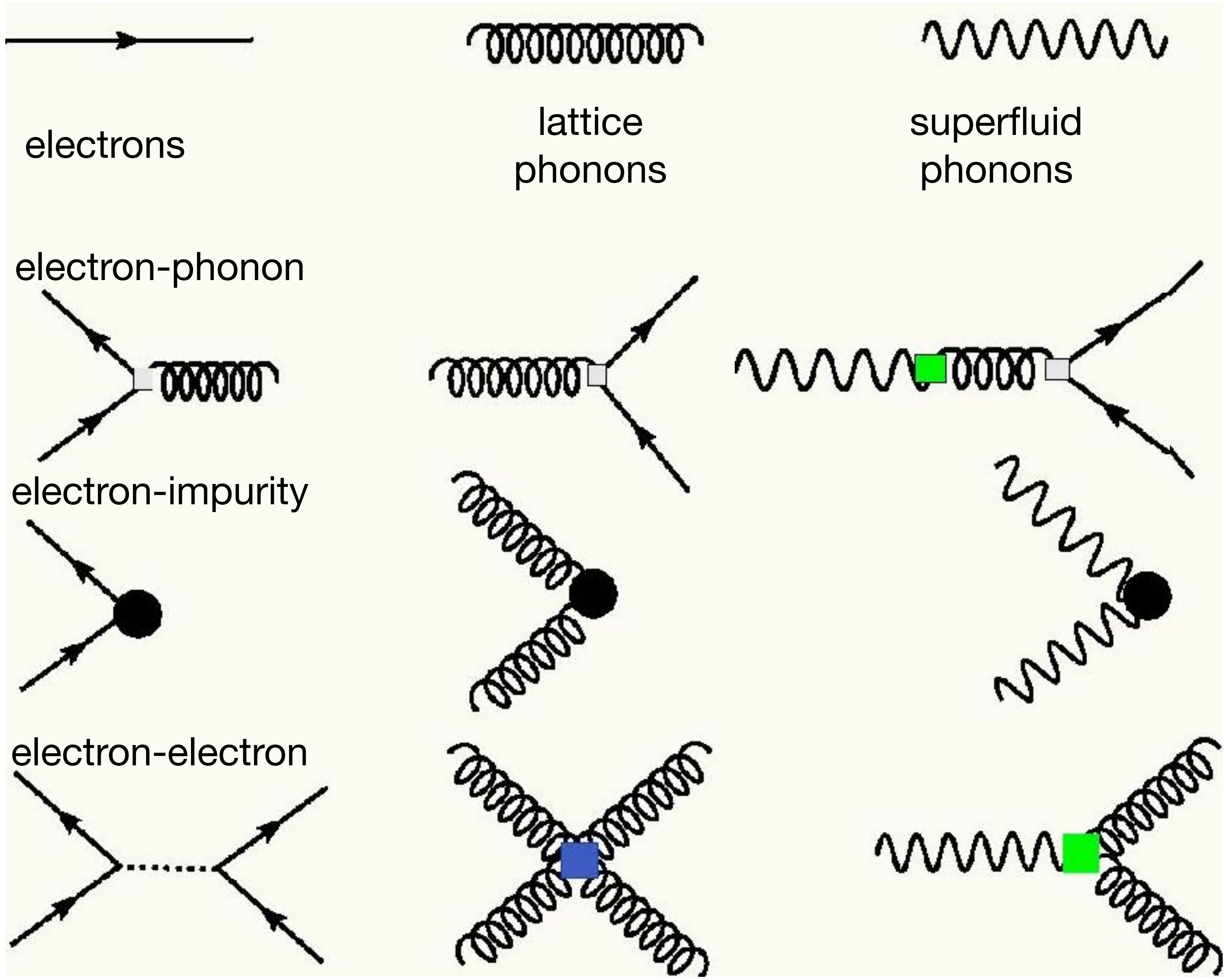


Figure from Rudy Wijnands (2013)



Cirigliano, Reddy & Sharma (2011), Page & Reddy (2012), Chamel, Page, & Reddy (2013)



Cirigliano, Reddy & Sharma (2011), Page & Reddy (2012), Chamel, Page, & Reddy (2013)

Phonon Mixing and Entrainment

$$\mathcal{L} = \frac{1}{2}(\partial_0\tilde{\phi})^2 - \frac{1}{2}v_\phi^2\,(\partial_i\tilde{\phi})^2 + \frac{1}{2}(\partial_0\tilde{\xi}_i)^2 - \frac{1}{2}\,v_l^2(\partial_i\tilde{\xi}_i)^2 + g_{\rm mix}\,\partial_0\tilde{\phi}\,\partial_i\tilde{\xi}_i$$

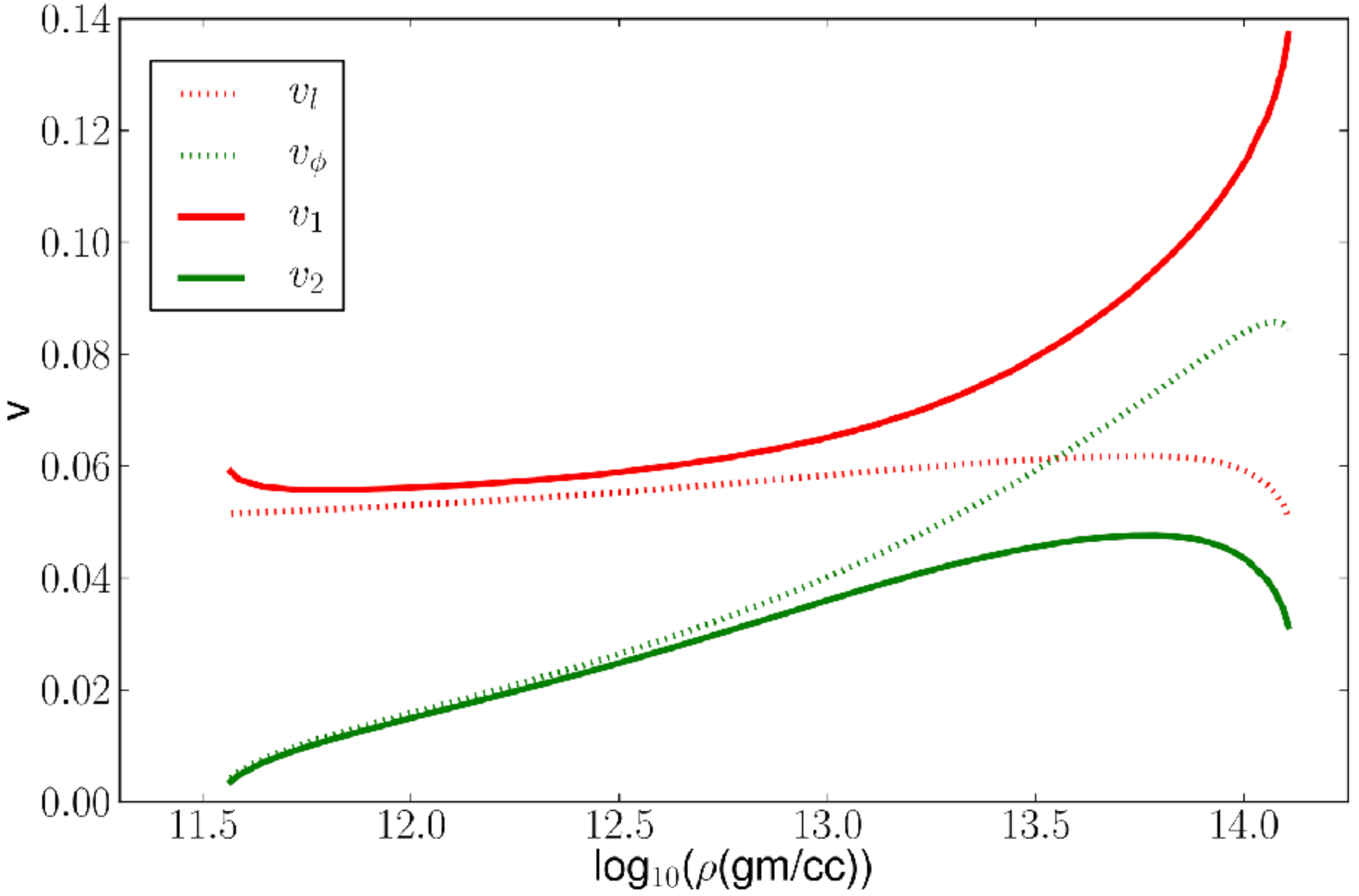
Low-energy constants can be obtained from the ground state thermodynamics and neutron band-structure calculations.

