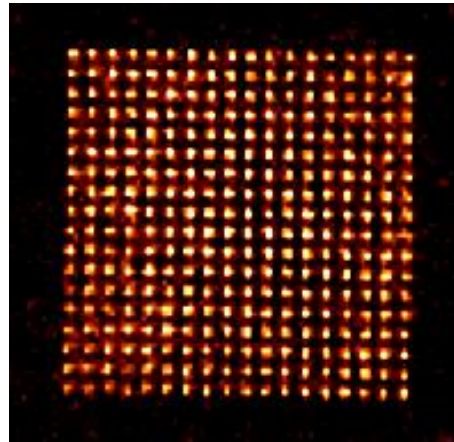


Exploring the dipolar XY model with arrays of single Rydberg atoms

Thierry Lahaye

Laboratoire Charles Fabry

CNRS & Institut d'Optique, Palaiseau, France



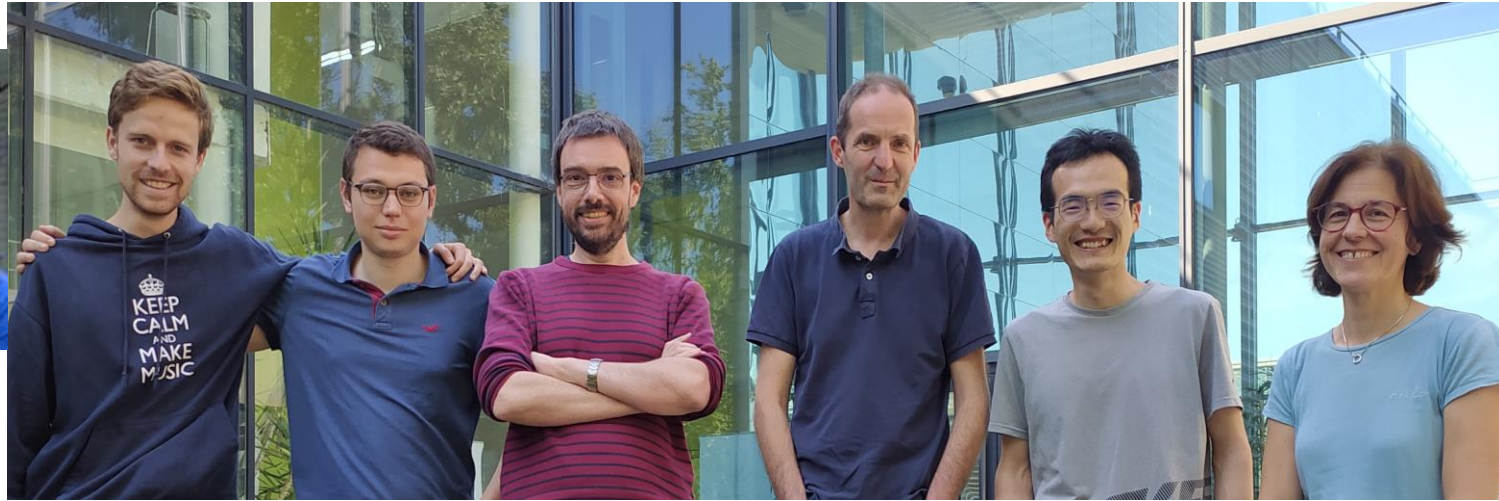
Precision Many-Body Physics 2023, Collège de France

June 14, 2023

The Rydberg team in Palaiseau



Daniel Barredo



Gabriel Emperauger

Guillaume Bornet

Thierry Lahaye

Antoine Browaeys

Cheng Chen

Florence Nogrette

Bastien Gély



Jamie Boyd



Collaborators (theory):

H.-P. Büchler (Stuttgart), A. Läuchli (Innsbruck),
N. Yao (Harvard), T. Roscilde (ENS Lyon)

<https://atom-tweezers-io.org/>

Funding:



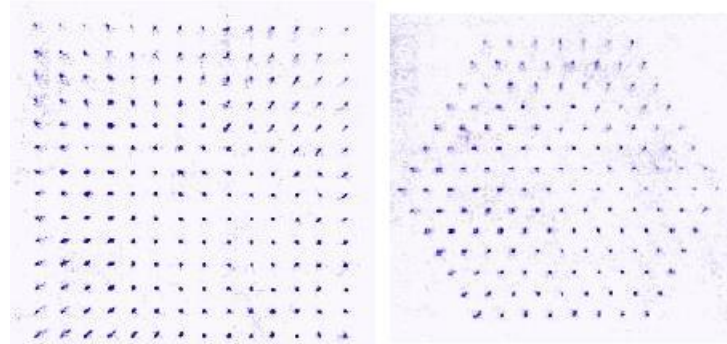
QUANTUM
FLAGSHIP



Arrays of single Rydberg atoms

- Arrays of single atoms with arbitrary geometries

Up to 300 atoms
Spacing: a few microns



- Strong interactions via Rydberg excitation

Interaction strength 1 to 10 MHz for $R \sim 5 \mu\text{m}$
Lifetime 100s of μs

- *Implement spin models*

Ising (vdW interactions)

$$\hat{H} \sim \sum_{i,j} J_{ij} \sigma_z^{(i)} \sigma_z^{(j)}$$

XY (resonant dipole-dipole interaction)

