Glassy dynamics versus localization in disordered interacting systems

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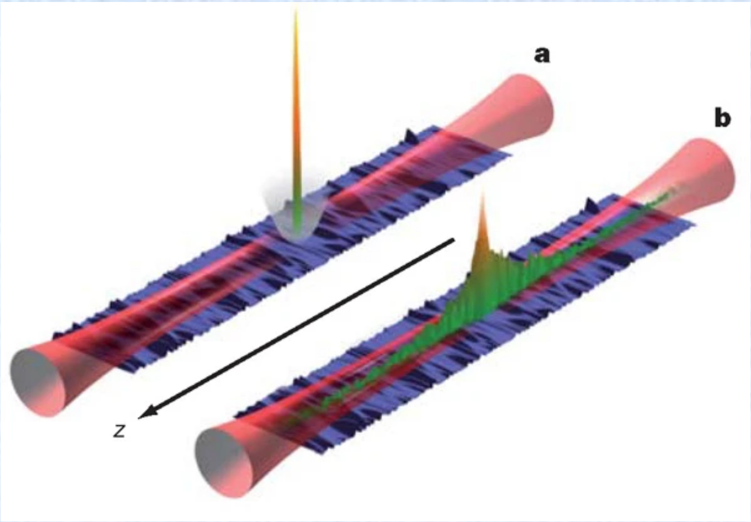
Precision many-body physics, College de France, Paris, **06/14/23**

Noninteracting systems.

Disorder leads to localization in low-dimensions.



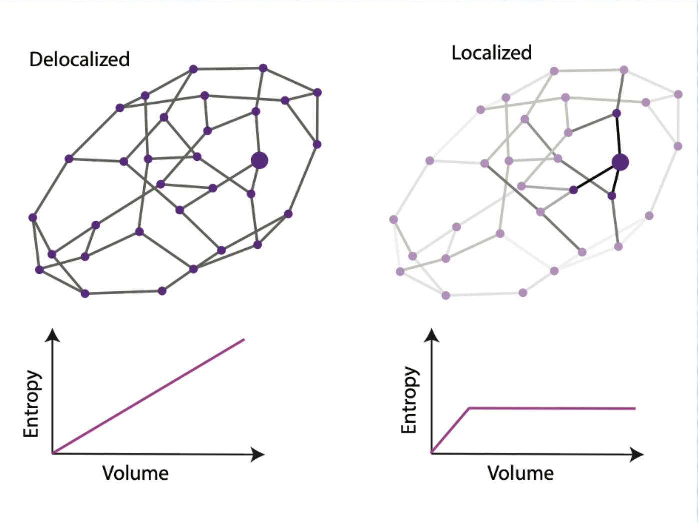
$$H = -t\sum_{\langle ij\rangle} c_i^\dagger c_j + h_j n_j, \quad h_j \in [-W, W]$$

1D and 2D - all states are localized.

Image from J. Billy,…, A. Aspect 2008 - Anderson Localization in 1D BECs

High dimensions, Bethe lattices, Random Regular graphs: phase transitions between delocalized and localized phases:

$W_c \sim \zeta, \quad$ the coordination number



MBL: localization in disordered systems are stable to short range interactions.D. Basko, I. Aleiner, B. Altshuler 2006, I. Gornyi, A. Mirlin, D. Polyakov, 2005; V. Oganesyan, D. Huse, 2007, … Onsager prize 2022.

Cartoon picture: MBL is like a Fock space localization (similar to RRG).
Sites are $1001010110, 0101010110, \ldots$

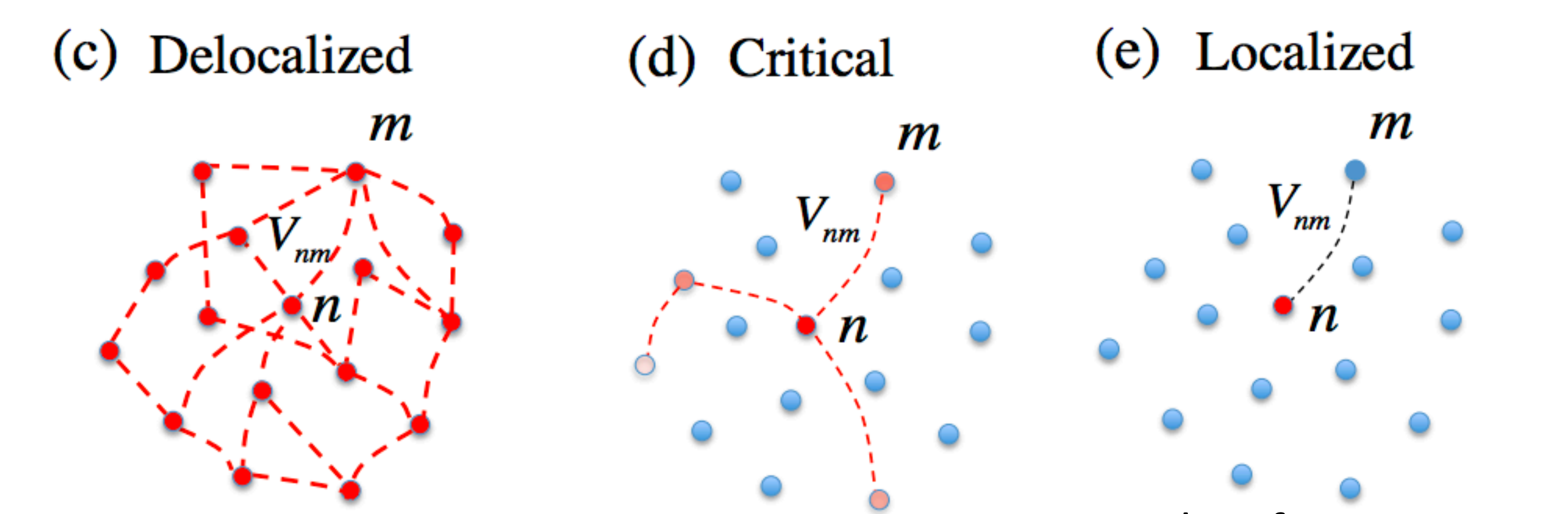


Image taken from Z. Papic talk

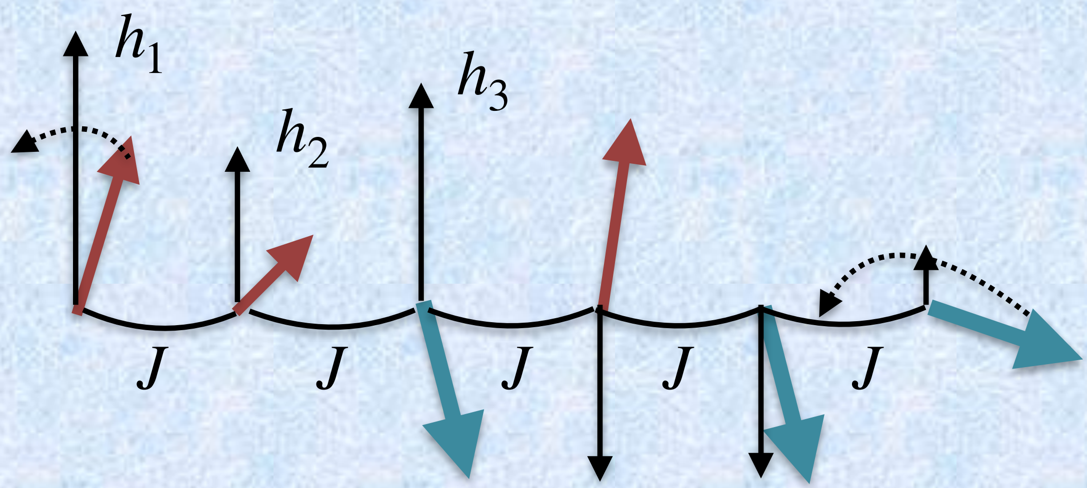
Competition between growing density of states and matrix elements. Claim: at strong disorder matrix elements decay faster than density of states grows.

Conventional mapping from particles to spins. Exact for hard core bosons in any D or fermions in 1D

The typical model

$$H = \sum_j h_j s_z^j + J \sum_{\langle ij \rangle} \vec{s}_i \cdot \vec{s}_j,$$

$$h_j \in [-W, W]$$



Classically: bad (off-resonant) transmission line. Exponentially suppressed transport $\kappa \sim \exp[-CW/J]$ (V. Oganesyan, D. Huse 2009). Intermediate time glassy subdiffusive dynamics (J. Wurtz, A.P., D. Sees, A. Sajna, 2018 +).

Quantum mechanics. Main claim: localization, no transport, no ergodicity in TD, time crystals, robust quantum memory,….

Punchline of this talk: no evidence that QM plays qualitative role at a finite temperature

Potential problems with mapping MBL to Anderson problem

Perturbation theory (from BAA paper):

$$\left[i\partial_t - \hat{H}\right] |\tilde{\Psi}_{k\alpha}(t)\rangle = \delta(t)\, \hat{c}_\alpha^\dagger |\Psi_k\rangle,$$

$$\hat{H} = \sum_\alpha \xi_\alpha\, \hat{c}_\alpha^\dagger \hat{c}_\alpha + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta}\, \hat{c}_\alpha^\dagger \hat{c}_\beta^\dagger \hat{c}_\gamma \hat{c}_\delta$$

$|\Psi_k\rangle$ is an eigenstate

Take the Fourier transform in time

$$|\tilde{\Psi}_{k\alpha}(\epsilon)\rangle = \frac{1}{\epsilon - \xi_\alpha} \left(\left|\psi_{k\alpha}^{(0)}(\epsilon)\right\rangle + \left|\psi_{k\alpha}^{(1)}(\epsilon)\right\rangle + \ldots \right).$$

$$\left|\psi_{k\alpha}^{(0)}\right\rangle = \hat{c}_\alpha^\dagger |\Psi_k\rangle.$$

$$\left|\psi_{k\alpha}^{(1)}\right\rangle = \sum_{\beta,\gamma,\delta} \frac{V_{\delta\gamma\beta\alpha}}{\epsilon - \Xi_{\gamma\delta}^\beta}\, \hat{c}_\delta^\dagger \hat{c}_\gamma^\dagger \hat{c}_\beta |\Psi_k\rangle,$$

$$\Xi_{\gamma\delta}^\beta = \xi_\gamma + \xi_\delta - \xi_\beta$$

$$\begin{aligned} \left|\psi_{k\alpha}^{(2)}\right\rangle = &\sum_{\alpha_1\beta_1} \sum_{\beta,\gamma,\delta} \frac{V_{\alpha_1\beta_1\gamma\delta}}{\epsilon - \Xi_{\alpha_1\beta_1}^\beta} \frac{V_{\delta\gamma\beta\alpha}}{\epsilon - \Xi_{\gamma\delta}^\beta}\, \hat{c}_{\alpha_1}^\dagger \hat{c}_{\beta_1}^\dagger \hat{c}_\beta |\Psi_k\rangle \\ &+ \sum_{\alpha_1,\beta_1,\gamma_1} \sum_{\beta,\gamma,\delta} \frac{2V_{\alpha_1\beta_1\gamma_1\delta}}{\epsilon - \Xi_{\gamma\alpha_1\beta_1}^{\beta\gamma_1}} \frac{V_{\delta\gamma\beta\alpha}}{\epsilon - \Xi_{\gamma\delta}^\beta}\, \hat{c}_{\alpha_1}^\dagger \hat{c}_{\beta_1}^\dagger \hat{c}_\gamma^\dagger \hat{c}_\beta \hat{c}_{\gamma_1} |\Psi_k\rangle \\ &+ \sum_{\alpha_1,\gamma_1,\delta_1} \sum_{\beta,\gamma,\delta} \frac{V_{\alpha_1\beta\gamma_1\delta_1}}{\epsilon - \Xi_{\gamma\delta\alpha_1}^{\gamma_1\delta_1}} \frac{V_{\delta\gamma\beta\alpha}}{\epsilon - \Xi_{\gamma\delta}^\beta}\, \hat{c}_{\alpha_1}^\dagger \hat{c}_\delta^\dagger \hat{c}_\gamma^\dagger \hat{c}_{\beta_1} \hat{c}_{\gamma_1} |\Psi_k\rangle, \end{aligned}$$