Cirigliano, Reddy, Sharma (2012)

A key feature is the entrainment of a large fraction of neutrons by the lattice.

Carter, Chamel, Haensel (2005)

$$g_{\rm mix} \simeq v_\phi\,\frac{n_b}{\sqrt{(n_b+n_p)n_f}}$$



Large entrainment and mixing have important consequences for superfluid heat conduction and lattice heat capacity.

Chamel, Page, Reddy (2012)

Evidence for a solid and superfluid inner crust.

Crustal Specific Heat

Crust Thickness

$$\tau_{\text{cool}} \simeq \frac{C_V}{\kappa} \Delta R^2$$

Thermal Conductivity

- 1000-day cooling timescale requires small specific heat and large thermal conductivity.
- The inner crust must be solid (for high electron conduction) and neutrons must be superfluid (to have low heat capacity)!

Shternin & Yakovlev (2007) Cumming & Brown (2009) Page & Reddy (2011)

Part III: Neutrino Transport

Neutrino transport and Nucleosynthesis

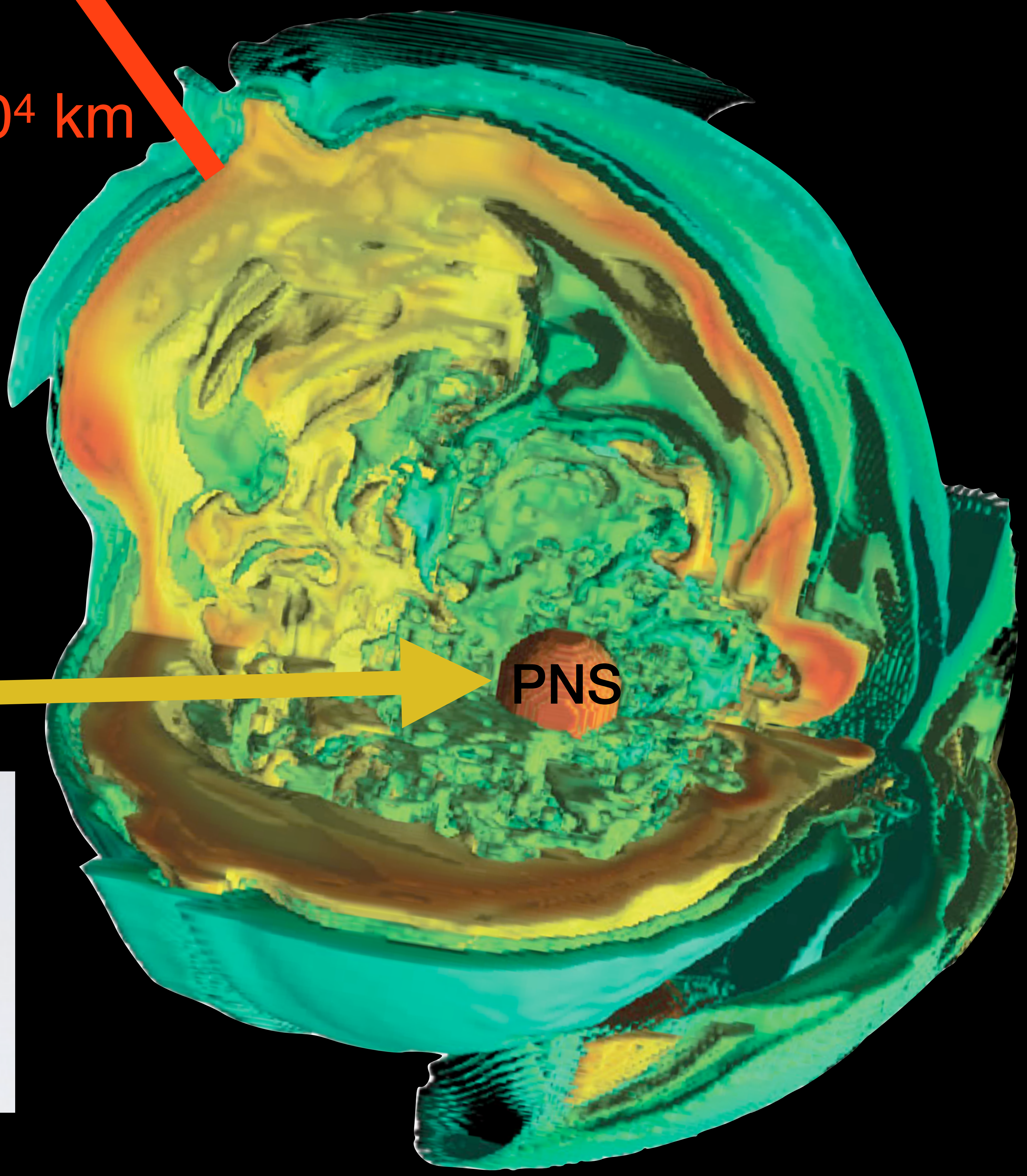
Neutron-rich ejecta from neutron star mergers and supernovae are potential sites of heavy-element nucleosynthesis.

A high neutron excess (>75%) is needed to make the heaviest elements like gold, platinum, and uranium.

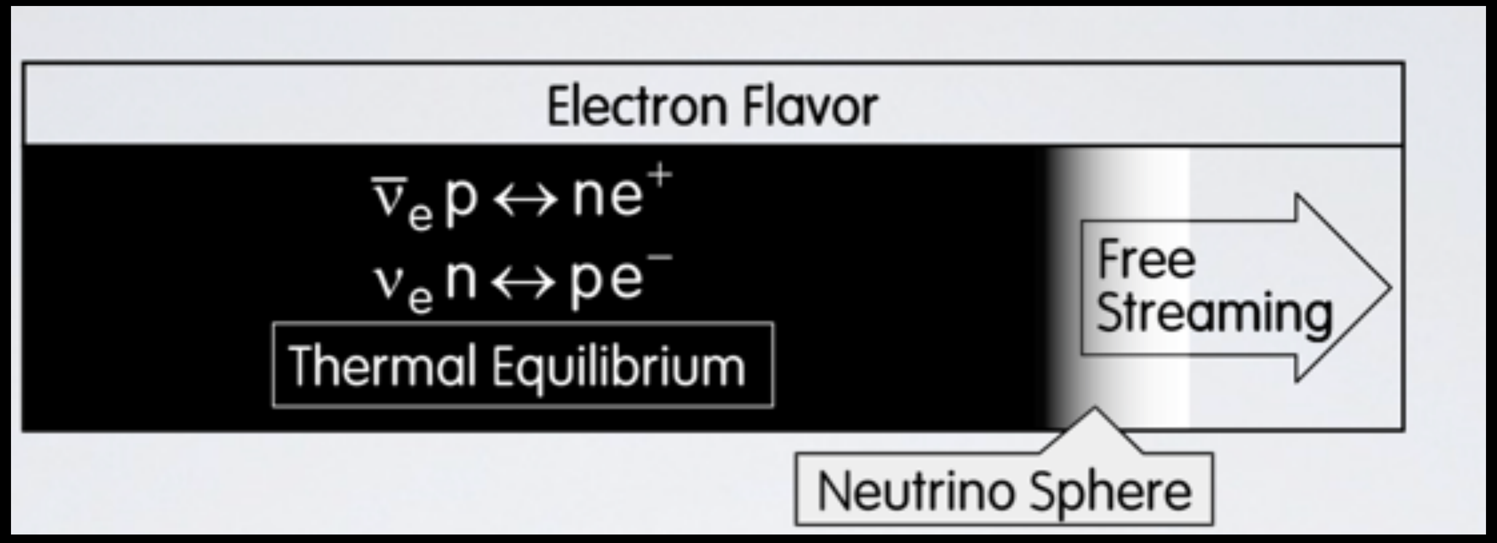
Neutrinos alter the composition of ejecta at low density and high entropy.