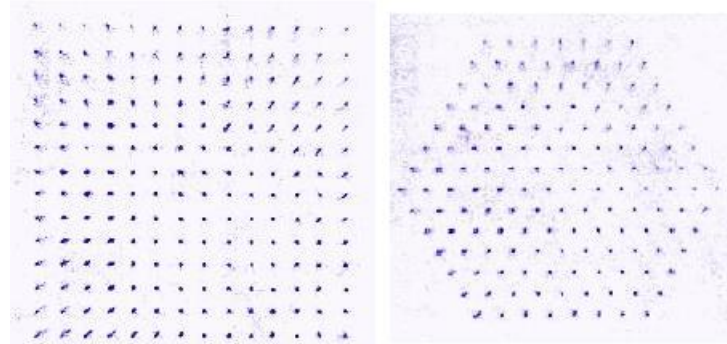
$$\hat{H} \sim \sum_{i,j} J_{ij} \sigma_+^{(i)} \sigma_-^{(j)}$$

Arrays of single Rydberg atoms

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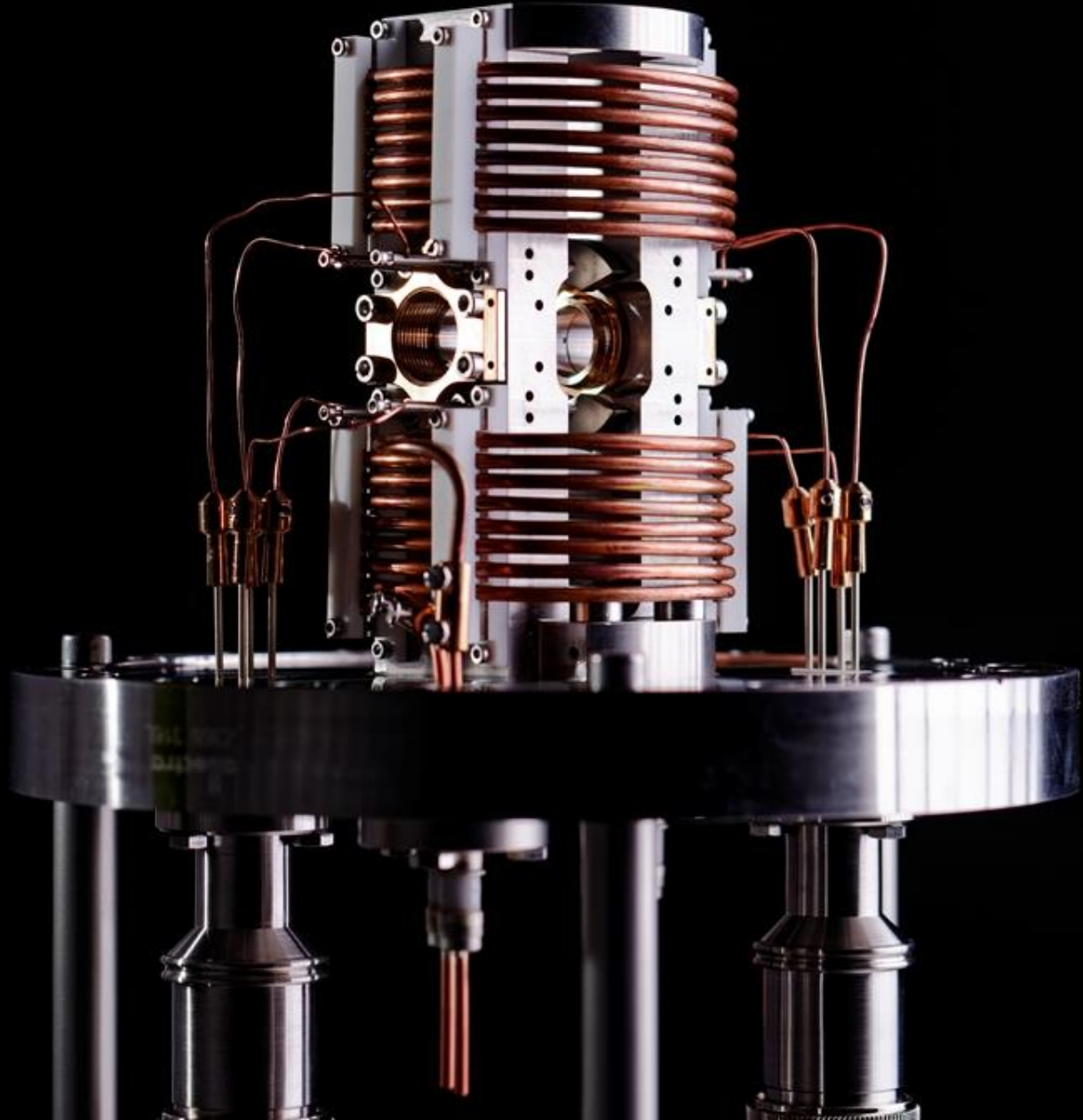
