Kardar-Parisi-Zhang universality in 1D exciton-polaritons

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Equilibrium Bose-Einstein condensation



[Anderson et al 1995]

transition controlled by tuning the temperature

order parameter : condensate fraction





In the out-of-equilibrium realm

Bose-Einstein condensates out of equilibrium

- an open quantum system
- flow in energy/momentum space
- non-equilibrium phase transition
- non-equilibrium steady-state



nature of the phase transition ? properties of the state ?



• KPZ in 2D

- Introduction I : Excitons-polaritons condensates
- Introduction II : The Kardar-Parisi-Zhang equation
- Emergence of KPZ universality in exciton-polaritons and its consequences : theory + experiment
- KPZ universality subclasses in 1D



[Squizzato, Canet and Minguzzi PRB 2018] [Deligiannis, Squizzato, Minguzzi and Canet, EPL 2021] [Fontaine et al, Nature 2022] [Deligiannis et al Phys. Rev. Research 2022]







What is a polariton ?

 Exciton-polaritons : hybrid light-matter particles from strong coupling of excitons and cavity photons
 Exciton : particle-hole bound state in a semiconductor

Cavity photon : has quadratic dispersion at small k

$$\mathcal{H}_{\text{exc}} = \int \frac{d^2 \mathbf{k}}{(2\pi)^2} \sum_{\sigma} \{ \hbar \omega_{\text{exc}}(\mathbf{k}) \hat{a}^{\dagger}_{X,\sigma}(\mathbf{k}) \hat{a}_{X,\sigma}(\mathbf{k}) + \hbar \Omega_R [\hat{a}^{\dagger}_{X,\sigma}(\mathbf{k}) \hat{a}_{C,\sigma}(\mathbf{k}) + \hat{a}^{\dagger}_{C,\sigma}(\mathbf{k}) \hat{a}_{X,\sigma}(\mathbf{k})] \}.$$



[Kasprzak et al, Nat 443, 209 (2006)]



$$\begin{aligned} \mathcal{L}_{cav} &= \int \frac{d^2 \mathbf{k}}{(2\pi)^2} \sum_{\sigma} \hbar \omega_{cav}(\mathbf{k}) \hat{a}^{\dagger}_{C,\sigma}(\mathbf{k}) \hat{a}_{C,\sigma}(\mathbf{k}) \\ \omega_{cav}(k) &= \frac{c}{n_0} \sqrt{q_z^2 + k^2} \simeq \omega_{cav}^o + \frac{\hbar k^2}{2m_{cav}} \\ q_z &= \pi M / \ell_z, \quad m_{cav} = \frac{\hbar n_0 q_z}{c} = \frac{\hbar \omega_{cav}^o}{c^2 / n_0^2} \end{aligned}$$

I. Carusotto and C. Ciuti, Rev. Mod. Phys. 85, 300 (2013)]



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$$\mathcal{H}_{cav} + \mathcal{H}_{exc} = \int \frac{d^2 \mathbf{k}}{(2\pi)^2} \sum_{\sigma} [\hbar \omega_{LP,\sigma}(\mathbf{k}) \hat{a}^{\dagger}_{LP,\sigma}(\mathbf{k}) \hat{a}_{LP,\sigma}(\mathbf{k}) + \hbar \omega_{UP,\sigma}(\mathbf{k}) \hat{a}^{\dagger}_{UP,\sigma}(\mathbf{k}) \hat{a}_{UP,\sigma}(\mathbf{k})]$$



polariton are bosons...

Bose-Einstein condensation !

incoherent pump : the phase of the condensate is chosen spontaneously

[J. J. Hopfield Phys. Rev. 112, 1555 (1958)

I. Carusotto and C. Ciuti, Rev. Mod. Phys. 85, 300 (2013)]



Polariton condensation

Bose-Einstein Condensation observed in experiments



[Kasprzak et al, Nat 443, 209 (2006)]



Non-equilibrium driven-dissipative conditions : laser pump compensates losses from mirrors

Control knob : the laser pump intensity Condensation for $P>P_{th}$

 \rightarrow Non-equilibrium phase transition

Exciton polariton condensates : the model

• Driven-dissipative stochastic Gross-Pitaevskii equation for the condensate wavefunction [Carusotto, Ciuti RMP 2013]

reservoir-

relaxation

condensate

resevoir

lifetime

$$i\hbar \frac{\partial}{\partial t}\psi = \begin{bmatrix} E(\hat{k}) - \frac{i\hbar}{2}\gamma(\hat{k}) + g|\psi|^2 + 2g_Rn_R + \frac{i\hbar}{2}Rn_R \end{bmatrix} \psi + \xi \text{ noise from pump/losses interactions with interactions with condensate interactions with lifetime $\langle \xi(x,t)\xi^*(x',t')\rangle = 2\xi_0\delta(x-x')\delta(t-t')$
Equation for the excitonic reservoir density, pumped by laser. It fills the condensate by collisions $\frac{\partial}{\partial t}n_R = P - (\gamma_R + R|\psi|^2) n_R$
pump$$