$$\nu_e + n \rightarrow p + e^-$$
$$\bar{\nu}_e + p \rightarrow n + e^+$$

R ~ 10^3-10^4 km



Neutrino-sphere at high density. R ~ 10-20 km



Neutrino Interactions in Dense Matter

Low energy Lagrangian: $\mathcal{L} = \frac{G_F}{\sqrt{2}}\ l_\mu\ j^\mu$ $\qquad l_1 + N_2 \to l_3 + N_4$

Absorption: $l^{cc}_\mu = \bar{l}\gamma_\mu(1-\gamma_5)\nu_l$ $\qquad j^\mu_{cc} = \bar{\Psi}_p \left(\gamma^\mu(g_V - g_A\gamma_5) + F_2 \frac{i\sigma^{\mu\alpha}q_\alpha}{2M}\right)\ \Psi_n$

Scattering: $l^{nc}_\mu = \bar{\nu}\gamma_\mu(1-\gamma_5)\nu$ $\qquad j^\mu_{nc} = \bar{\Psi}_i\ (\gamma^\mu(C^i_V - C^i_A\gamma_5) + F^i_2 \frac{i\sigma^{\mu\alpha}q_\alpha}{2M})\ \Psi_i$

Neutrino scattering and absorption rates: $\frac{d\Gamma(E_1)}{dE_3 d\mu_{13}} = \frac{G_F^2}{32\pi^2}\frac{p_3}{E_1}(1-f_3(E_3))L_{\mu\nu}\ \mathcal{S}^{\mu\nu}(q_0,q)$

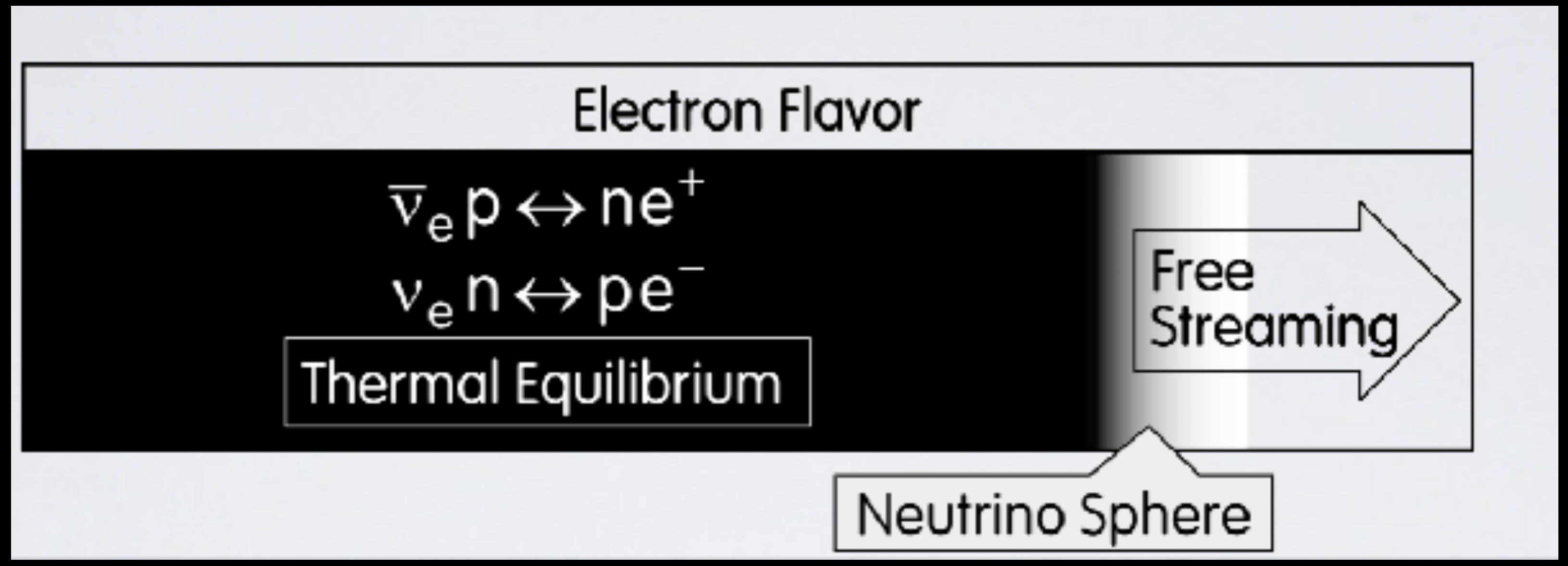
Dynamic structure function: $\mathcal{S}^{\mu\nu}(q_0,q) = \frac{-2\ \mathrm{Im}\ \mathbf{\Pi}^{\mu\nu}(q_0,q)}{1-\exp\left(-(q_0+\Delta\mu)/T\right)}$

Current-current correlations functions: $\mathbf{\Pi}^{\mu\nu}(q_0,q) = -i\int dt\ d^3x\ \theta(t)\ e^{i(q_0 t - \vec{q}\cdot\vec{x})}\langle\ |[j_\mu\ (\vec{x},t), j_\nu(\vec{0},0)]|\ \rangle$

difficult to calculate in general due to the non-perturbative nature of strong interactions.

Sawyer (1970s), Iwamoto & Pethick (1980s), Horowitz & Wherberger, Raffelt et al., Reddy et al. (1990s), Carlson, Gandolfi, Horowitz, Pethick, Reddy, Roberts, Schwenk, Shen, and others (2000s)

Charged current reactions in the neutrino-sphere



Absorption of ν_e and $\bar{\nu}_e$ by non-relativistic nucleons is described by

Spin-isospin density response

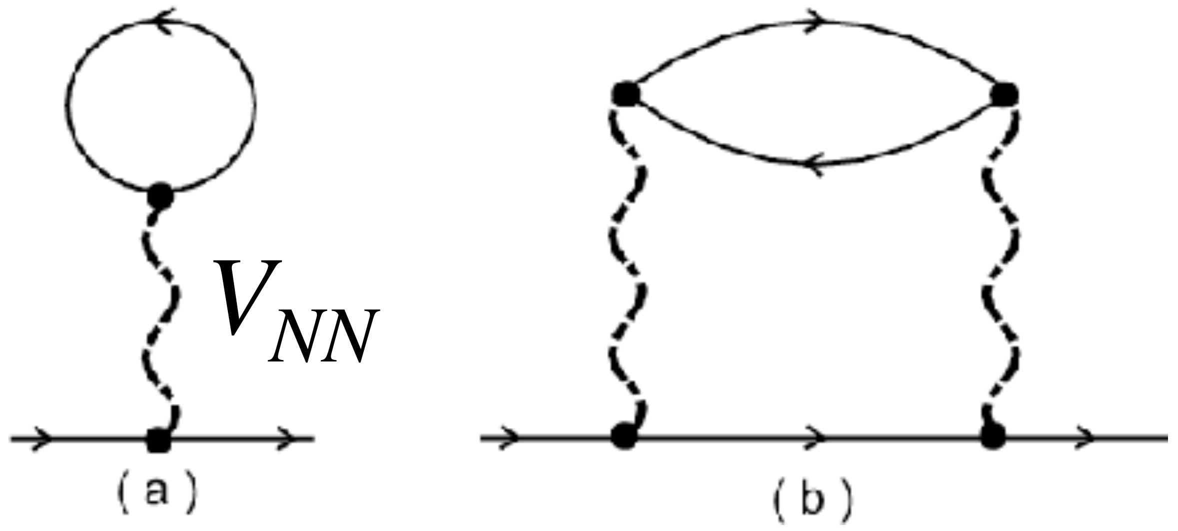
$$\frac{d\Gamma}{d\omega\ d\cos\theta} = \frac{G_F^2 \cos^2\theta_C}{4\pi^2} p_e E_e (1 - f_e)\ \Big((1+\cos\theta) S_\tau(\omega, q) + (3 - \cos\theta) S_{\tau\sigma}(\omega, q)\Big)$$

Isospin density response

These response functions are strongly modified due to nuclear interactions:

- Difference in the dispersion relations of neutrons and protons.
- Correlations and collective excitations

Dressed nucleons:



$$E_n(p) \approx m_n + \frac{p^2}{2m_n^*} + U_n + i\,\Gamma_n$$

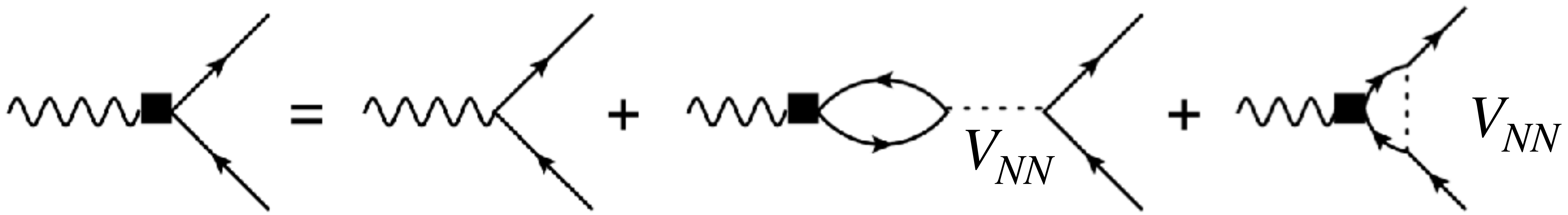
$$E_p(p+q) \approx m_p + \frac{(p+q)^2}{2m_n^*} + U_p + i\,\Gamma_p$$

$$\text{Q} = \varepsilon_n(\vec{k}) - \varepsilon_p(\vec{k} - \vec{q})$$

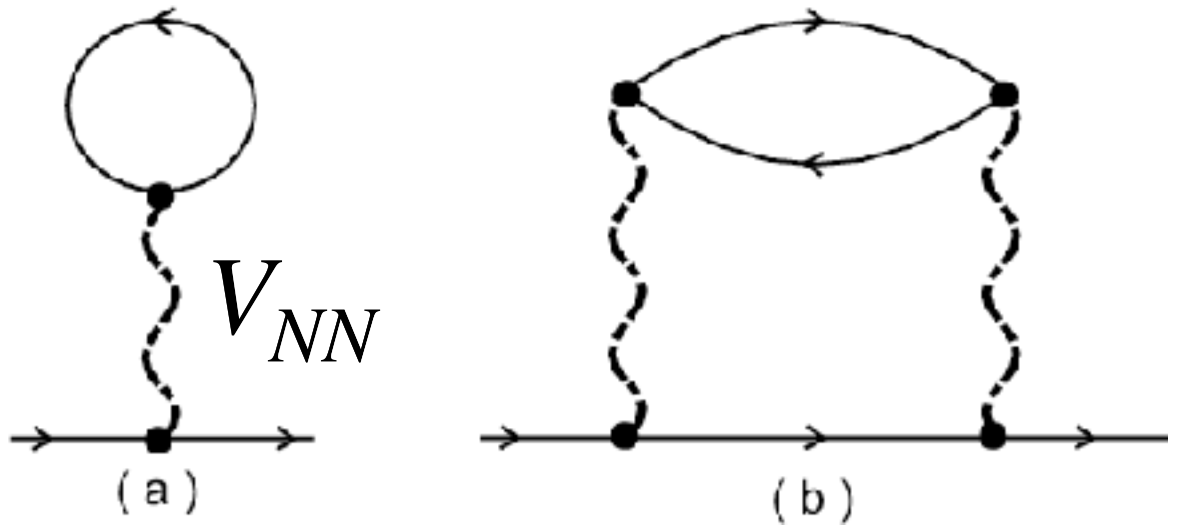
$$= M_n - M_p + \Sigma_n(k) - \Sigma_n(k-q)$$

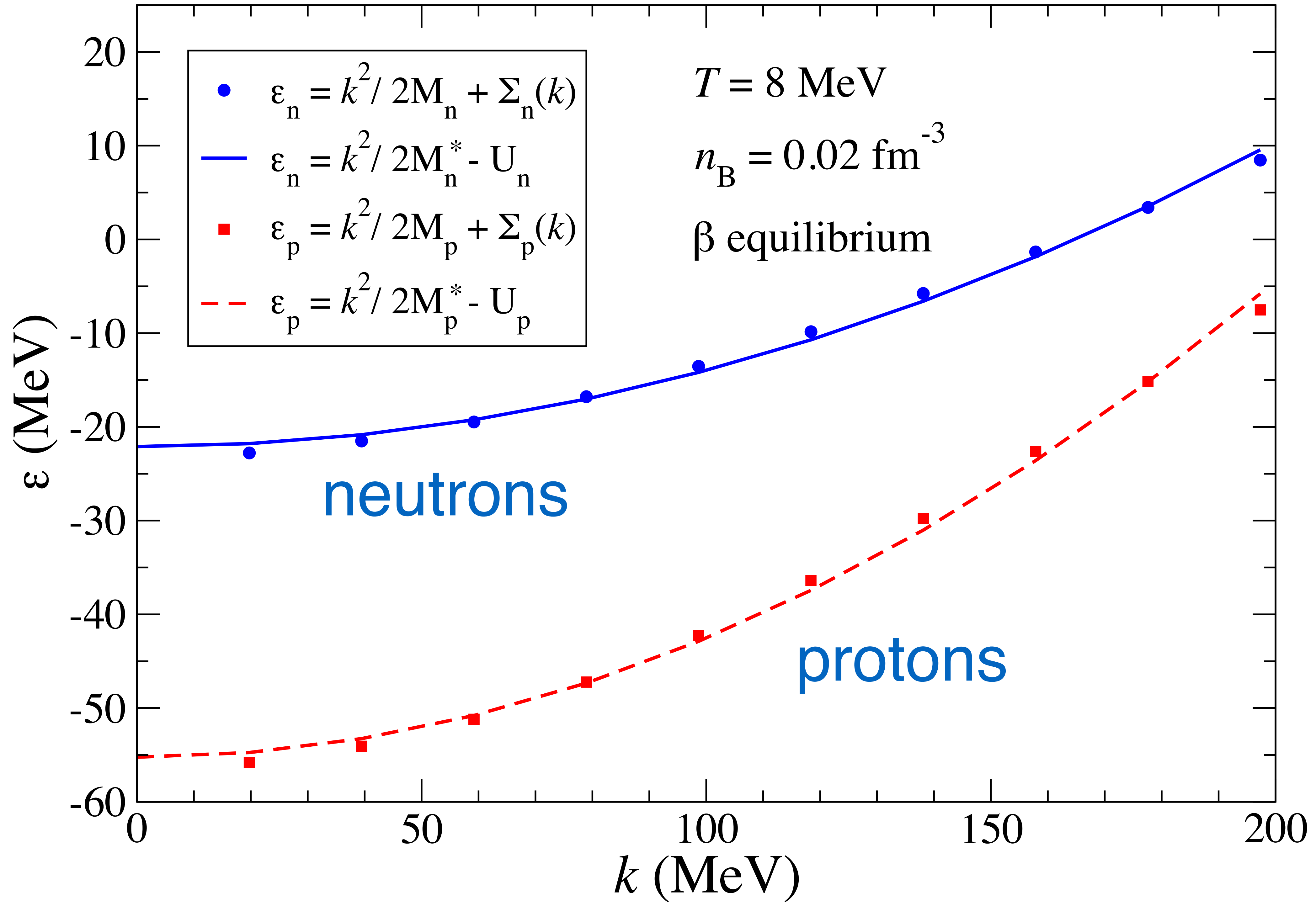
Reddy, Prakash & Lattimer (1998), Martinez-Pinedo et al. (2012), Roberts & Reddy (2012), Rrapaj, Bartl, Holt, Reddy, Schwenk (2015)

Consistent vertices (RPA):



Dressed nucleons:





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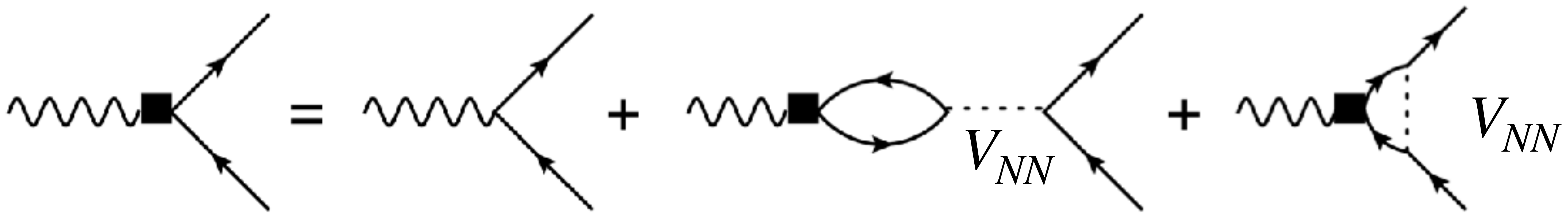
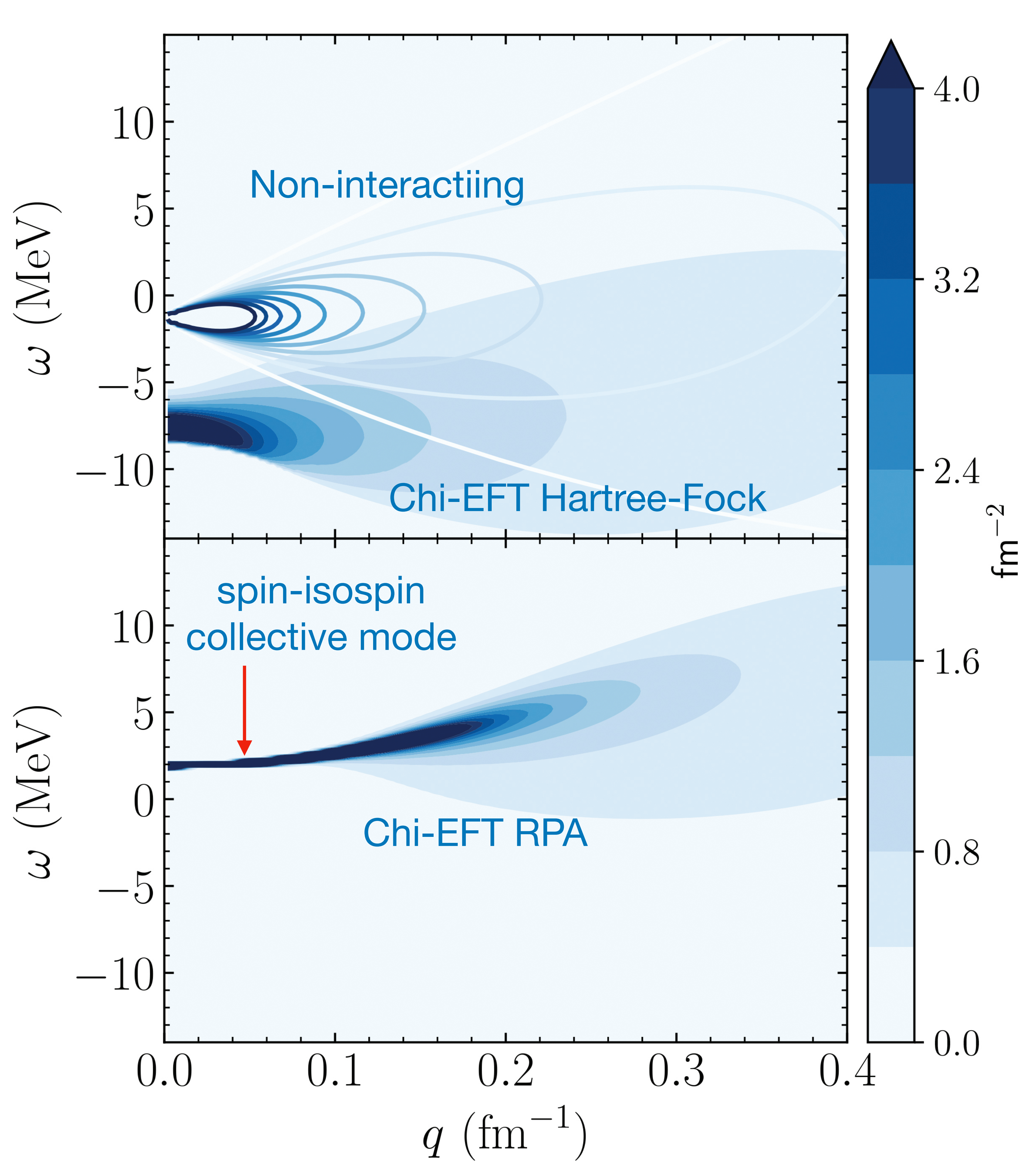
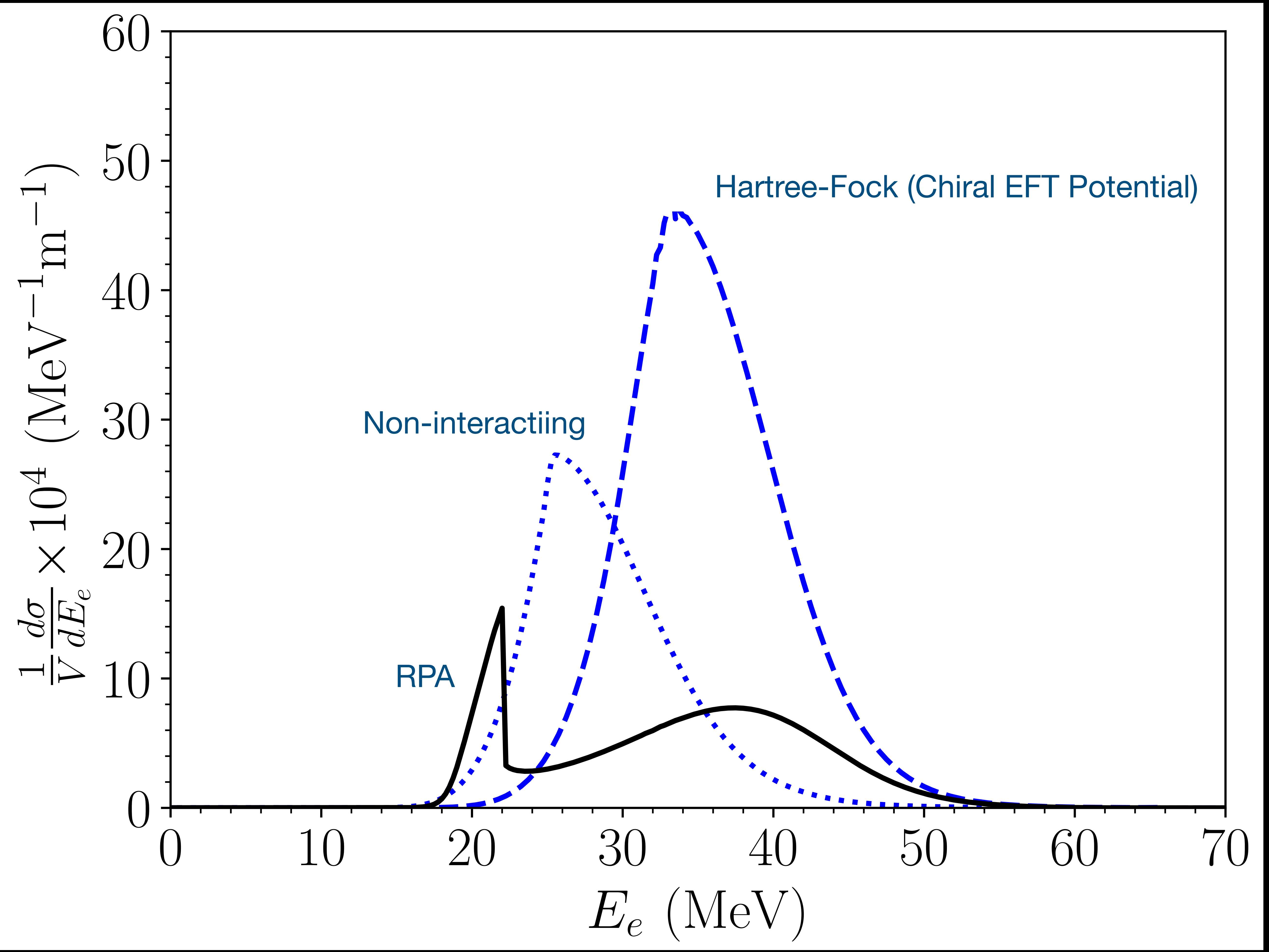