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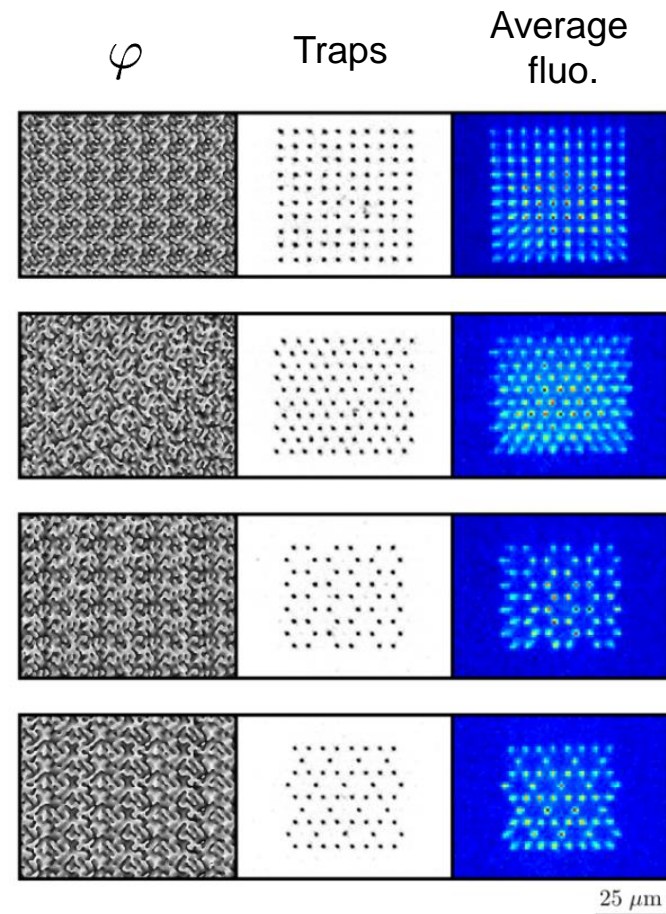
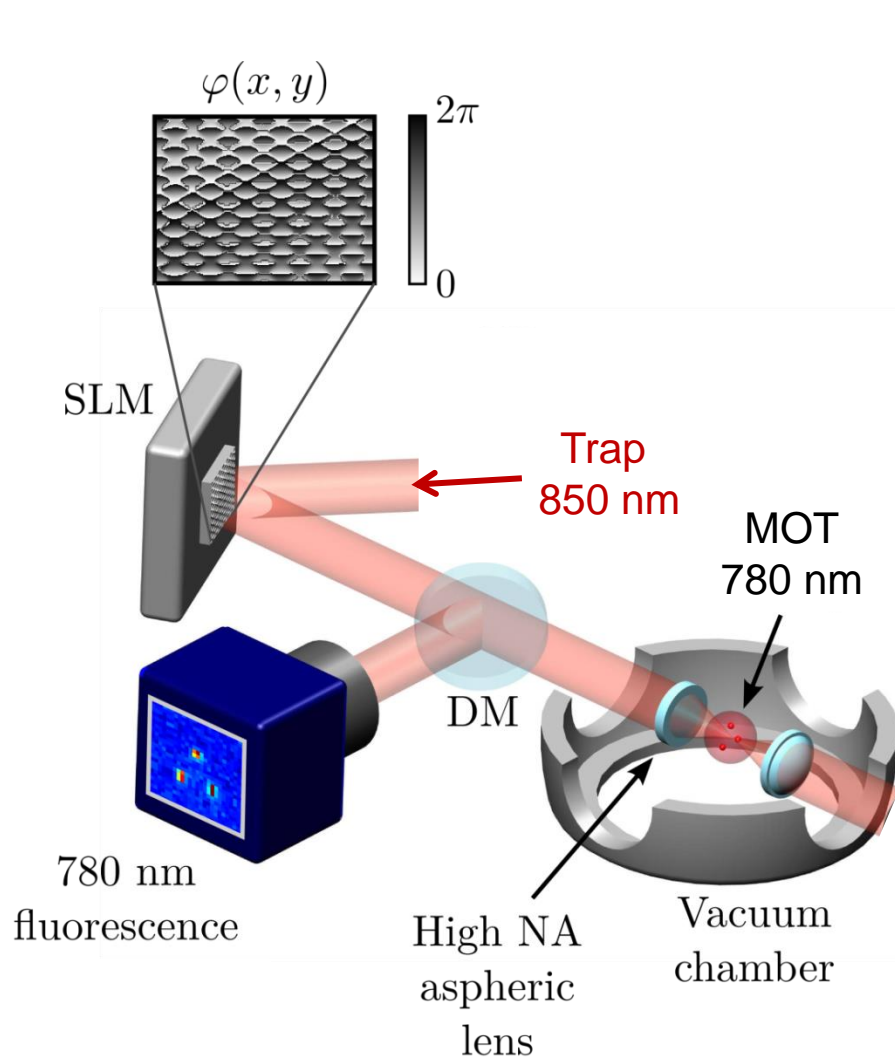
XY (resonant dipole-dipole interaction)