They also show $|\psi_{k\alpha}^{(3)}\rangle$

Problems with this expansion:

1. Too many terms in the perturbative expansion $N_n \sim \min(n!, V^n)$
2. Have to deal with small denominators

Solution:

1) argue that $N_n \sim K^n, \quad K \sim T/\delta_\xi$, δ_ξ is the single-particle level spacing within ξ^d; lattices $K \sim$ coordination number

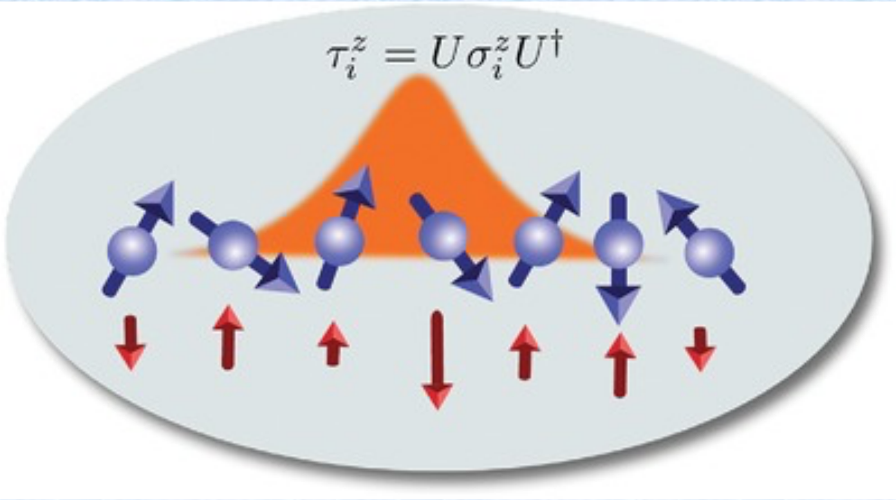
2) Argue that the problem maps to an Anderson localization problem with this *K*.

Developed a more detailed formalism to justify these assumptions.

A similar argument by A. Mirlin et. al. Real connectivity is $K \sim V$ but most terms lead to large denominators.

No proof or numerical test of this statement was ever given (to our knowledge). Many theory papers site this argument as an evidence that the factorial growth is absent.

LIOM (local integrals of motion) picture of MBL. M. Serbyn, Z. Papi´c, D. A. Abanin (2013); V. Oganesyan and D. Huse (2013).

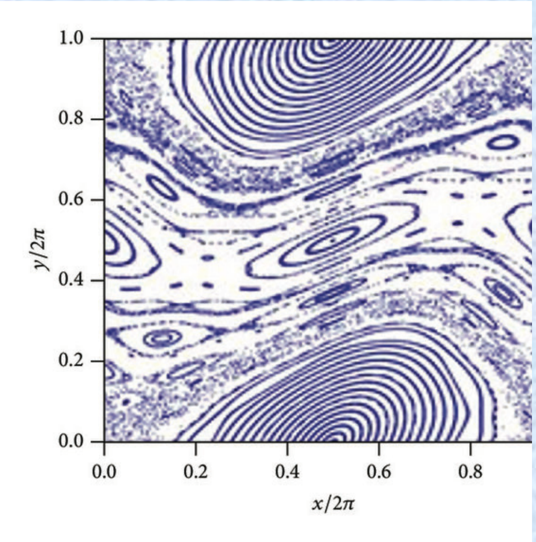


In MBL systems there exist an extensive set of localized conserved operators τ_i^z (dressed localized spins/particles).

The argument is a bit circular if there is MBL, there must be such states and vice versa.

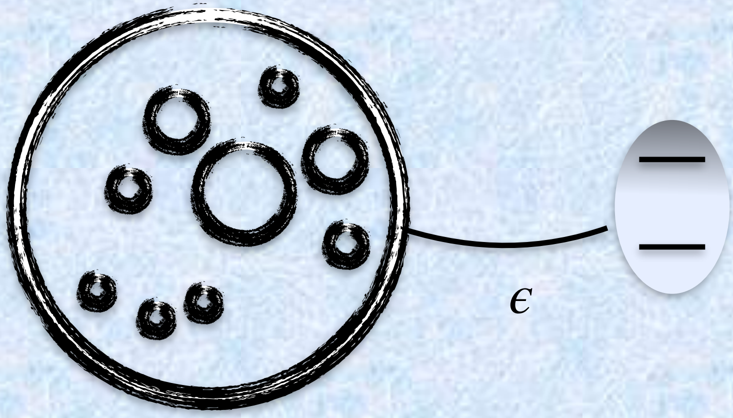
In retrospect this idea came from insufficient understanding of classical chaos and a difference between localization in phase space and integrability.

At best LIOMs are eigenstate-dependent operators, i.e. they do not exist in TD limit.



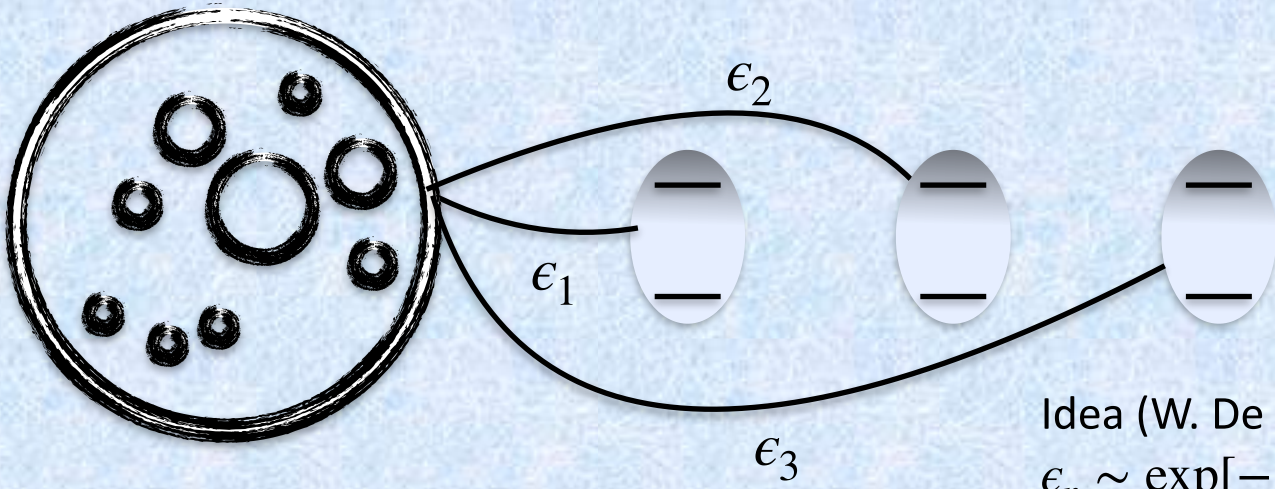
Chirikov map (kicked rotor). KAM guarantees localization but there are no LIOMs.

Problems with initial arguments. The avalanche instability.



In TD limit there is always a nonzero probability of an ergodic (ETH) island.

If $\epsilon > \exp[-S/2] \sim \exp[-N\log(2)/2]$ the impurity will hybridize with the ETH island absorbing it.



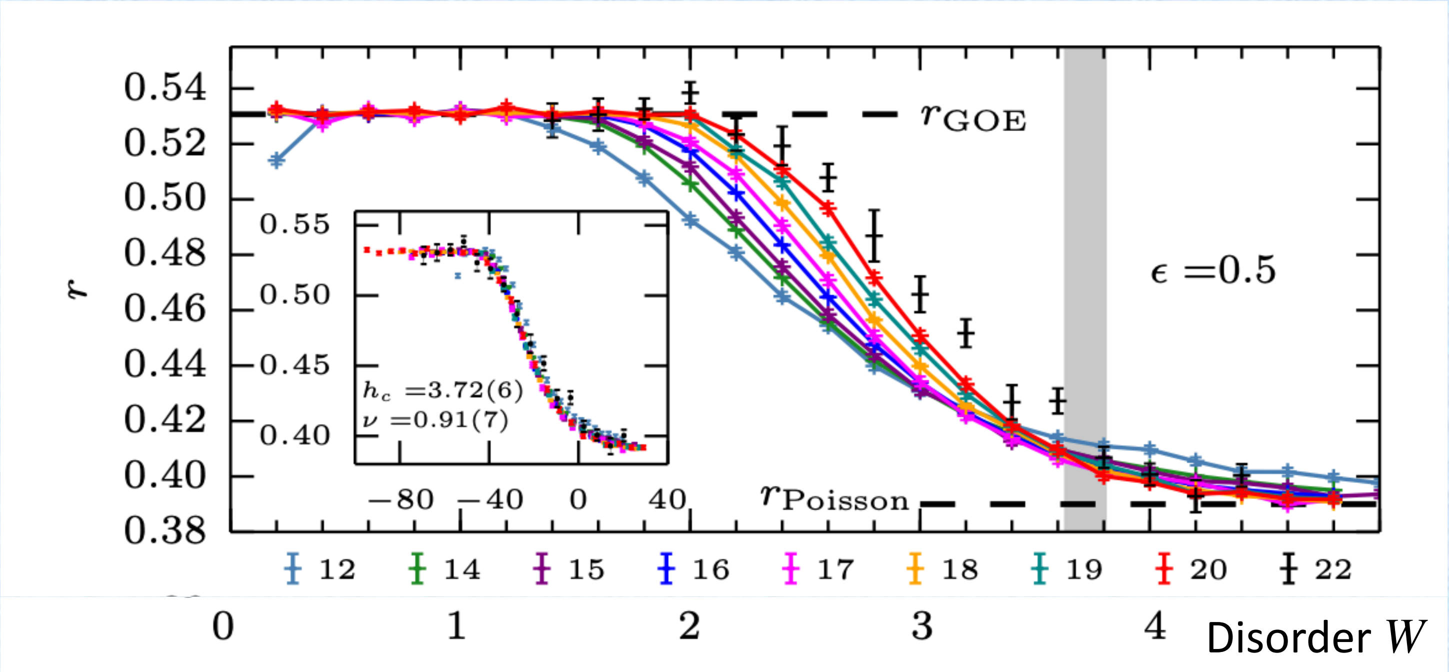
Idea (W. De Roeck, F. Huveneers 2016):
$\epsilon_n \sim \exp[-na/\zeta]$, $S = n^d \log(2)$,

Localization cannot exist in d>1. Localization is a UV effect: $\epsilon_n < \exp[-S/2] \;\rightarrow\; \zeta < 2a/\log(2)$.
Either this argument or the original BAA argument, which has no UV or finite d physics, must contain an error. What is ζ - the localization length?