Large corrections to neutrino interactions at sub-nuclear density



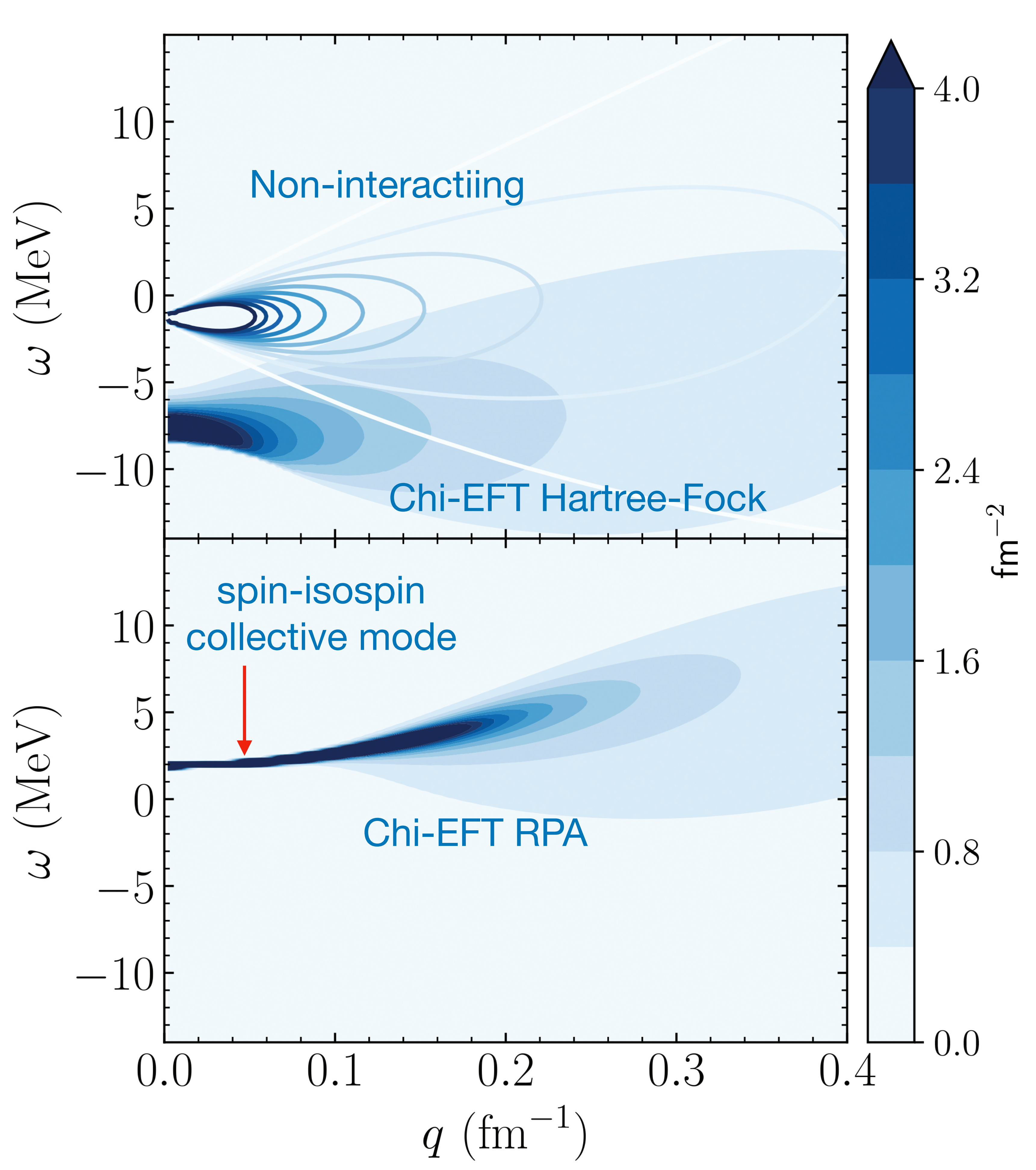
Response Function



Neutrino Cross-Section

Shin, Rrapaj, Holt, and Reddy (2023)

Large corrections to neutrino interactions at sub-nuclear density



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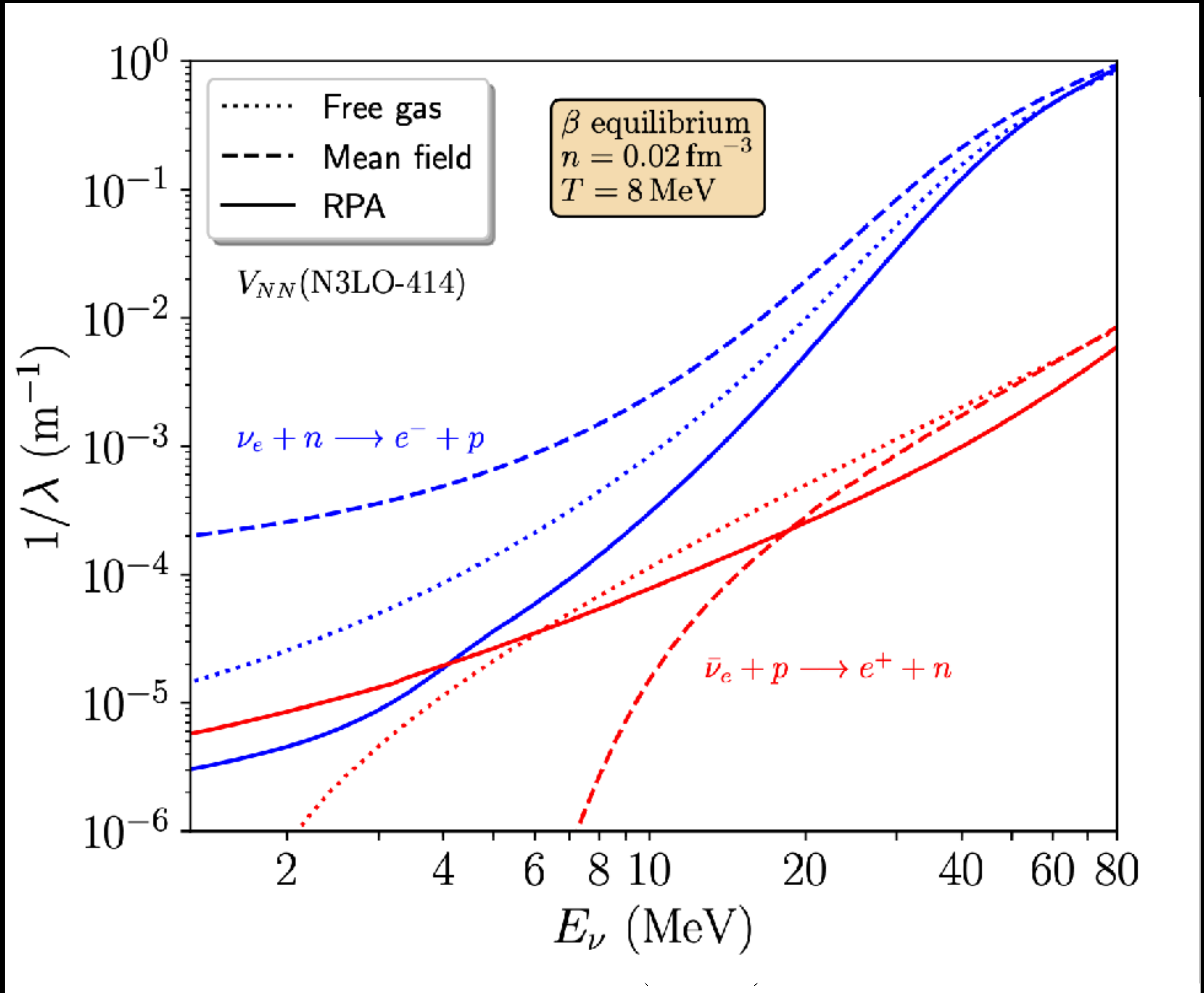