$$\hat{H} \sim \sum_{i,j} J_{ij} \sigma_+^{(i)} \sigma_-^{(j)}$$

Experimental setup

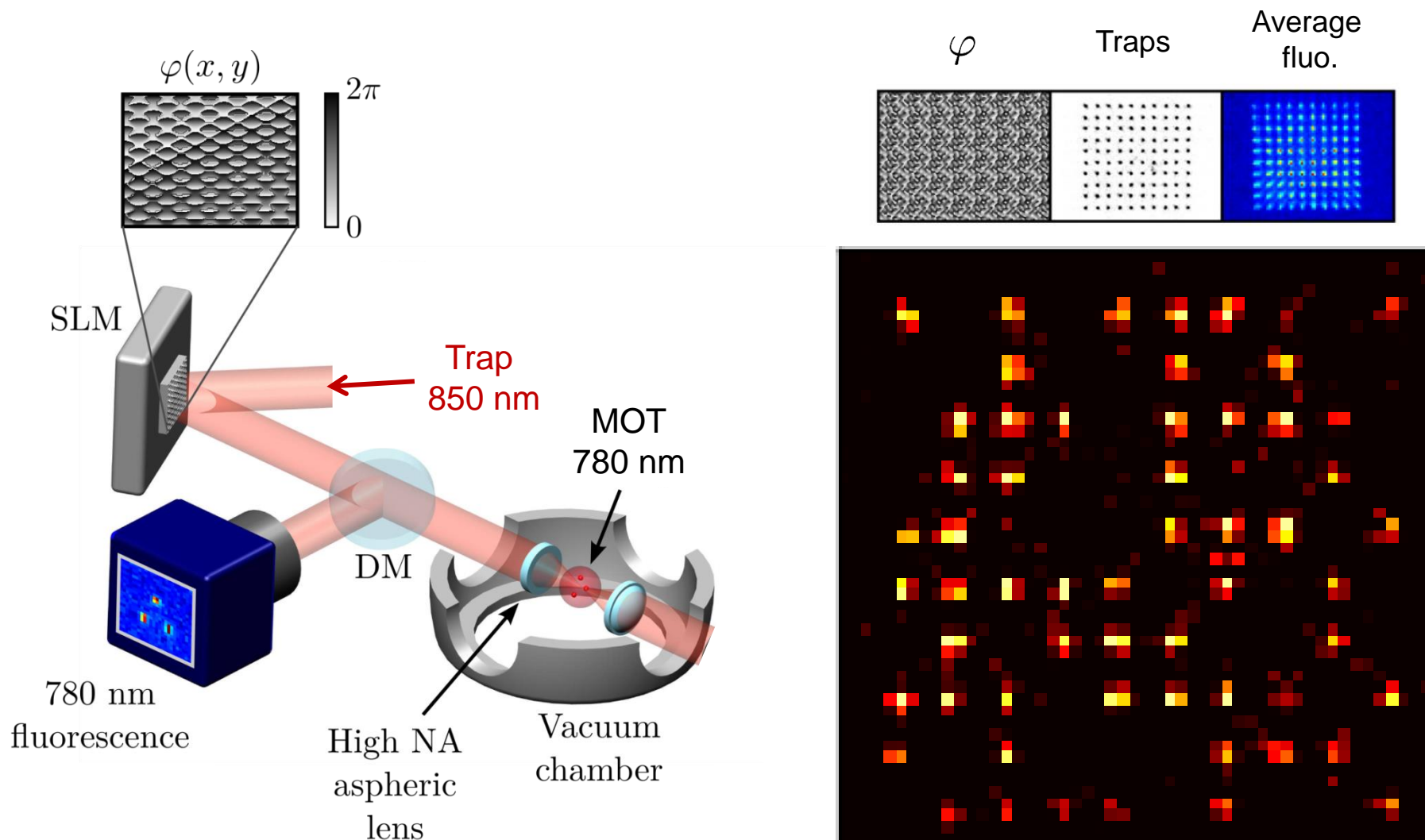


Arrays of single atoms

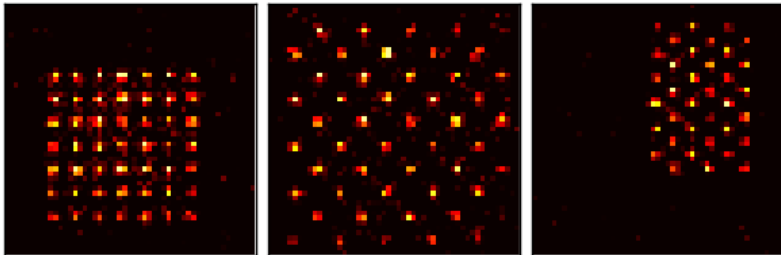
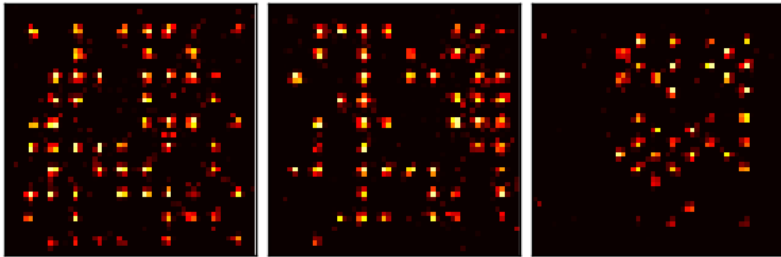