Extensive state of the art numerical work since 2007. “Strong” support for MBL

Standard Model for MBL amenable to numerics: disordered Heisenberg (XXZ) chain:

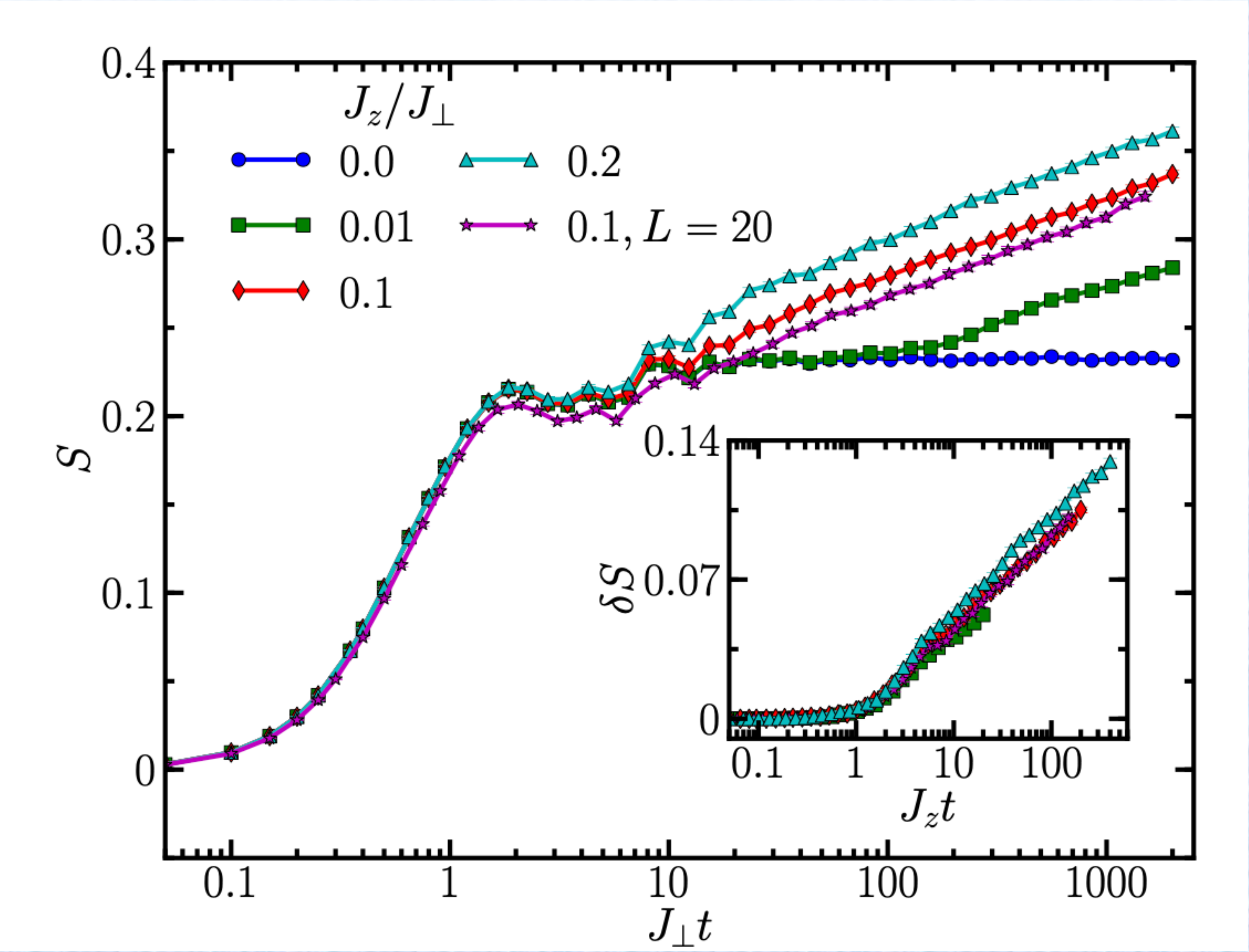
$$H = \sum \mathbf{S}_j \mathbf{S}_{j+1} + h_j S_j^z, \quad h_j \in [-W, W]$$



D. J. Luitz, N, Laflorencie, and F. Alet, PRB 2015. Critical disorder $W_c \approx 3.72$ from level statistics

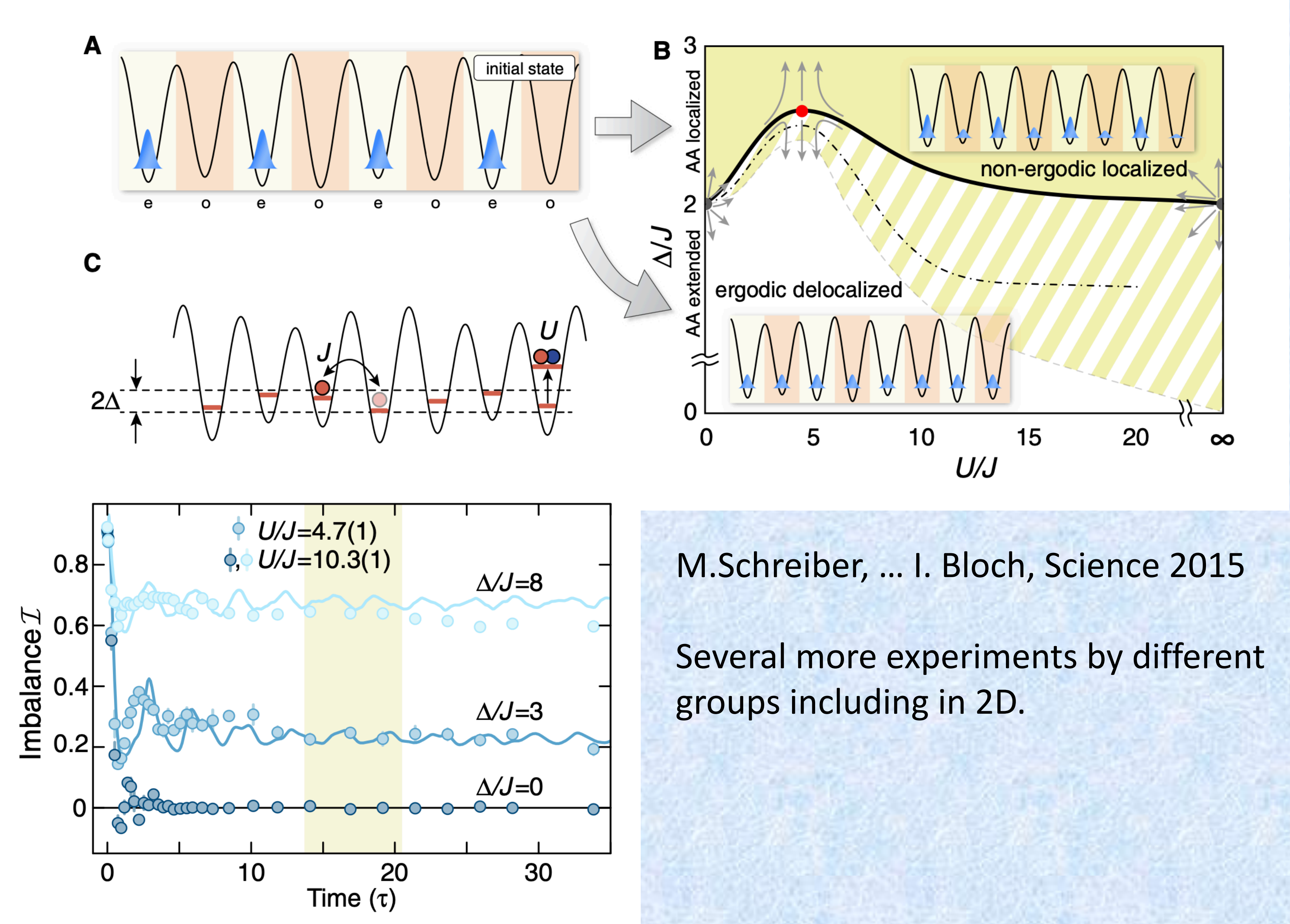
Many other papers “confirming” MBL transition near $W_c \approx 3.6$.
Analytic “proof” of stability of MBL phase by J. Imbrie with few extra assumptions (limited level attractions) for stability of MBL phase at weak interactions (2016).

A “key” difference between the Anderson localization and MBL: logarithmic entanglement growth (in simple words growth of transverse correlations)



J. H. Bardarson, F. Pollmann, and J. E. Moore (2012)

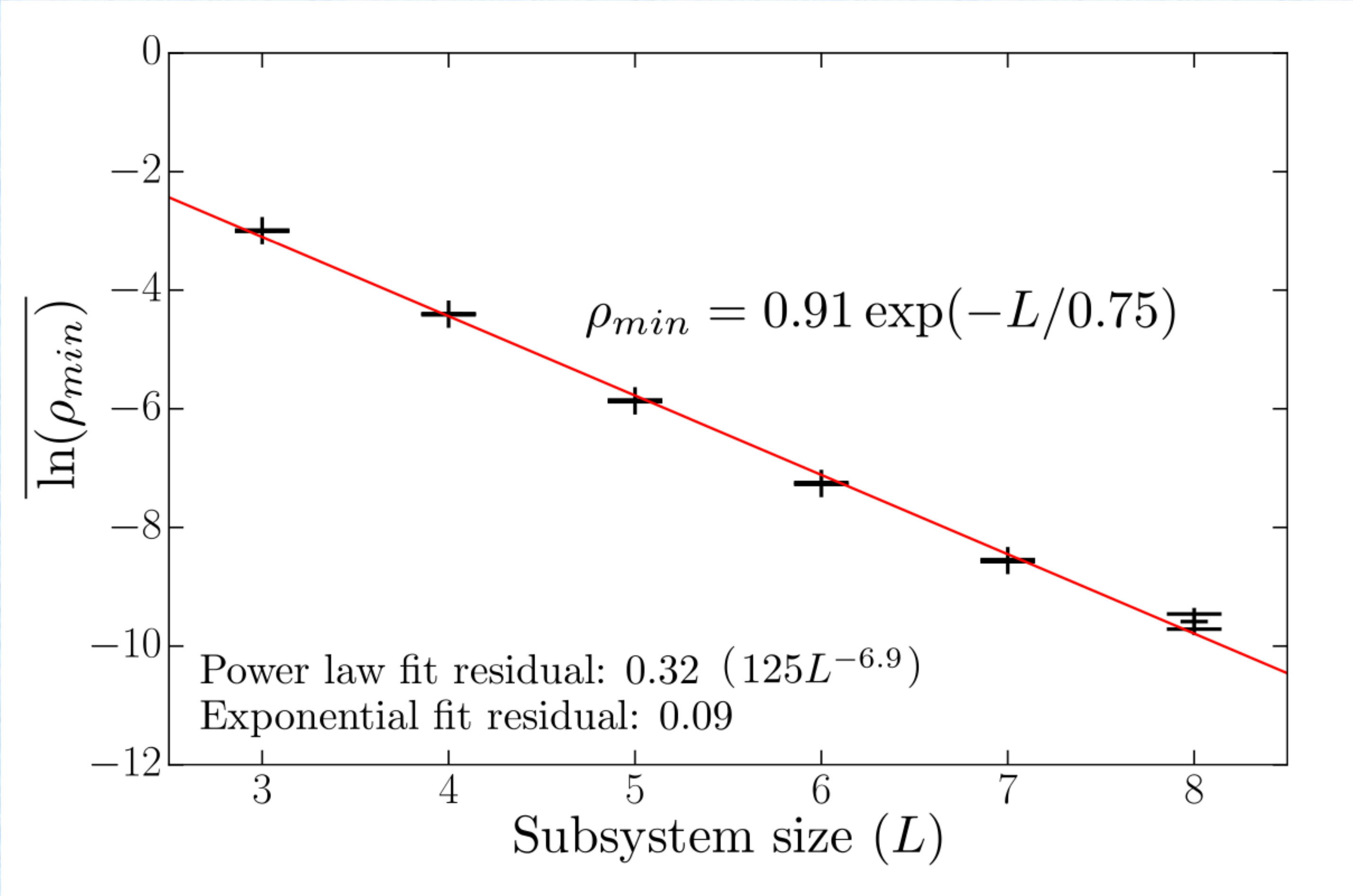
Beautiful experiments in cold atoms: interacting fermions with quasi-periodic incommensurate potential



M.Schreiber, … I. Bloch, Science 2015

Several more experiments by different groups including in 2D.