A famous statistical physics model

• The Kardar-Parisi-Zhang equation describes the kinetic roughening during stochastic interface growth



Frost on a window



Combustion front



Bacteria growth



Lichen on a rock



The Kardar-Parisi-Zhang equation

• Describing the growth of a classical interface [Kardar, Parisi, Zhang PRL 1986]

$$\partial_t h = \nu \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + \eta$$

with random white noise

$$\langle \eta(x,t)\eta(x',t')\rangle = 2D\delta(x-x')\delta(t-t')$$



competition of *smoothening* from surface tension (diffusion term) and *roughening* due nonlinear growth normal to the interface





Universal scaling properties of the KPZ phase

Self-organized criticality : the whole parameter region is critical

• Scaling properties : power law increase of fluctuations of the height field

$$\langle |h(x,t) - h(x',t')|^2 \rangle \sim |x - x'|^{2\chi}$$
 for $|t - t'| = 0$
 $\langle |h(x,t) - h(x',t')|^2 \rangle \sim |t - t'|^{2\beta}$ for $|x - x'| = 0$

 $\begin{array}{ll} \mbox{critical exponents :} & \mbox{in 1D :} \\ \mbox{saturation exponent } \chi & & \mbox{$\chi = 1/2$} & \beta = 1/3 \\ \mbox{roughness exponent } \beta & & \mbox{in 2D :} \\ \mbox{dynamical exponent } z = \chi/\beta & & \mbox{$\chi \simeq 0.38$} & \beta \simeq 0.24 \end{array}$

• Space-time scaling : collapse on a single scaling function

$$\langle |h(x,t) - h(x',t')|^2 \rangle \sim |x - x'|^{2\chi} f(|t - t'|/|x - x'|^2)$$



An emergent statistical physics model



...but also KPZ in quantum systems !



Heisenberg chains [Ljuobotina et al Nat Phys 2017]

- 1D antiferromagnets (KCuF₃) [Schele et al Nat Phys 2021]
- Ultracold bosons in optical lattices
 [Wei el al Science 2022]
- Strongly repulsive fermions in trap [Pecci et al PRR 2022]



- Excitons polaritons in semiconductors
- Which interface ?
- Conditions and parameters for KPZ ?
- Observables ?
- Is it *really* the same physics?



Polariton phase dynamics : emergence of KPZ

- Using the Gross-Pitaevskii equation for the condensate wavefunction, $\psi = \sqrt{\rho}e^{i\theta}$
 - \rightarrow KPZ equation for the condensate phase [Altman et al, PRX 2015]

$$\partial_t \theta = \nu \nabla^2 \theta + \frac{\lambda}{2} (\nabla \theta)^2 + \eta$$

with $\langle \eta(x,t)\eta(x',t')\rangle = 2D\delta(x-x')\delta(t-t')$

 u, λ, D related to to the parameters of the Gross-Pitaevskii equation

• Observable in experiments via the first-order correlation function of the condensate

$$g^{(1)}(\Delta x, \Delta t) = \frac{\langle \psi^*(x, t)\psi(x', t')\rangle}{\sqrt{\langle \rho(x, t)\rangle\langle \rho(x', t')\rangle}} \left[|g^{(1)}(\Delta x, \Delta t)|^2 \simeq \exp(-\langle \operatorname{Var}[\theta(x, t) - \theta(x', t')]\rangle) \right]$$

- at difference from an interface, the phase is a compact variable
 - \rightarrow we unwind the phase trajectories
 - \rightarrow some differences may occur...



- Early theoretical works predicted emergence of KPZ in polaritons, but in a too large system (not realistic parameter conditions) [He, Sieberer, Altman, Diehl PRB 2015]
- Our work : momentum-dependent condensate lifetime, experimentally relevant, stabilizes the KPZ phase within experimentally accessible parameters !

[Squizzato, Canet and Minguzzi PRB 2018]

Our theoretical predictions of KPZ scaling in polaritons :

 $C(\Delta x, \Delta t) = -2\log(|g^{(1)}(\Delta x, \Delta t)|) \quad |g^{(1)}(\Delta x, \Delta t)|^2 \simeq \exp(-\langle \operatorname{Var}[\theta(x, t) - \theta(x', t')] \rangle)$





Implications of KPZ behaviour: the Penrose-Onsager criterion

• Bose-Einstein condensation : condensate density n_0 from the offdiagonal long range order (ODRLO) of the one-body density matrix [Penrose, Onsager PR 104, 576 (1956)]