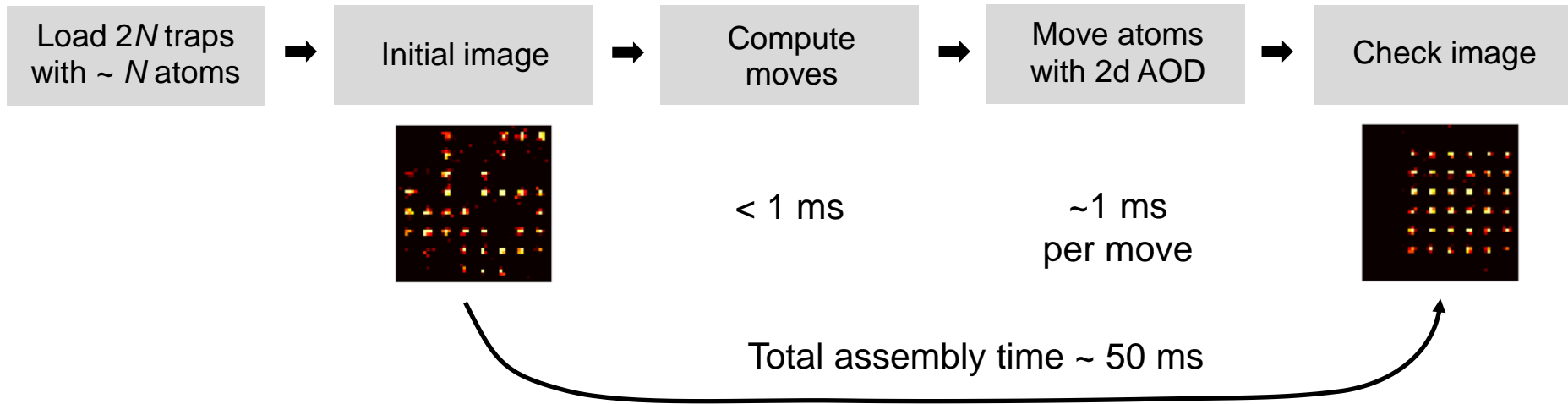


Single shot: 50% filling

Arrays of single atoms



Atom-by-atom assembly



- Fully loaded arrays up to 50 atoms
- 98% filling fraction
- Rep. rate up to ~ 4 Hz

Barredo *et al.*, [Science](#) **354**, 1021 (2016)

See also:

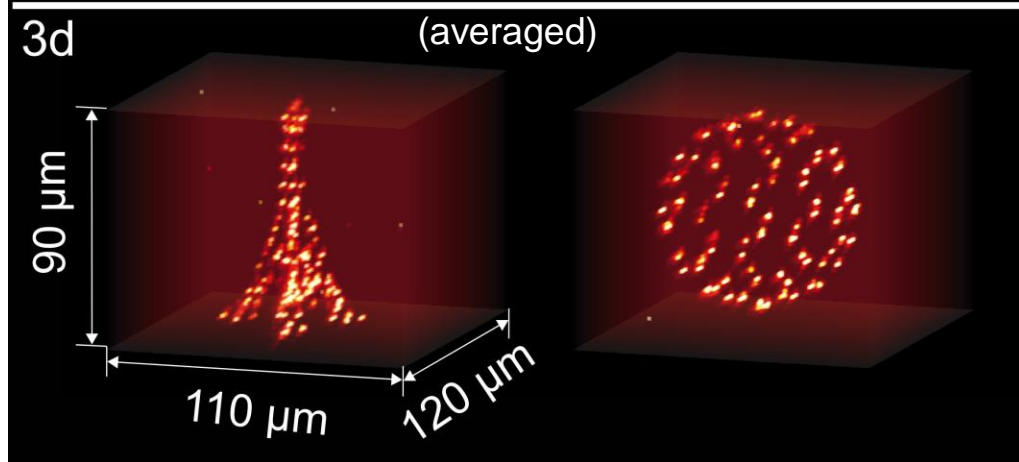
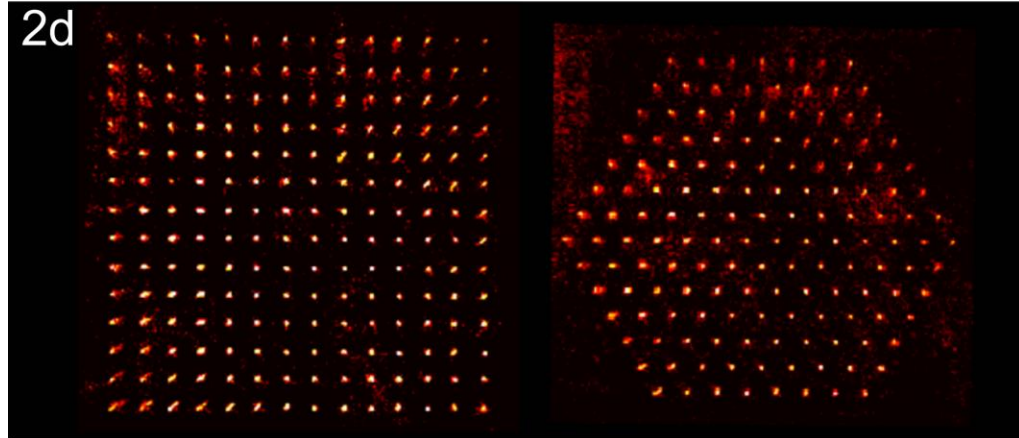
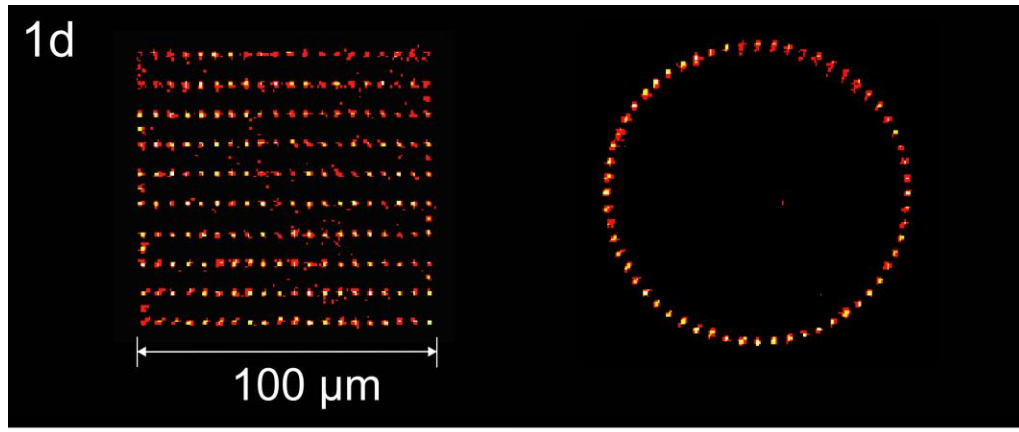
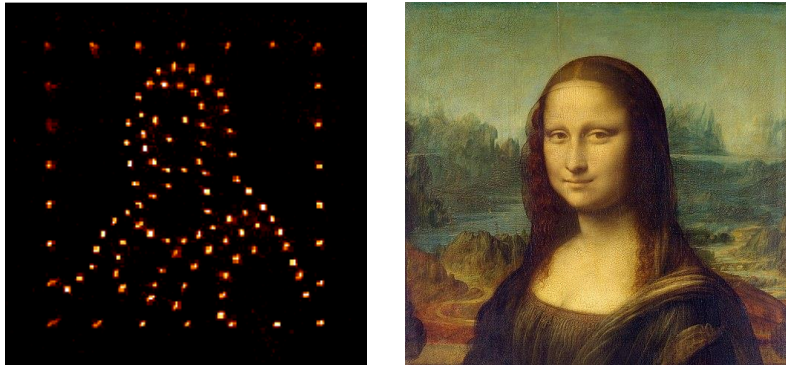
Endres *et al.*, [Science](#) **354**, 1024 (2016)