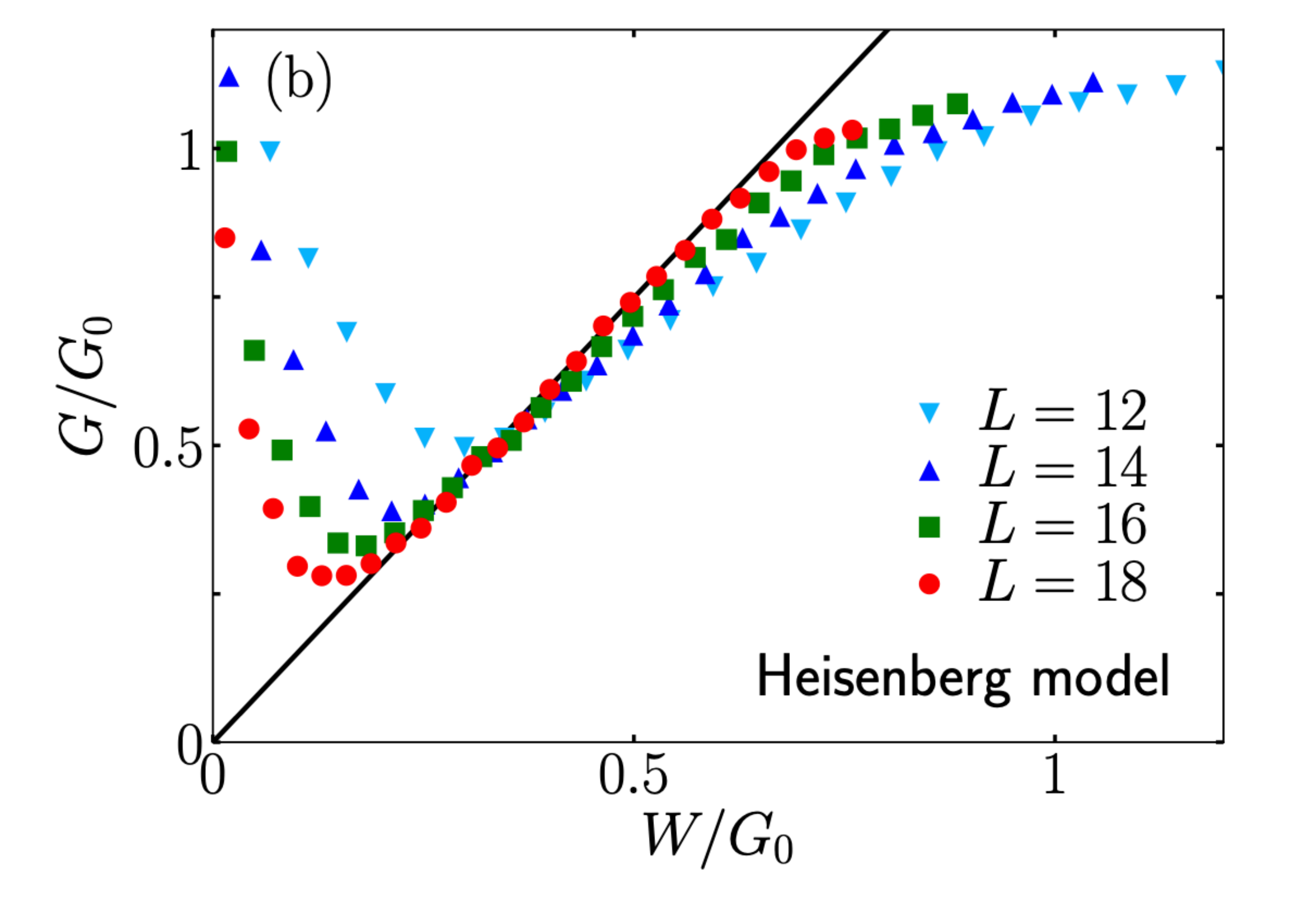
Numerical confirmation of LIOM theory: exponential scaling of the slowest operator with the system size (vs. expected diffusive in ergodic systems)



“Exponential scaling” of the slowest operator in the MBL phase (T O'Brien, D Abanin, G. Vidal, and Z. Papic, PRB, 2016)

Situation is largely clear by 2018. Only details are missing like correct critical exponents (RG schemes predicted KT), sub-diffusion exponents on the ergodic side, reconciliation of the avalanche instability and BAA ....

The Thouless time extracted from the spectral form factor

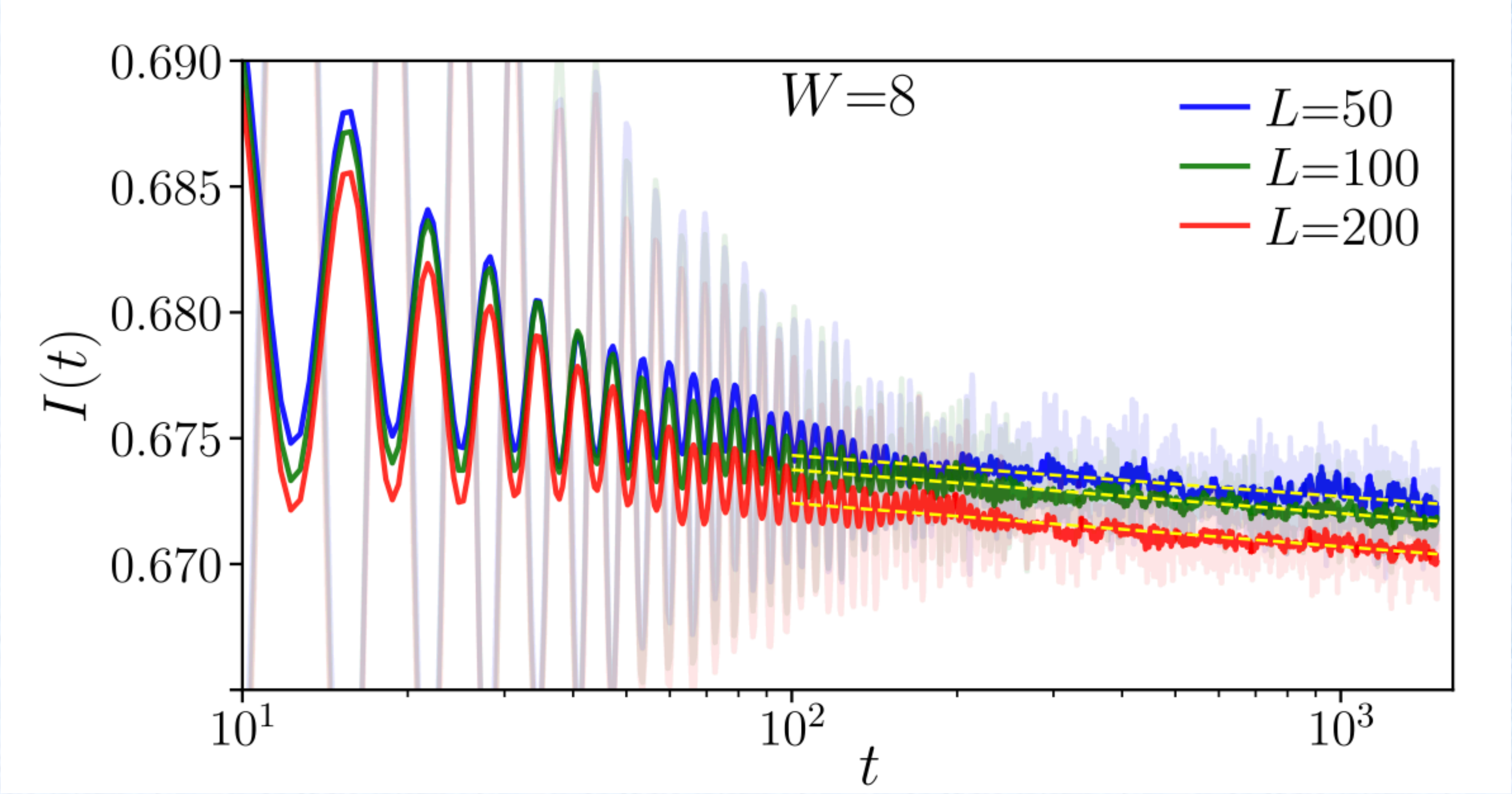


J. Suntajs, J. Bonca, Tomaz Prosen, and L. Vidmar (2019).

No signature of transition with increasing L. Maybe the elephant is not there!

Further numerical tests showed that previous numerical extrapolations were grossly wrong.

Imbalance decay, similar to the experiments in the group of I. Bloch (2016).

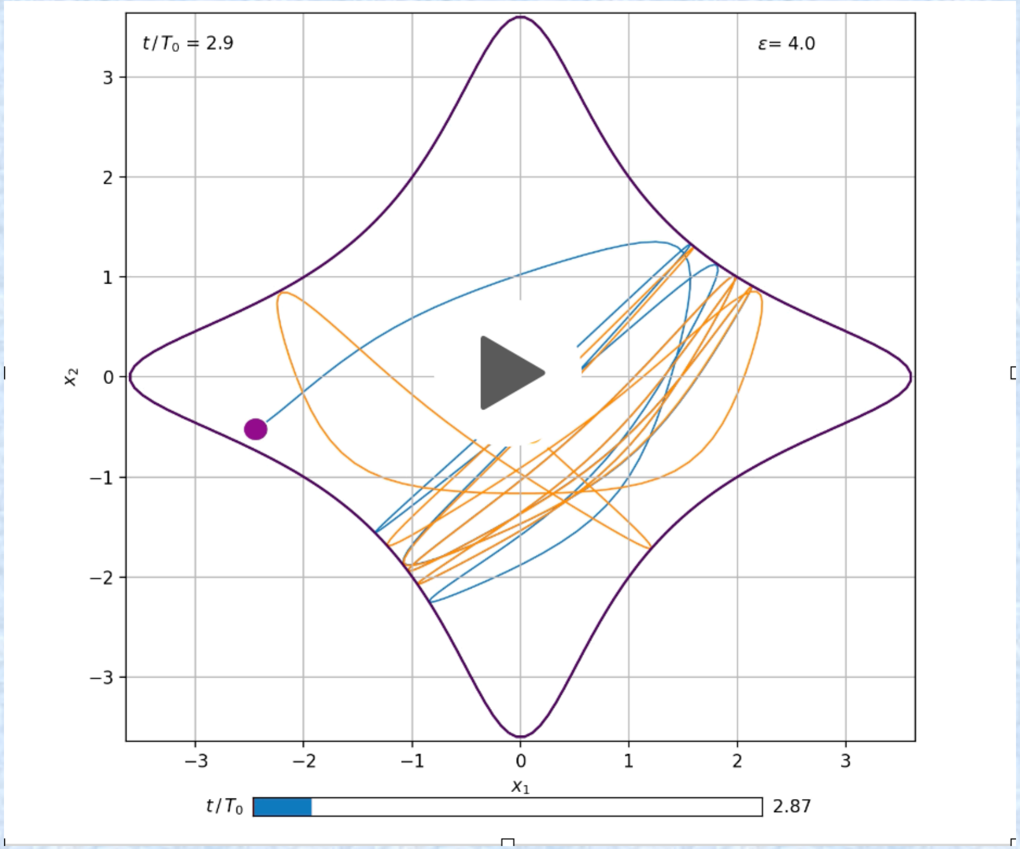


P. Sierant, and J. Zakrzewski, Can we observe the many-body localization?, 2021. Rule out transition for $W < 10$. This disorder was thought to be deep inside MBL phase

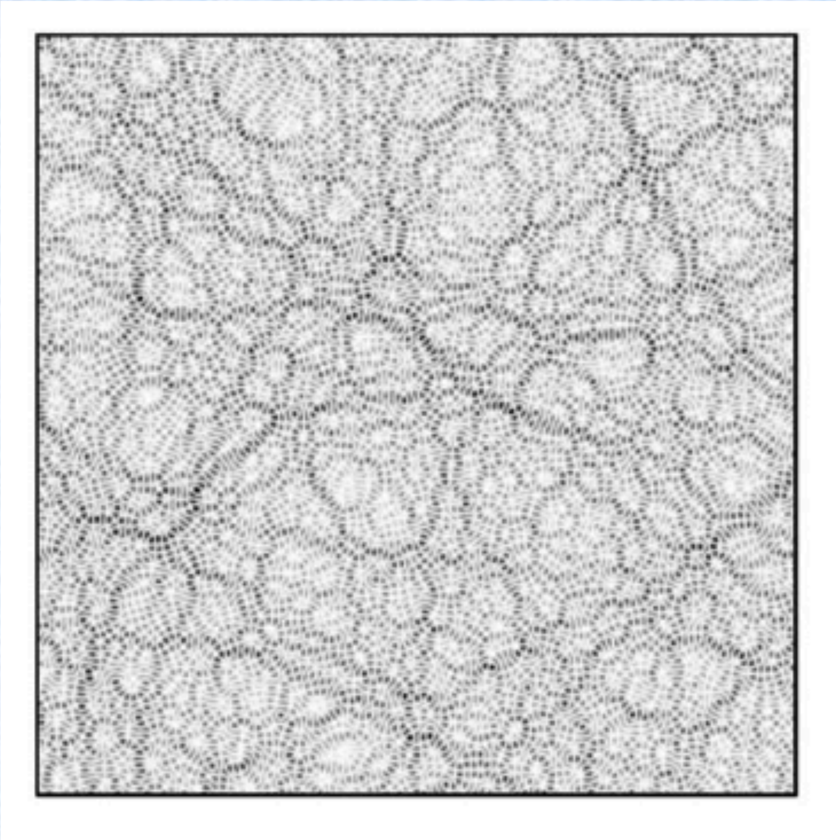
Thouless time in the ergodic phase (before critical slowing down): $t_{\mathrm{Th}} \gtrsim \exp[35]$

Idea of defining chaos separate from ergodicity: use sensitivity of eigenstates as a measure of chaos.