 $\langle \psi^*(x)\psi(x')\rangle \to n_0 \quad \text{ for } \quad |x-x'| \to \infty$



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 KPZ correlations of the phase-phase correlations imply that driven-dissipative polariton condensates are not true Bose-Einstein condensates !

$$|g^{(1)}(\Delta x, \Delta t)|^2 = \frac{\langle \psi^*(x, t)\psi(x', t')\rangle}{\sqrt{\langle \rho(x, t)\rangle\langle \rho(x', t')\rangle}} \simeq \exp(-\langle \operatorname{Var}[\theta(x, t) - \theta(x', t')]\rangle) \quad \text{phase profiles}$$

KPZ:
$$\langle |\theta(x,t) - \theta(x',t')|^2 \rangle \sim |x - x'|^{2\chi} f(|t - t'|/|x - x'|^2)$$

 \rightarrow growing phase fluctuations destroy ODRLO

 $\rightarrow\,$ the polariton condensates belong to a different universality class than equilibrium ones

\rightarrow fundamental limit on the coherence of open quantum systems





• Jacqueline Bloch team @CN2, Palaiseau : control over decoherence and instability effects by band engineering in a lattice : condensates with negative mass



KPZ evidence : space and time correlations

$$g^{(1)}(\Delta x, \Delta t) = \frac{\langle \psi^*(x, t_0)\psi(-x, t_0 + \Delta t) \rangle}{\sqrt{\langle |\psi(x, t_0)|^2 \rangle} \sqrt{\langle |\psi(-x, t_0 + \Delta t)|^2 \rangle}}$$

from Michelson interferometry

• Spatial correlations





- Evidence for KPZ window both in space and time
- Extracted critical exponents $\chi = 0.51 \pm 0.08$ $\beta = 0.35 \pm 0.02$

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• Deviation from KPZ behaviour at large time and space

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• Deviation from KPZ behaviour at large time and space



• Observed spread of slopes for the phase trajectories

 $\Delta \theta(t_0, \Delta t) \equiv \theta(t_0 + \Delta t) - \theta(t_0) \sim \omega_0 \Delta t + (|\Gamma| \Delta t)^{\chi/z} \tilde{\theta}(\Delta t)$



• The observed disruption of KPZ scaling at large time differences is due to the spread of slopes, ie an inhomogeneous broadening



KPZ evidence : scaling function and data collapse



- Excellent agreement with the theoretical KPZ scaling function
- Normalization non-universal parameters are very close to microscopic ones



Robustness to phase slips

• simulations show phase slips corresponding to vortices in space time



Vortex-antivortex pairs perturb only locally the phase map



 \rightarrow first order correlation robust to few phase slips :

jumps close to multiples of 2π they do not affect it !

 $g^{(1)}(\Delta x, \Delta t) \sim \langle e^{i(\theta(x,t) - \theta(x',t'))} \rangle$

theoretical analysis full supports observation of KPZ scaling



• By increasing noise, pump or interactions, three main phases



Beyond scaling : KPZ probability distribution of height fluctuations

• In KPZ universality class : non-Gaussian probability distribution for height fluctuations [Corwin, Rand. Mat. 2012]

$$h(x,t) \sim v_{\infty}t + (\Gamma t)^{1/3}\chi(x,t)$$

- various universality *subclasses* (all with the same critical exponents)
- \rightarrow depending on the initial conditions for the interface, different shapes of the distribution



Flat : Tracy-Widom GOE



Curved : Tracy-Widom GUE



Disordered : Baik-Rains





Beyond scaling : probability distribution of the condensate phase fluctuations

• Distribution of the rescaled phase of a polariton condensate – from numerical simulations of the stochastic generalized Gross-Pitaevskii equation



temporal phase trajectory



- Excellent agreement with the theoretical prediction Tracy-Widom GOE \rightarrow confirms KPZ universality in polaritons



[Squizzato, Canet and Minguzzi PRB 2018]



Possibility to tune the universality subclass by an external confining potential: realization with polaritons of curved case, Tracy-Widom GUE!

$$V(x) = \frac{1}{2}m\omega^2 x^2$$

KPZ mapping still holds

$$\theta(x,t) \sim \omega_{\infty} t + (\Gamma t)^{1/3} \chi$$







• KPZ : phase profile becomes more and more rough with time



[[]Squizzato, Canet and Minguzzi PRB 2018]

PMC Probability distribution of the condensate phase fluctations – experimental parameters

• Phase slips give rise to copies of the Tracy-Widom-GOE phase distribution





Exciton-polaritons in 2D : KPZ ??