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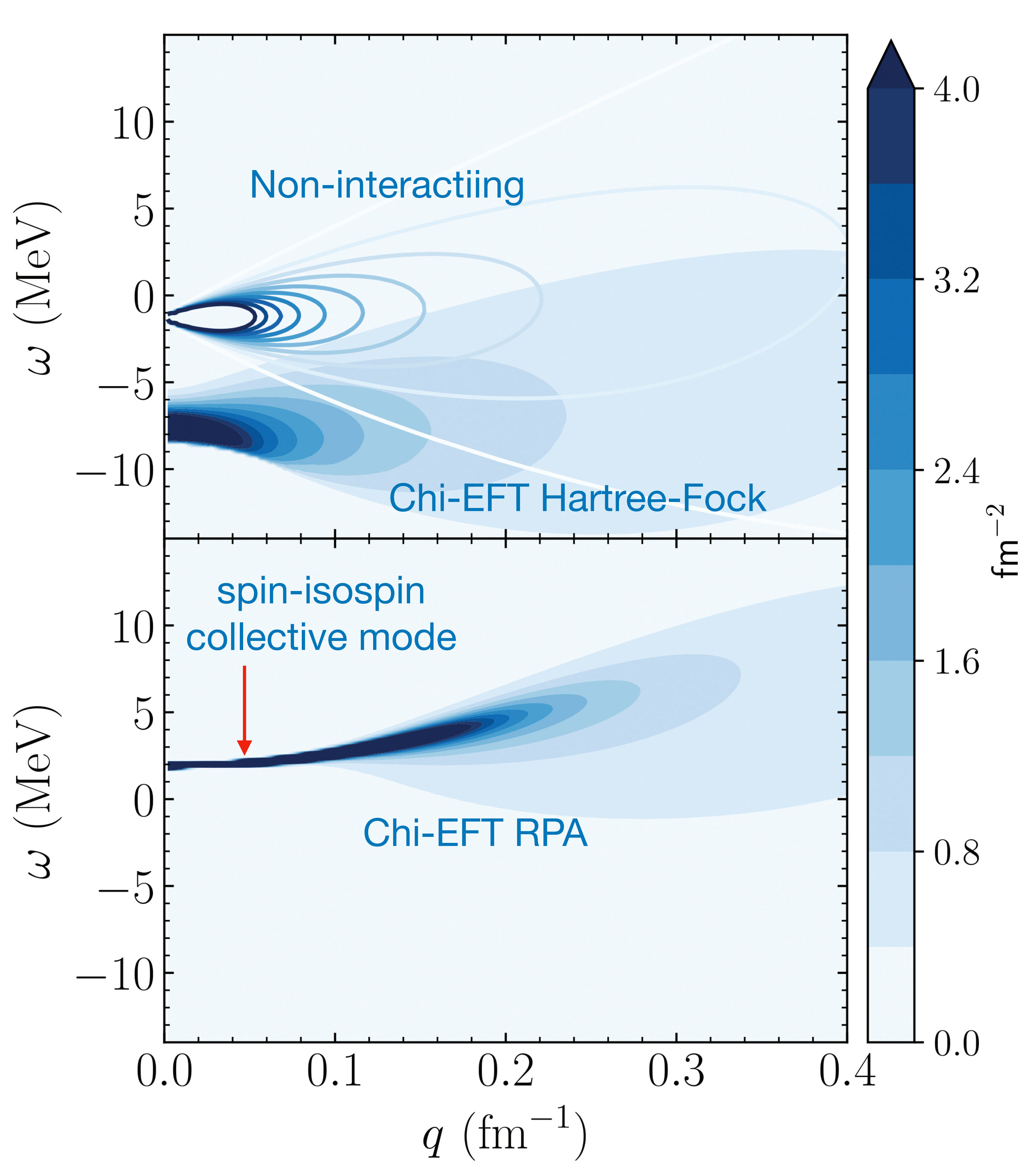
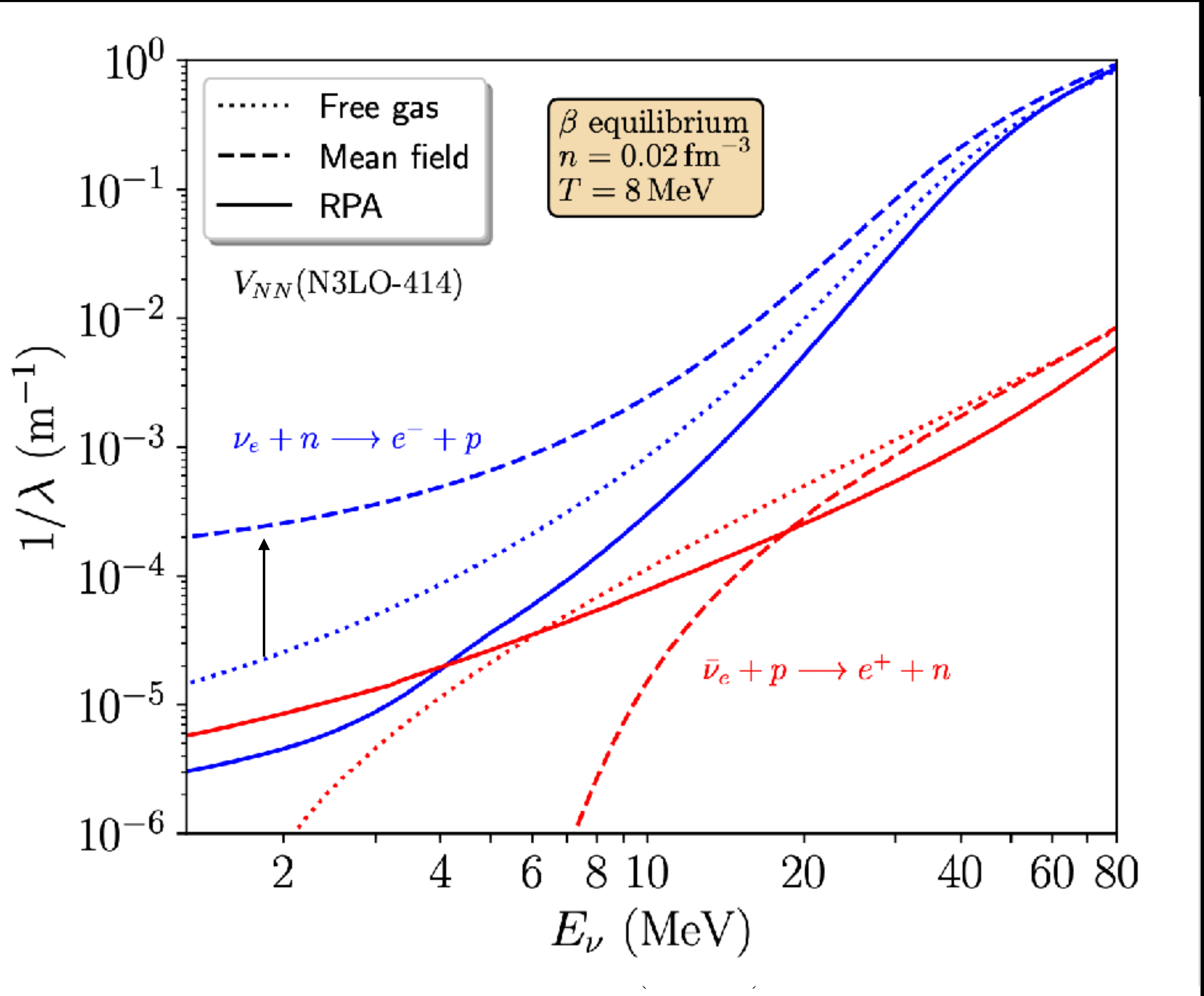