Classical systems: very fragile trajectories



Quantum systems: very fragile eigenstates



$$|\delta\vec{x}(t)| \sim |\delta\vec{x}_0| \exp[\lambda t], \quad \lambda$$ is the Lyapunov exponent

Quantum geometric tensor (fidelity susceptibility, Fisher information)

$$\chi_\lambda = \langle n | \overleftarrow{\partial_\lambda} \partial_\lambda | n \rangle - |\langle n | \partial_\lambda | n \rangle|^2 \qquad \chi_\lambda = \frac{1}{D} \sum_{m \neq n} \frac{|\langle n | \partial_\lambda H | m \rangle|^2}{(E_m - E_n)^2} = \int d\omega \frac{\Phi_\lambda(\omega)}{\omega^2}$$

$$\Phi_\lambda(\omega) = \frac{1}{2\pi} \int dt\, e^{i\omega t}\, \overline{\langle n | \partial_\lambda H(t) \partial_\lambda H(0) | n \rangle_c} \sim \epsilon''(\omega) \sim \Gamma_{FGR}(\omega)$$

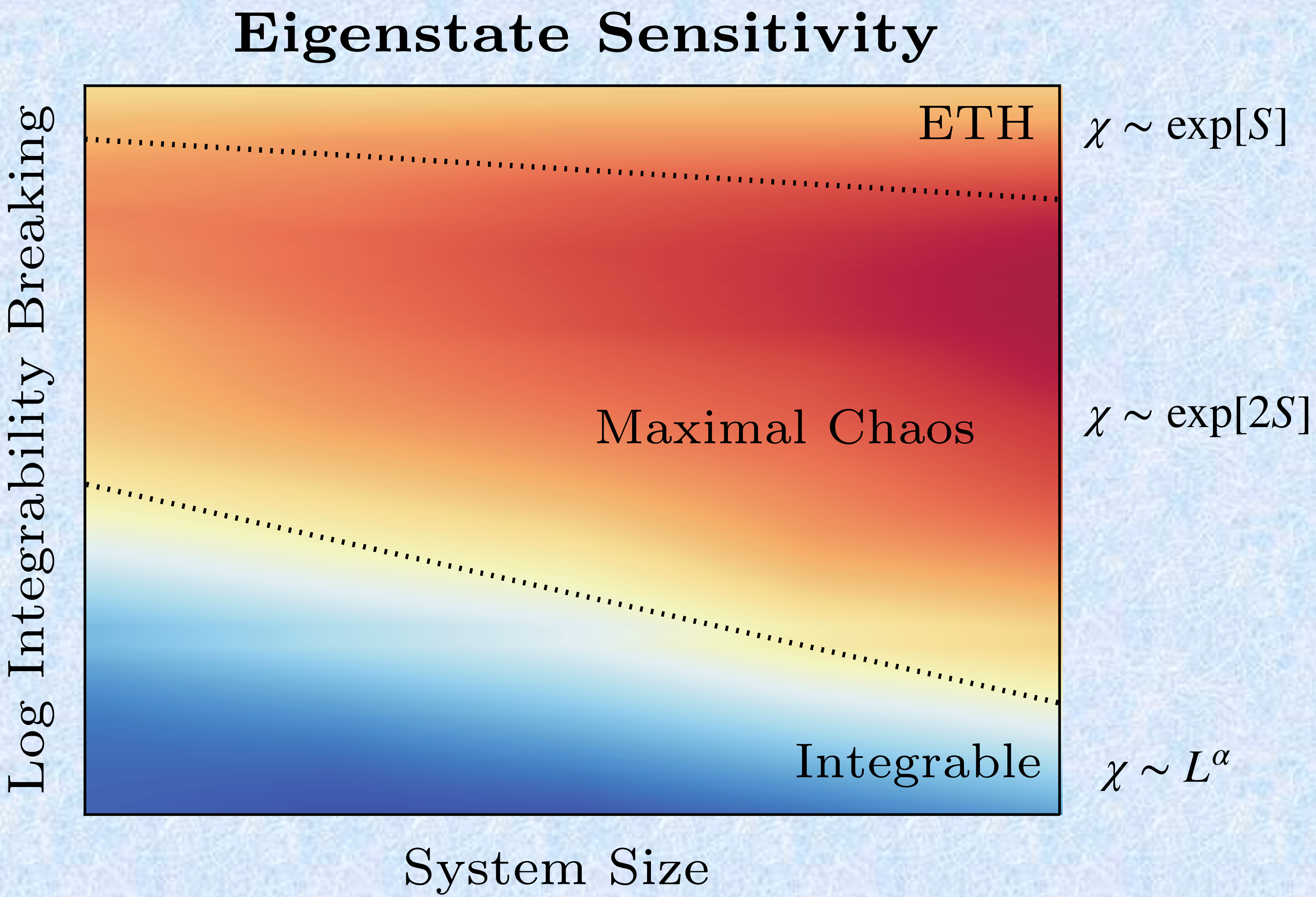
RMT, diffusion, kinetic theories,… $\rightarrow \Phi_\lambda(\omega) = \text{const}, \quad \omega < \omega_{\text{Th}} = D/L^2.$

Conclusion: $\chi_\lambda \sim \int \frac{d\omega}{\omega^2} \sim \frac{1}{\omega_H} \sim \exp[S]$

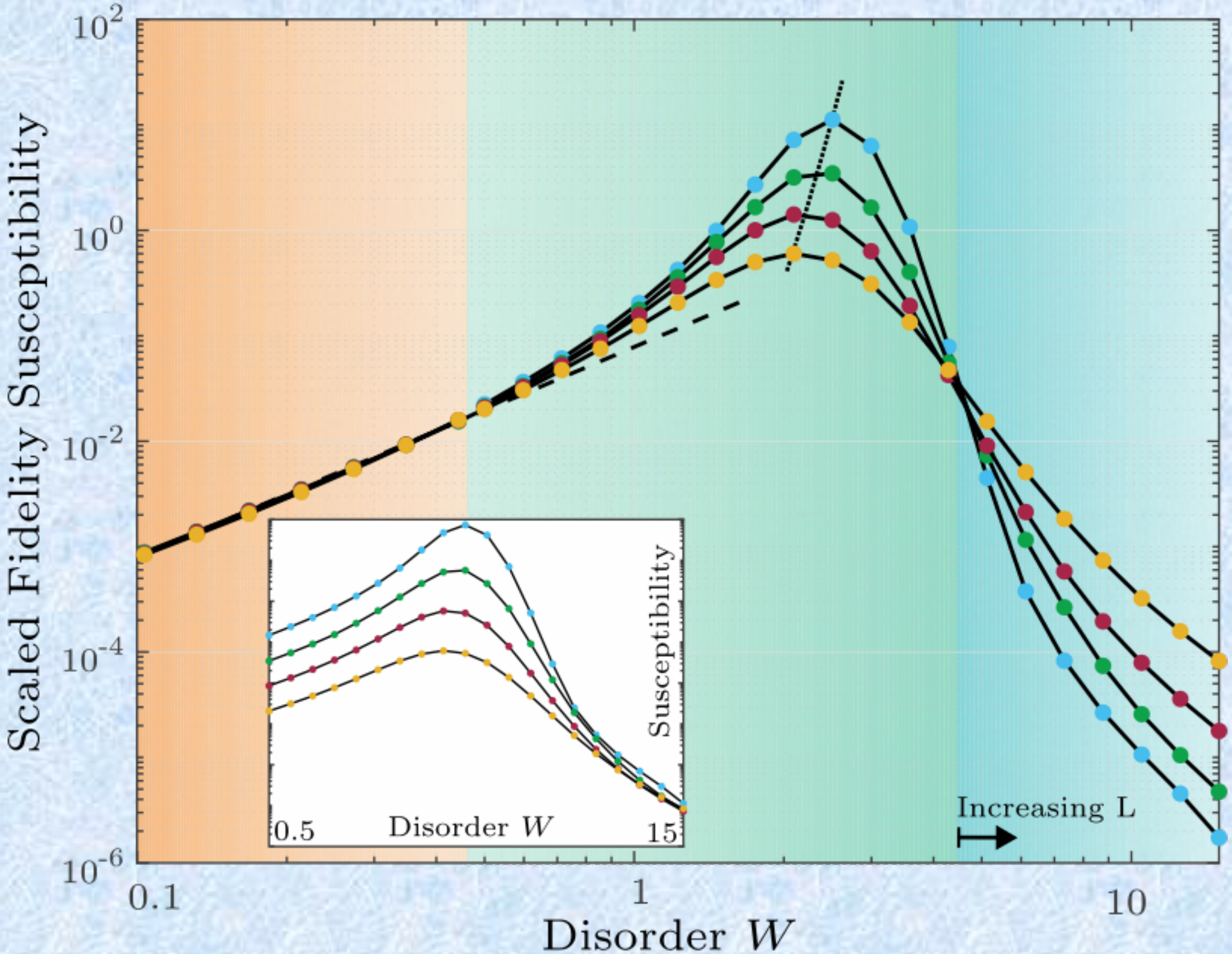
The problem of small denominators leads to (exponentially) high in system size sensitivity of eigenstates. Expect finite χ_λ in integrable systems.

Classically χ_λ has a meaning of the norm of the generator of the complexity preserving canonical transformation.

Qualitative chaos phase diagram for generic models



Strong disorder: MBL seems unstable (D. Sels, A.P. 2020,)



Linear drift of the maximum of fidelity with the system size. Consistent with J.Suntajs et. al. 2019.

$t_{\rm Th} \sim \exp[3.5W]$

Transition occurs when

$t_{\rm Th} \sim 1/\omega_H \sim 2^L$

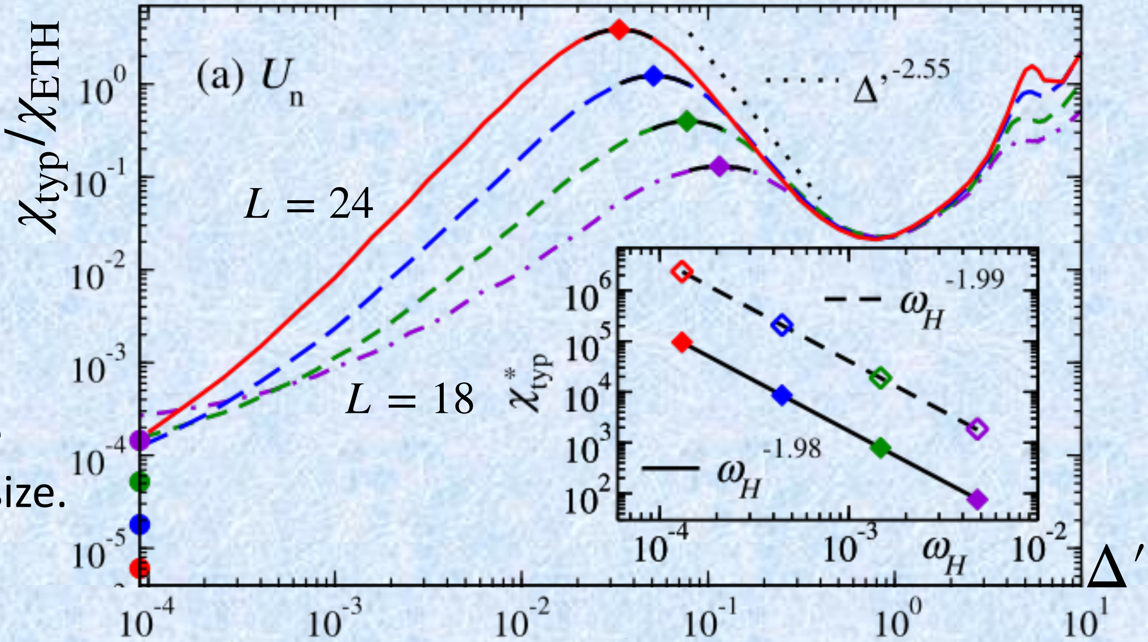
Comparison of clean and disordered systems

Clean:

$$\hat{H}_{\rm cln} = \sum_{i=1}^{L}\left[\frac{J}{2}\left(\hat{S}_i^+\hat{S}_{i+1}^- + {\rm H.c.}\right) + \Delta \hat{S}_i^z\hat{S}_{i+1}^z + \Delta'\hat{S}_i^z\hat{S}_{i+2}^z\right]$$

$$J=\sqrt{2}, \qquad \Delta = (\sqrt{5}+1)/4, \qquad \Delta' \in [10^{-4}, 10^{1}]$$

Strong (nearly exponential) drift of the ergodicity threshold with the system size.