- 2D at equilibrium : BKT transition vortex unbinding and proliferation
- In driven-dissipative condensates : spatial vortices may hinder KPZ
- Perturbative RG argument : always vortex unbinding at large distances [Altman et al PRX 1015]
- Power-law decay of correlations reported close to pump threshold with nonequilibrium power-law exponent [Comaron et al EPL 2021]



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- Power-law decay of correlations reported close to pump threshold with nonequilibrium power-law exponent [Comaron et al EPL 2021]
- Numerical evidence of KPZ scaling with artificially low noise and very high pump $P/P_{th} = 10$ [Mei, Ji, Wouters PRB 2021]
- Possibility to reach KPZ regime in OPO configuration [Zamora et al PRX 2017, Ferrier et al PRB 2022]
- \rightarrow our work : lattice model, realistic parameters



Various topological defects...

• Spatial vortices



• Space-time vortices





Control over topological defects



Spatial vortices decrease at increasing pump !

Space-time vortices are close to multiples of 2π \rightarrow as in 1D, they should do not disrupt KPZ universality...





Evidence of KPZ universality in 2D polaritons !

• Space-time scaling with KPZ critical exponents in 2D



[Deligiannis et al Phys. Rev. Research 2022]



Evidence of KPZ universality in 2D polaritons !

• Comparison to the theoretical KPZ scaling function obtained by Functional Renormalization Group [Kloss et al PRE 2012]



Probability distribution of phase fluctuations

No analytical prediction available for the KPZ distribution in 2D



[Deligiannis et al Phys. Rev. Research 2022]



Conclusions & Outlook

- Exciton-polaritons : driven-dissipative condensates, the phase dynamics belongs to the KPZ universality class → fundamental limit of coherence
- Experimental observation KPZ scaling function & critical exponents

 $-2 \ln g_{1,\psi}(\Delta t, \Delta r)/C_0 \Delta t^{2t}$

8 10

 10^{-2}

• Phase distributions in 1D

• KPZ in 2D



Effects of more phase slips ?

How to access to phase distributions ?

Experiment & full phase diagram in 2D ?

[Squizzato, Canet and Minguzzi PRB 97, 195453 (2018)] [Deligiannis, Squizzato, Minguzzi and Canet, EPL 132, 67004 (2021)] [Fontaine et al, Nature 608, 687 (2022)] [Deligiannis et al, PRR 4, 043207 (2022)]



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Other recent results

Exact spectral function of a Tonks-Girardeau gas on a lattice

[Jacopo Settino, Nicola Lo Gullo, Francesco Plastina and Anna Minguzzi, PRL 126, 065301, 2021]



Persistent currents in a strongly interacting multicomponent Bose gas on a ring

[Giovanni Pecci, Gianni Aupetit-Diallo, Mathias Albert, Patrizia Vignolo and Anna Minguzzi, arXiv:2211.16194]





January 8th to March 22nd, 2024

Organisers:

Rosario Fazio (ICTP - Trieste) Thierry Giamarchi (University of Geneva) Anna Minguzzi (LPMMC, University Grenoble-Alpes, CNRS) Patrizia Vignolo (InPhyNi, University Côte d'Azur, CNRS)



Quantum many-body systems out-of-equilibrium

Thematic programme with short courses, seminars and workshops

IESC Introductory school November 26th to December 2nd, 2023

Quantum simulators February 5th to 9th, 2024

Driven quantum systems March 18th to 20th, 2024



Program coordinated by the Centre Emile Borel (CEB) at IHP (Paris) and also accessible online Participation of postdocs and PhD students is strongly encouraged Registration is free however mandatory

Scientific program and registration on: https://indico.math.cnrs.fr/category/615/ Deadline for financial support: June 15th, 2023 Contact: negmb2024@ihp.fr

CEB organisation assistant: Sofiia Minasian

CEB manager: Sylvie Lhermitte

Also supported by:

INPHYNI

Deadline for participants application : June 15th



Thank you for your attention !



to read-out the KPZ phase-phase correlator

$$\left< |\theta(x,t) - \theta(x',t')|^2 \right> \sim |x - x'|^{2\chi} f\left(|t - t'|/|x - x'|^z\right)$$

we assumed

$$|g^{(1)}(\Delta x, \Delta t)|^2 = \frac{\langle \psi^*(x, t)\psi(x', t')\rangle}{\sqrt{\langle \rho(x, t)\rangle \langle \rho(x', t')\rangle}} \simeq \exp(-\langle \operatorname{Var}[\theta(x, t) - \theta(x', t')]\rangle)$$

 \rightarrow check this hypothesis by numerical simulations



Role of compactness of the phase : space-time vortices

• simulations show phase slips corresponding to *vortices in space time*





Role of compactness of the phase : space-time vortices

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Role of compactness of the phase : space-time vortices

• simulations show phase slips corresponding to vortices in space time



[[]Fontaine et al, Nature 2022]



Robustness of KPZ in 2D

• KPZ region at varying pump strength $P/P_{th} = 1.6...2.5$



[Deligiannis et al Phys. Rev. Research 2022]