Response Function



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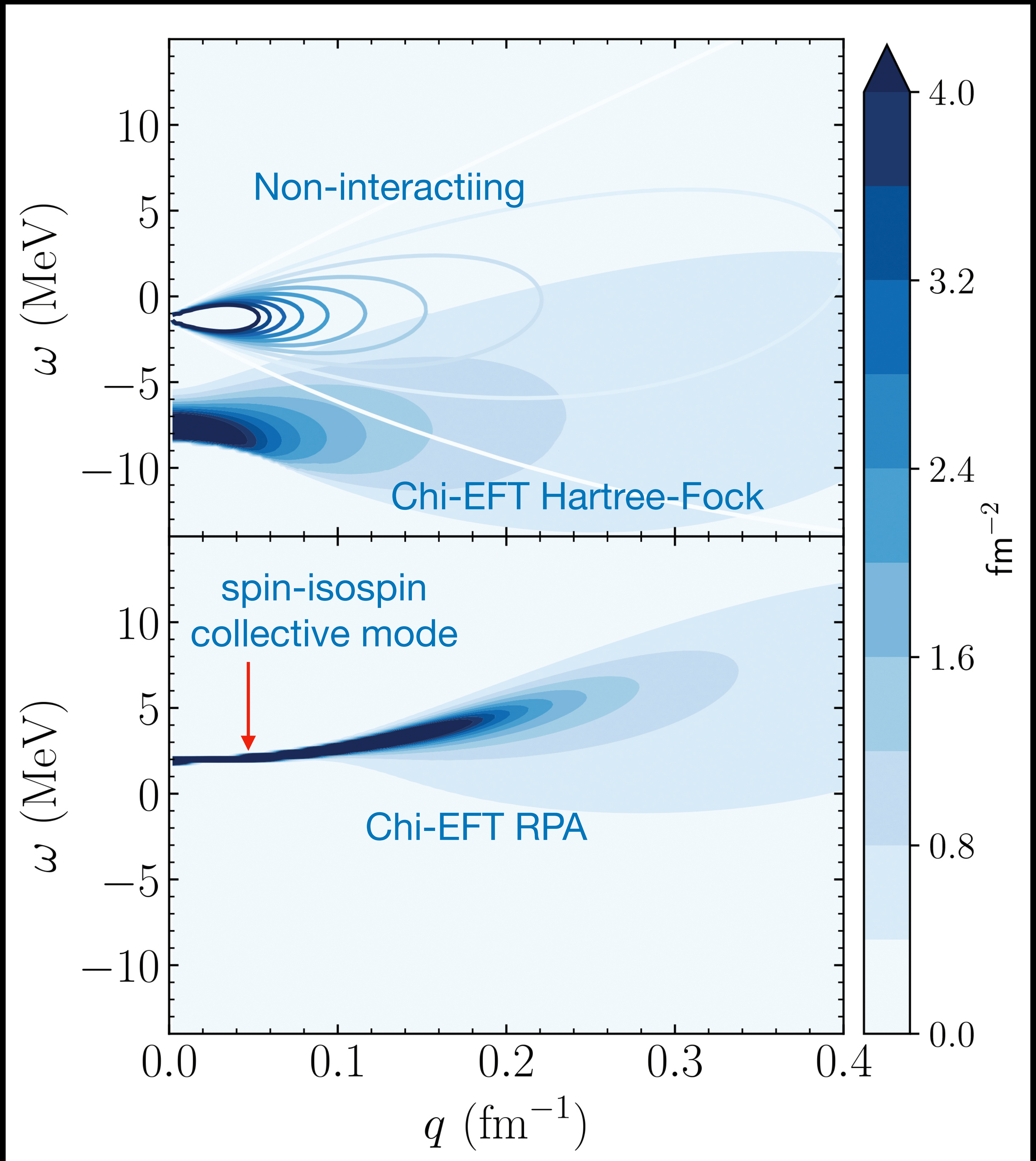
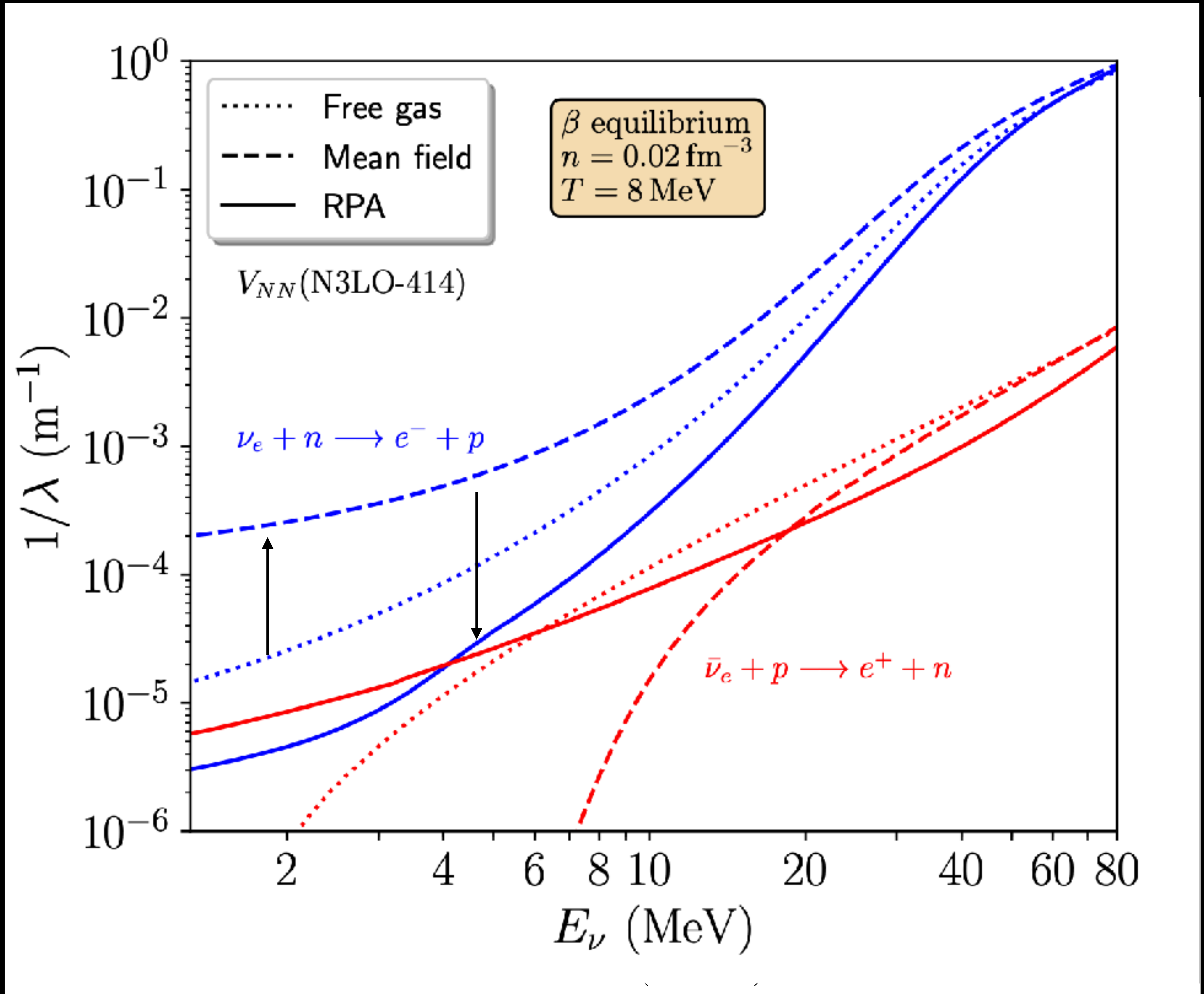


Response Function



Neutrino Cross-Section

Shin, Rrapaj, Holt, and Reddy (2023)

Closing Remarks

- Many-body theory for dense neutron-rich matter has broad implications for neutron stars and nuclear astrophysics.
- Interplay between EFT and RG inspired potentials and many-body approximations needs to be better understand and could lead to improved error estimates.
- The equation of state at moderate density encountered in the outer core is calculable and it strongly constrains neutron star structure.
- Thermal and transport propertied of the solid and superfluid inner crust plays a role in thermal evolution. We now have a qualitative understanding of entrainment and mixing.
- Response functions (density, spin, isospin, spin-isospin) all play an essential role in neutrino transport in supernovae (and neutron star mergers). We now have a qualitative understanding of the importance of correlations but reliable error estimates are lacking.