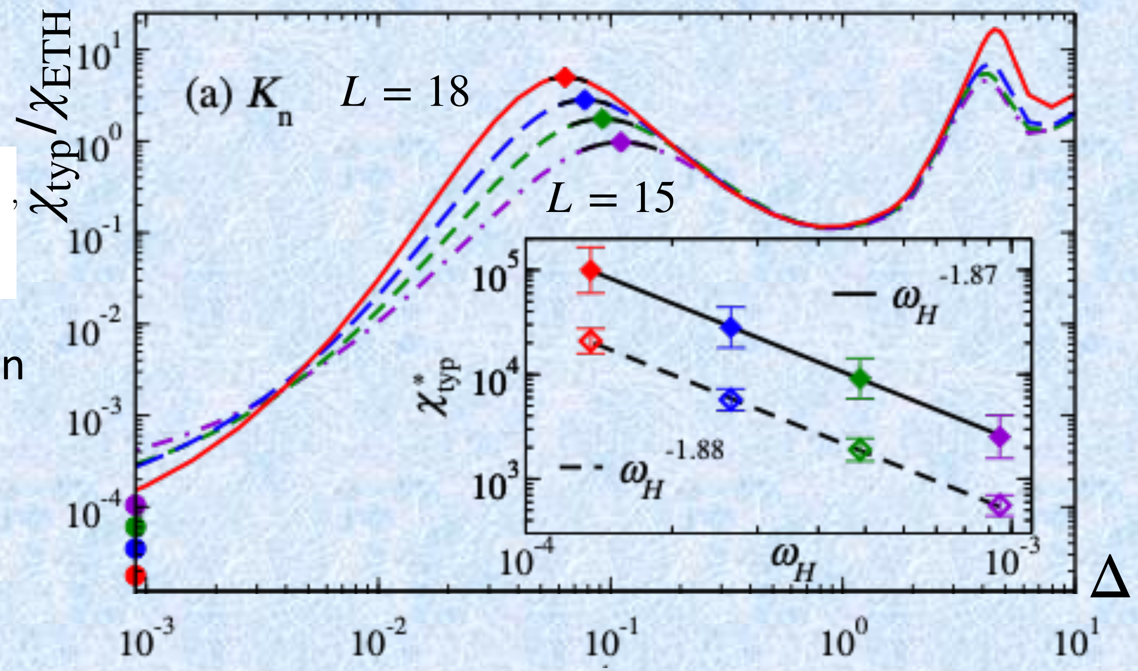


Anderson insulator with interactions

$$\hat{H}_{\rm dsr} = \sum_{i=1}^{L}\left[\frac{J}{2}\left(\hat{S}_i^+\hat{S}_{i-1}^- + H.c.\right) + h_i\hat{S}_i^z + \Delta \hat{S}_i^z\hat{S}_{i+1}^z\right],$$

$$J=\sqrt{2},\ h_i \in [-0.81, 0.81],\ \Delta \in [10^{-3}, 10]$$

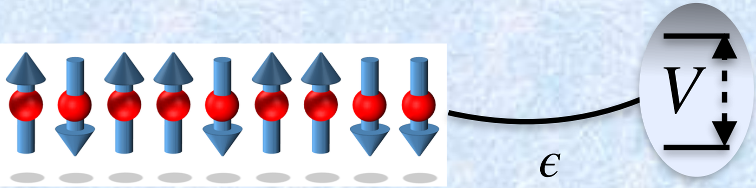
No qualitative difference between clean and disordered models. Very strong approach to ETH with the system size.



Highly nontrivial (strongly chaotic) localized side

Mechanism of delocalization: drift of the correlation length.

$$H = VS_z^0 + \epsilon H_{\rm int} + H_{\rm bath}$$



Idea: find recursively LIOM: $[Q, H] = 0$

Use perturbation theory (Birkhoff normal form). First order in ϵ and all orders in $1/V$. Work in the TD limit. Make no assumptions about eigenstates, gaps,…

$$Q = S_z^0 + \frac{1}{V} q_1(\epsilon) + \frac{1}{V^2} q_2(\epsilon) + \ldots,$$

Can solve analytically in the linear order in ϵ

$$Q = S_0^z + \frac{\epsilon}{V} H_{\rm int} + \frac{\epsilon}{V^2} \sigma_0^z [H_{\rm bath}, H_{\rm int}] + \frac{\epsilon}{V^3} [H_{\rm bath}, [H_{\rm bath}, H_{\rm int}]] + \ldots$$

This is an expansion of the conserved charge (and the AGP) in the Krylov space.

Stop at N-th order:

$$[Q_N, H] = \frac{\epsilon}{V^{2N+1}} \mathscr{L}^{2N+1} H_{\rm int} \qquad \Gamma_N^2 = \|i[Q_N, H]\|^2 \approx \epsilon^2 \frac{\|\mathscr{L}^{2N+1} H_{\rm int}\|^2}{V^{4N+2}}$$

This is a convergent procedure for any finite-dimensional matrices.

Generic chaotic models (no selection rules): D. E. Parker et. al. 2019; A. Avdoshkin, A. Dymarsky 2020; X. Cao 2021, … Disorder plays no role!

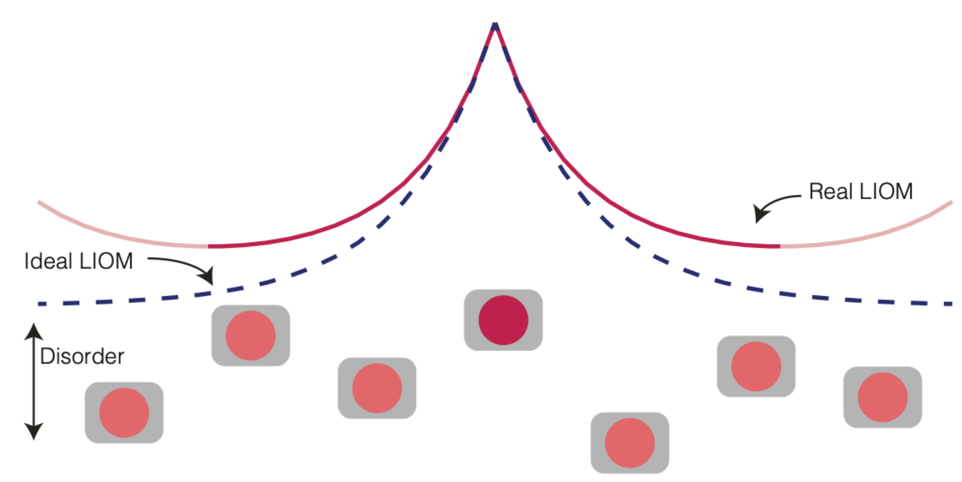
$$\|\mathscr{L}^k O\|^2 \approx \left(\frac{2k}{e\tau}\right)^{2k}, \quad \Rightarrow \quad \Gamma_N \sim \frac{N!}{(\tau V)^N}$$

Minimal decay rate when

$$N^* \sim \tau V \log(V\tau) \quad \leftrightarrow$$

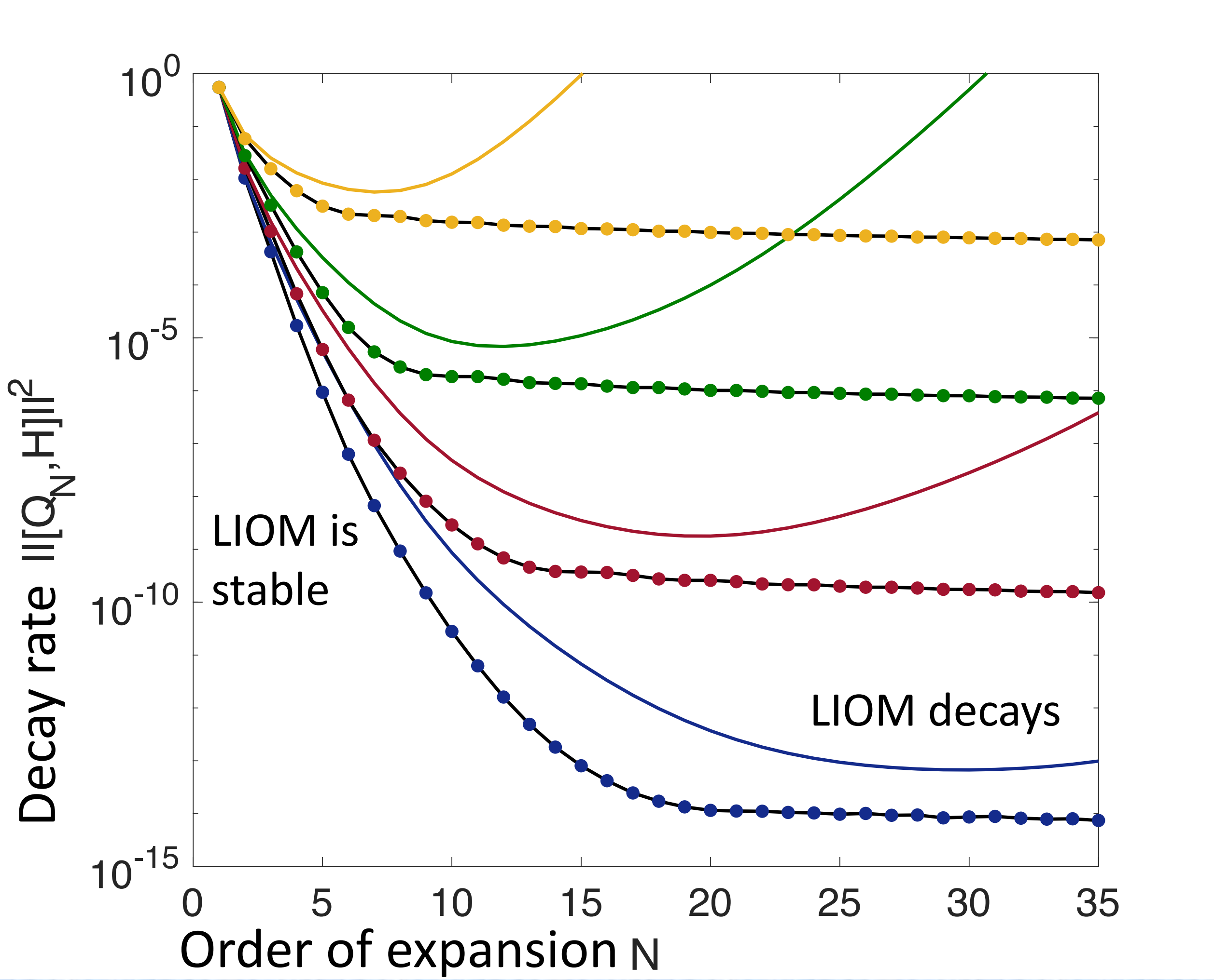
$$\Gamma_{N^*}^2 \sim \exp[-\tau V \log V\tau]$$

Divergence is due to virtual UV processes! LIOM correlation length flows with the distance, no exponential tails!



Can do the variational minimization in the Krylov space instead of perturbative approach

$$Q_{\rm var} = S_0^z + \alpha_0 H_{\rm int} + \alpha_1 \sigma_0^z [H_{\rm bath}, H_{\rm int}] + \alpha_2 [H_{\rm bath}, [H_{\rm bath}, H_{\rm int}]] + \ldots, \quad \|[Q_{\rm var}, H]\| = \min$$

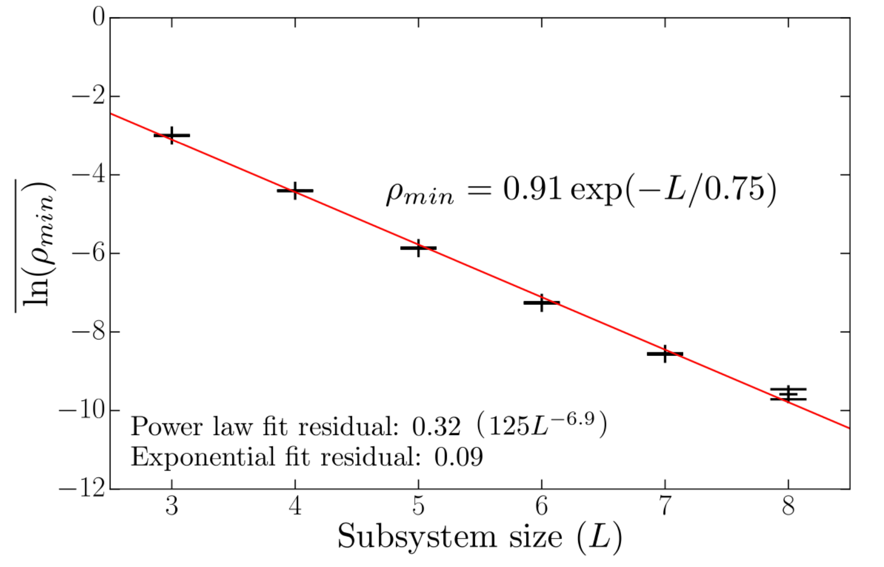


The variational approach agrees with perturbative at small N and then crossovers to a very slow asymptotic regime.

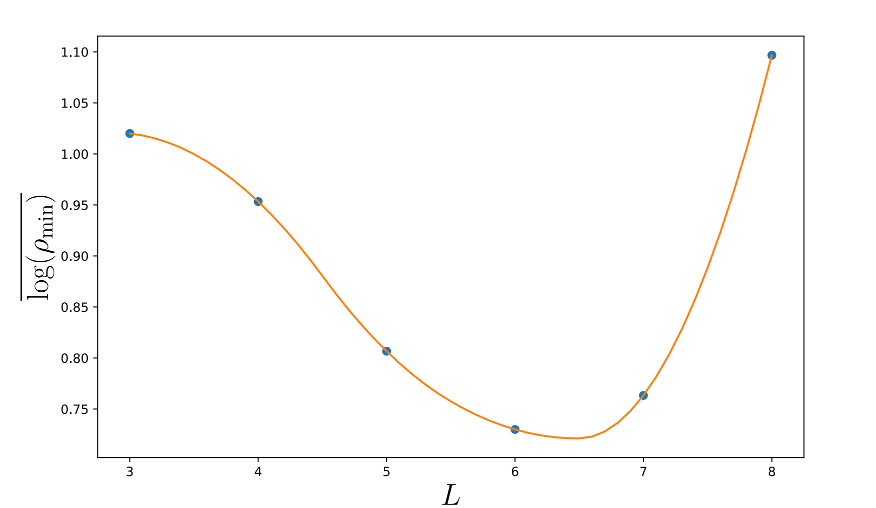
Many nearly degenerate solutions in the slow regime.

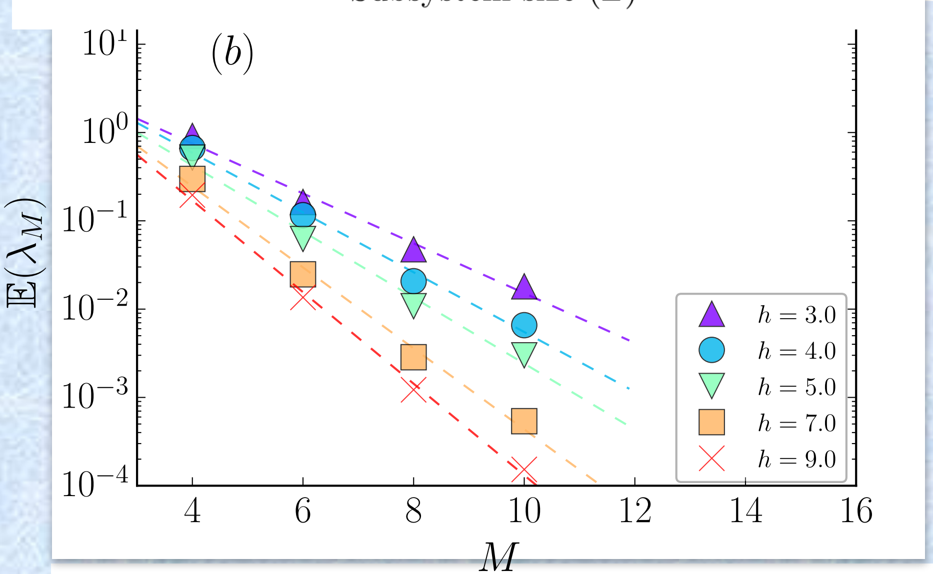
What about earlier studies showing exponential L-bits

“Exponential scaling” of the slowest operator in the MBL phase (T O'Brien, D Abanin, G. Vidal, and Z. Papic, PRB, 2016)

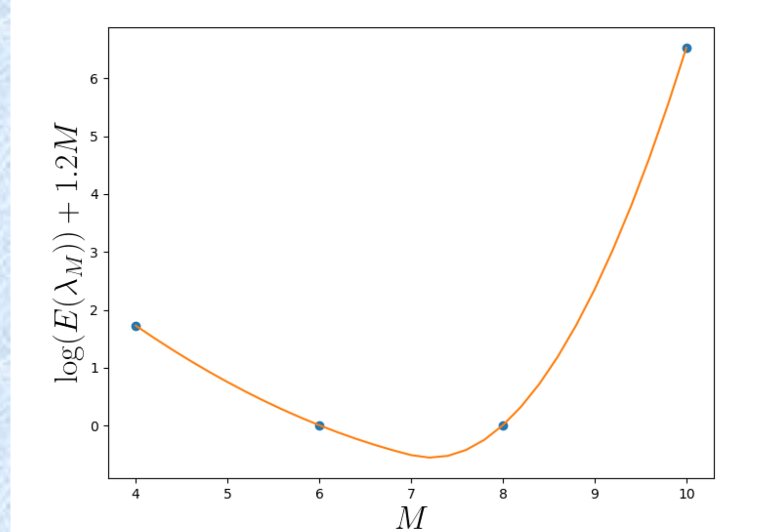


Digitized data with subtracted slope. Strong drift of slope. MBL is unstable using arguments by W. De Roeck, F. Huveneers (2015)



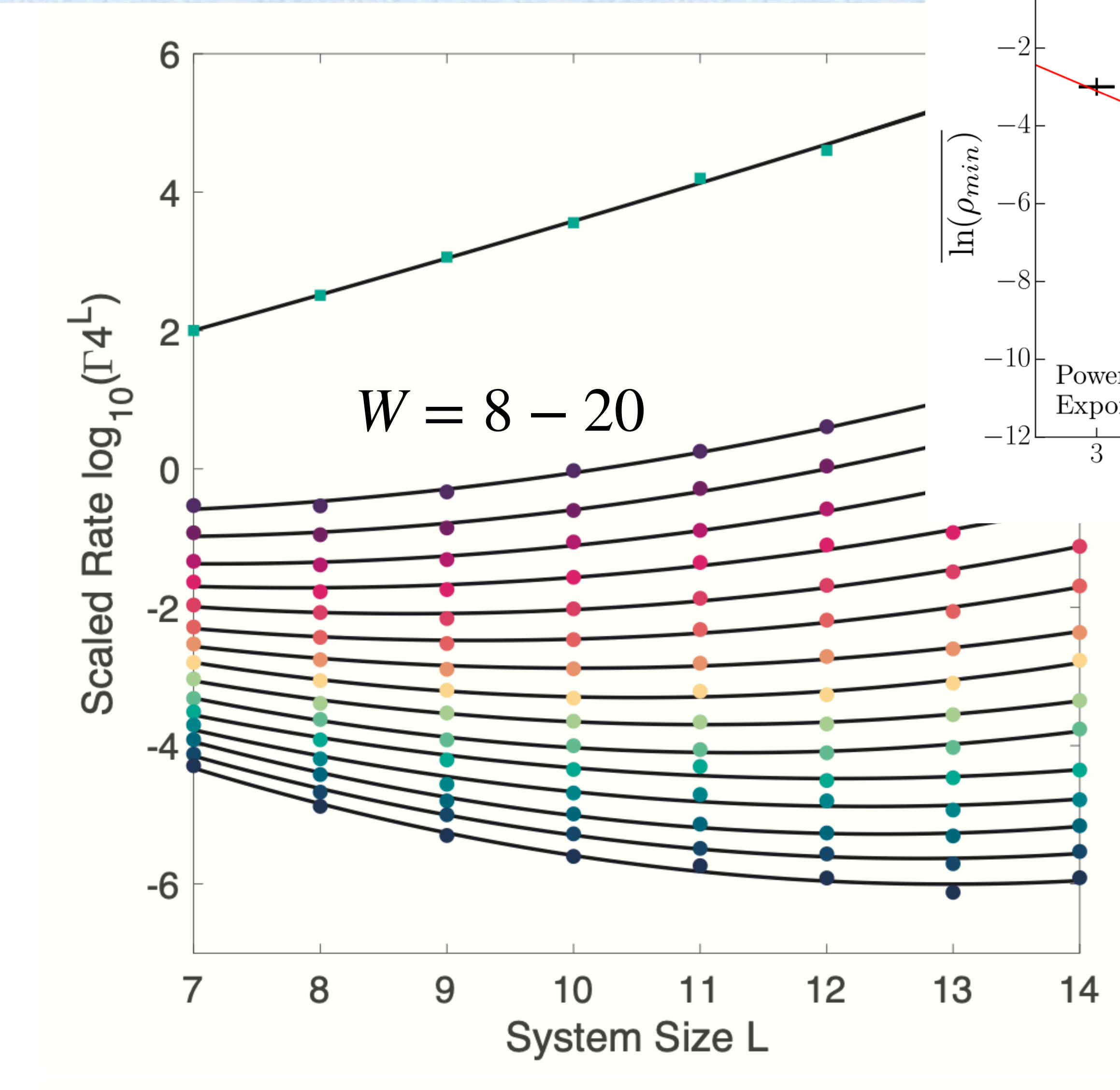


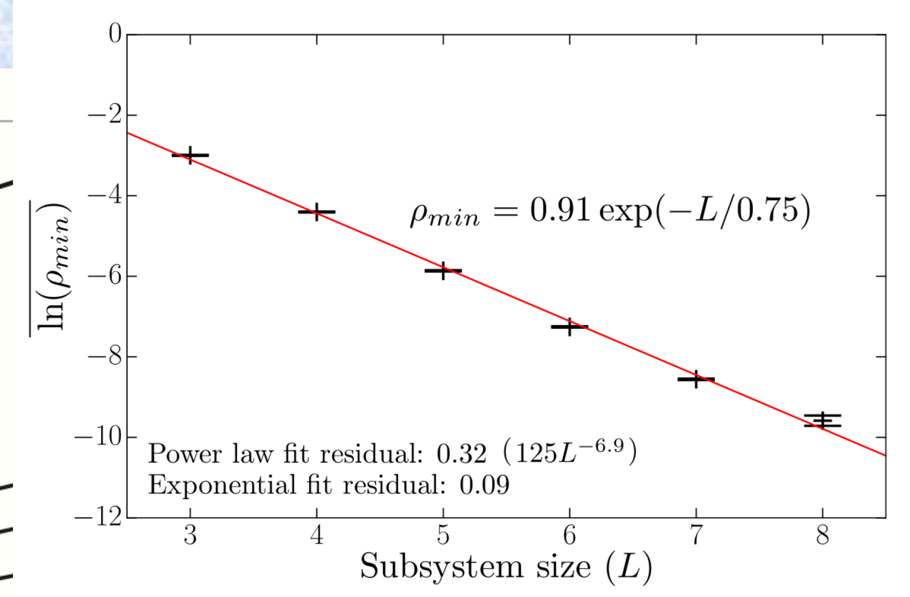
N. Pancotti, M. Knap, D. A. Huse, I. Cirac, and M. Banuls, PRB 2018. “For strong disorder, the decay of λ_M is compatible with an exponential form, $e^{-M/\xi}$ ”



The digitized data for $h = 9$ with a subtracted mean slope. True statement: “For strong disorder, the decay of λ_M is incompatible with an exponential form, $e^{-M/\xi.}$ ”

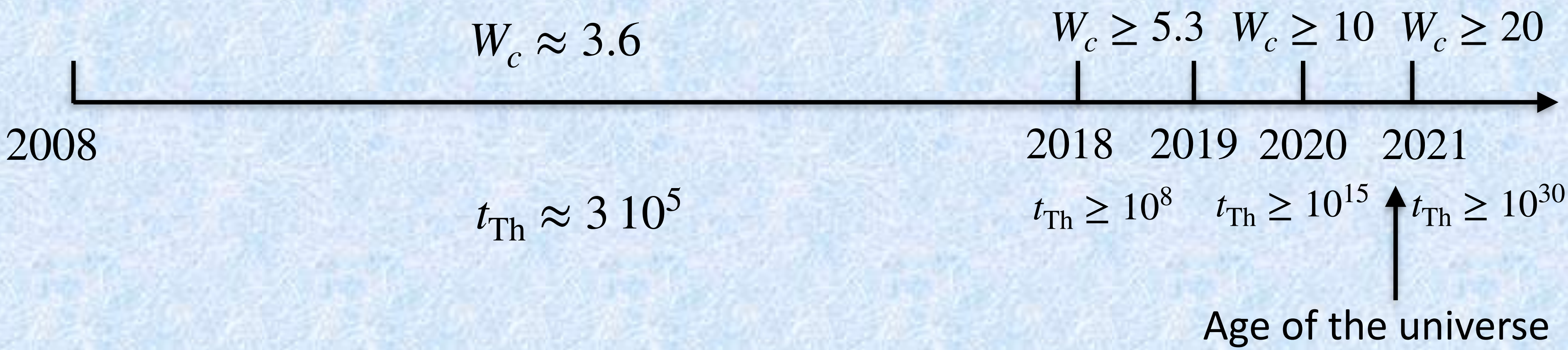
A. Morningstar, L. Colmenarez, V. Khemani, D. J. Luitz, and D. A. Huse, 2021,D. Sels 2021, no signatures of localization for $W \lesssim 20$.



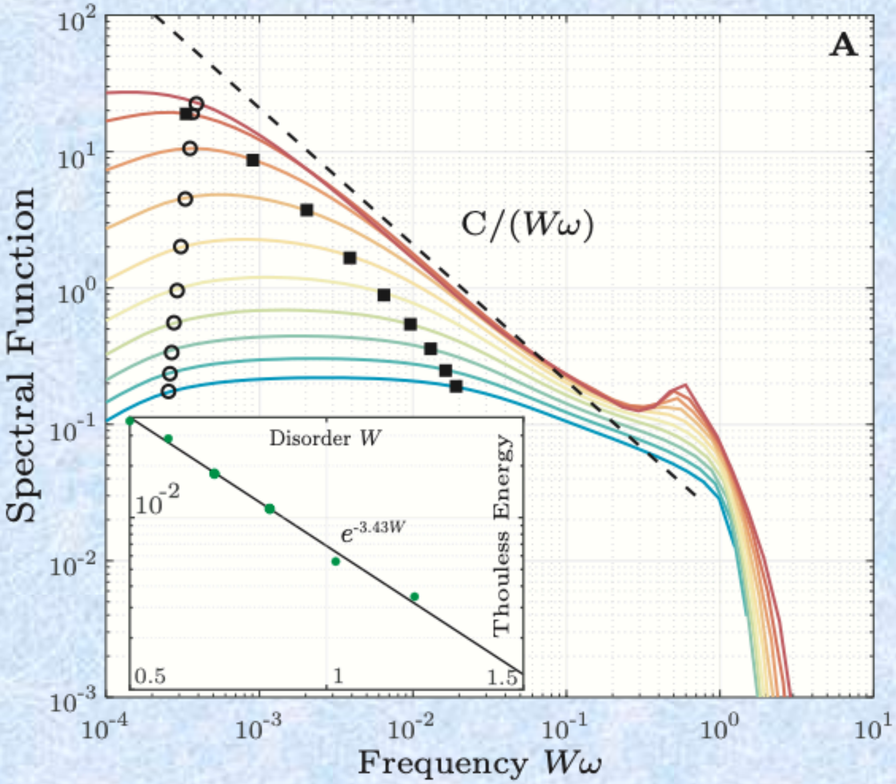


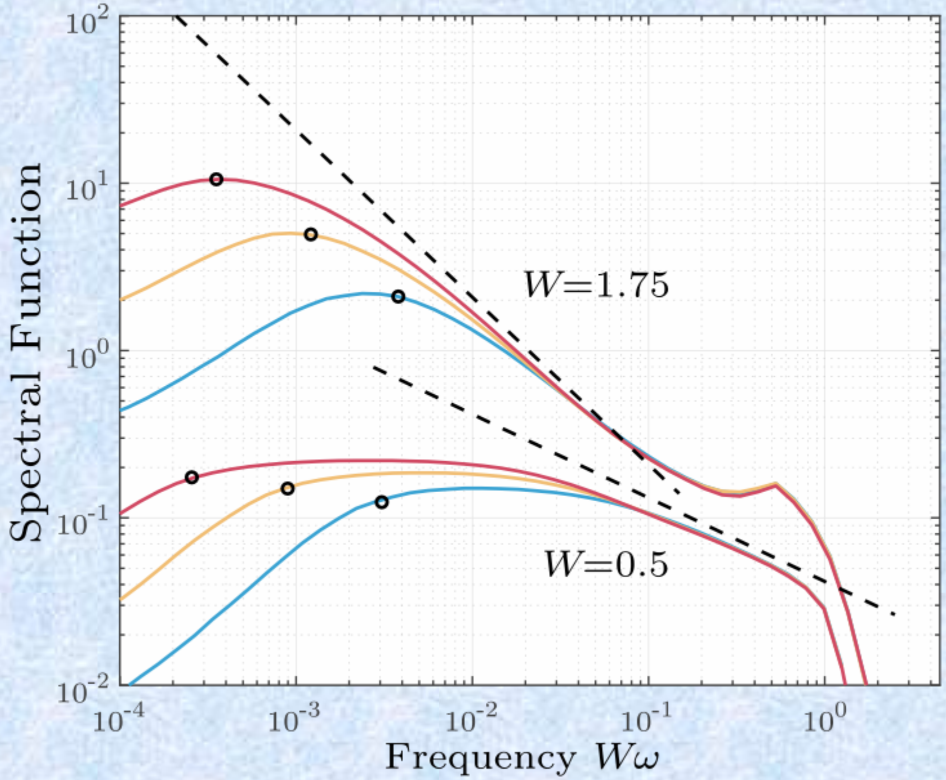
Visible factorial growth. Agrees with Birkhoff construction. $t_{\mathrm{Th}} > \exp[70]$.

Numerical progress in MBL disorder/time scales



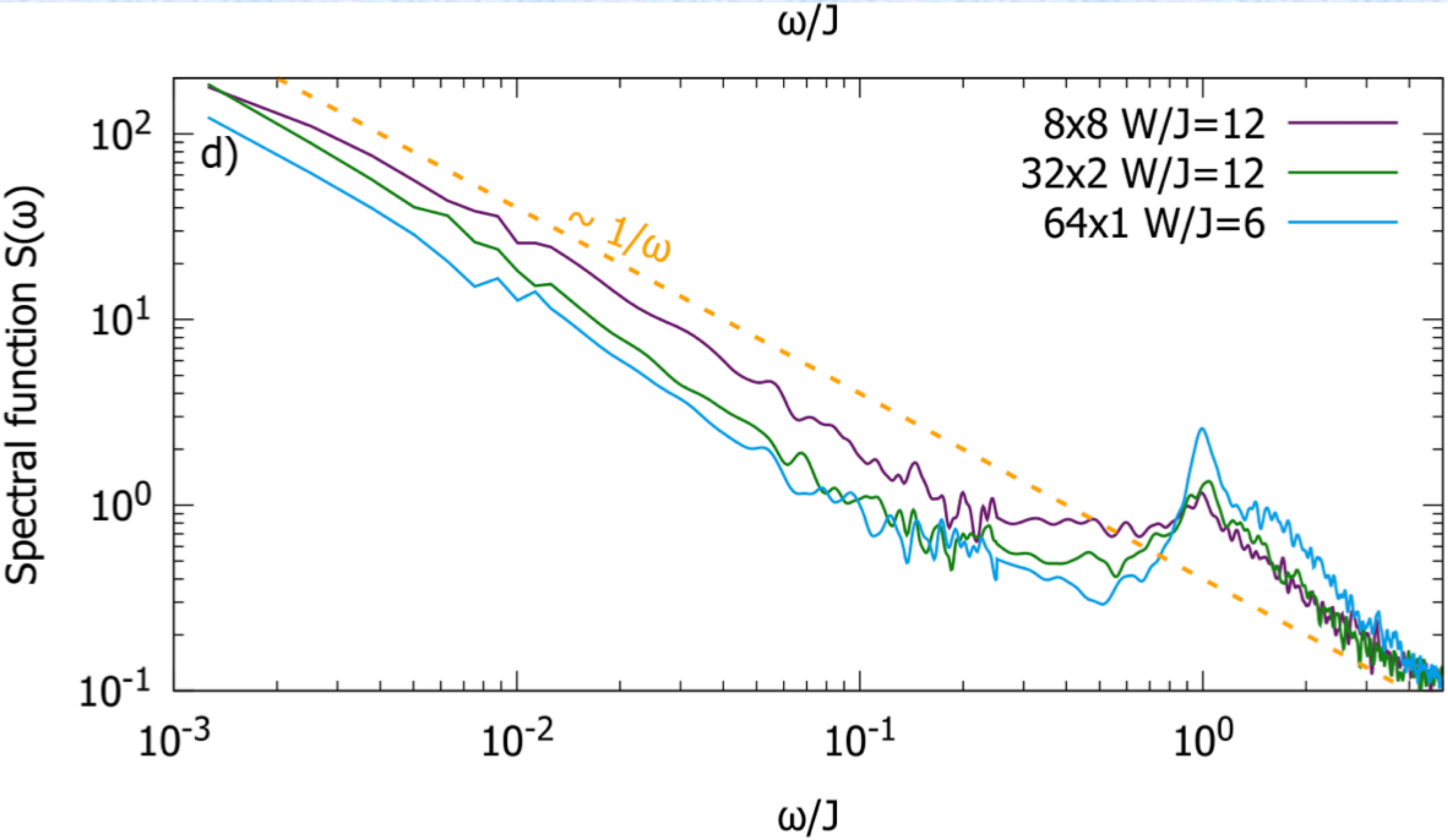
MBL is a glass. Spectral function for the disordered XXZ chain





- Indication for ω^{-1}scaling ($1/f$ noise). No variable subdiffusion exponents.
- Consistent with $\log(t)$ spreading of the correlation functions up to the Thouless time $t_{\mathrm{Th}} \sim \exp[\tau W]$ and then crossover to diffusion.
- Empirical explanation through FGR with a broad distribution of relaxation times: Griffiths in energy space: L. Vidmar, B. Krajewski, J. Bonca, M. Mierzejewski, PRL, 2021

Semiclassical dynamics of interacting fermions: treat fermion bi-linears as classical SU(N) spins (Ł. Iwanek, M. Mierzejewski, A. P., D. Sels, A. Sajna, 2022)



Similar results hold for classical spins (J. Wurtz, A.P., D. Sels, 2018)

Conclusions

- No real evidence for localization transition in TD limit analytical or numerical.
- Existing and earlier numerical results all point to absence of localization. Incorrect finite size scaling, mistakes, unjustified assumptions in theoretical arguments
- Quantum mechanics is not qualitatively important at high temperatures.
- MBL seems to be a transient glassy regime, which is very similar to known other examples: FPU, Floquet,…
- Positive developments: gained a lot of intuition on a transition/crossover from integrable to ergodic systems.
- Open questions: reconciling classical nearly integrable dynamics (Arnold diffusion, KAM,…) with quantum (proliferation of resonances, divergence of χ, emergence of RMT).
- Main lesson: we must remain skeptical and not be biased by even accepted theories.