Kim *et al.*, [Nature Comm.](#) **7**, 13317 (2016)

Flexible geometries

New assembler algorithms:

Schymik *et al.*, [PRA 102, 063107 \(2020\)](#)

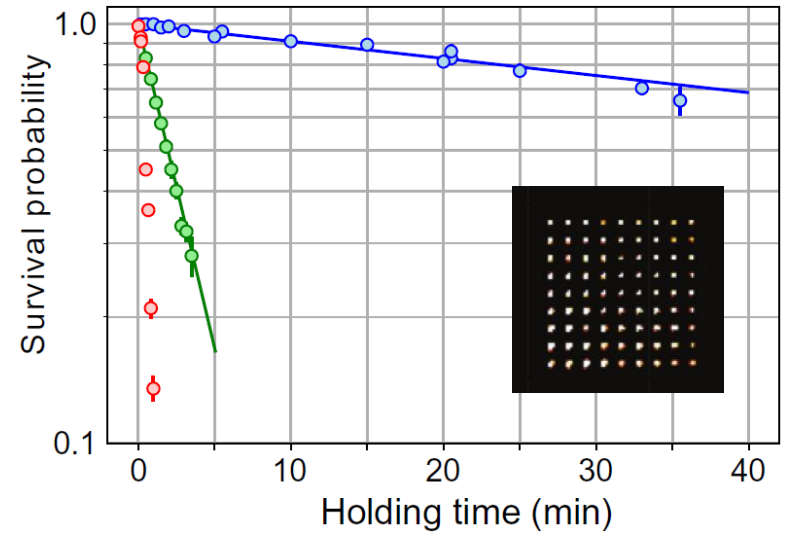
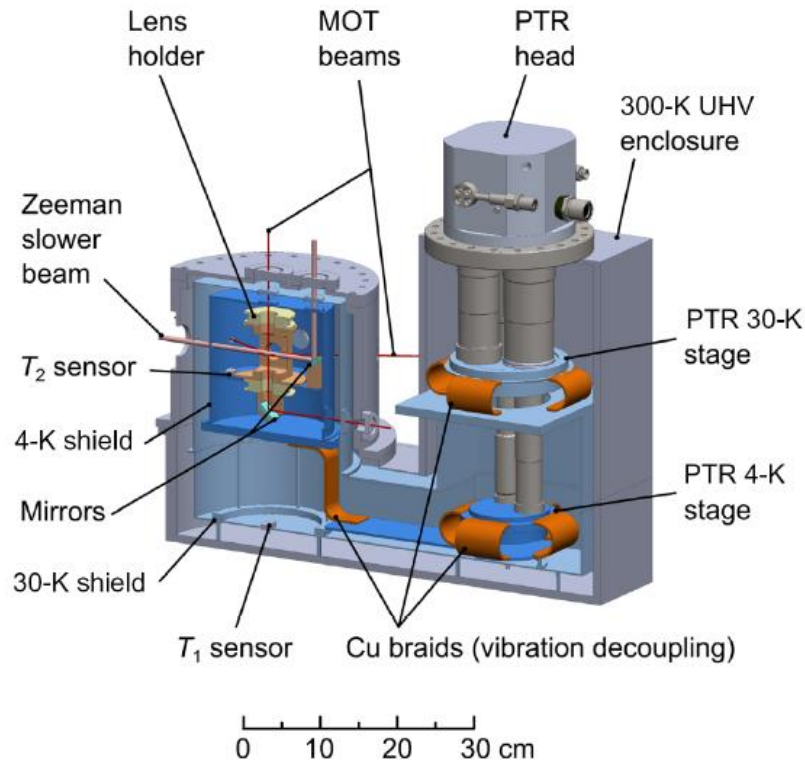


Up to $N = 200$ atoms:

Filling fraction > 99 %

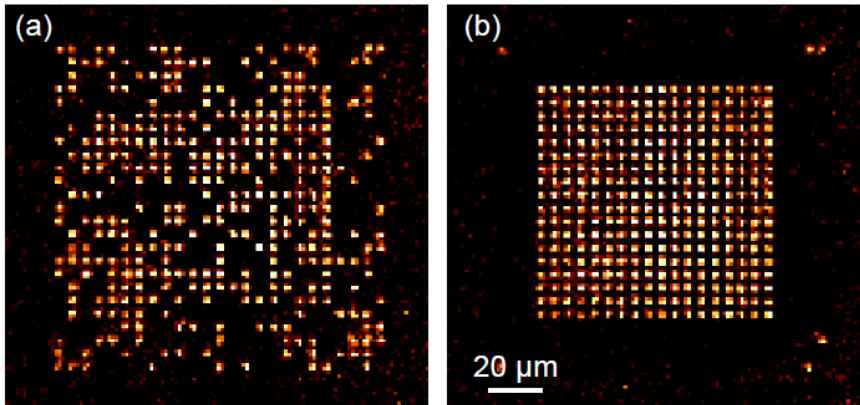
Probability of defect-free shots ~ 20 %

A cryogenic setup

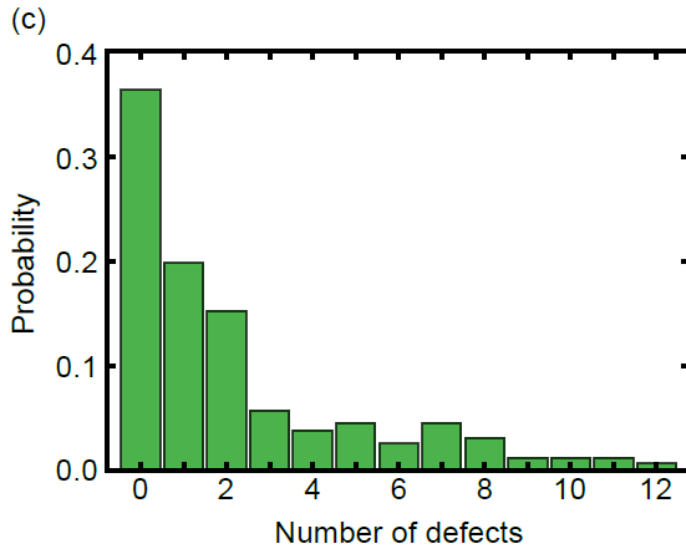


Trapping lifetime > 6000 s !

Defect-free arrays with 324 atoms



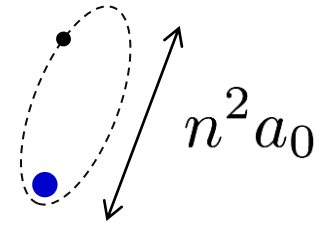
- New procedure to optimize trap loading
- Main limitation: field of view of objectives



K.-N. Schymik *et al.*, *Phys. Rev. A* **106**, 022611 (2022).

Rydberg atoms

Large principal quantum number: $n \gg 1$
 $n \sim 50 - 100$



Exaggerated properties:

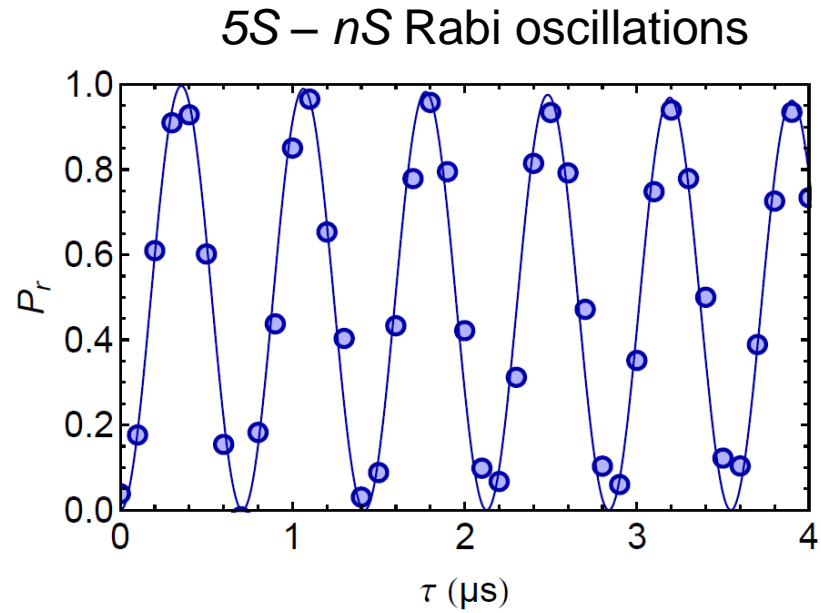
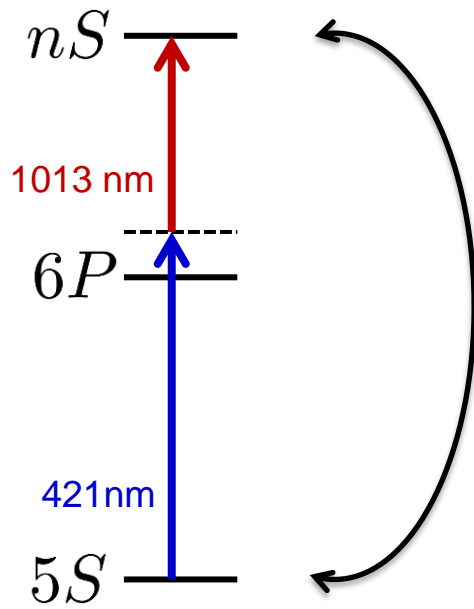
Electric dipole $\langle nS | d | nP \rangle \sim n^2$

Lifetime $\tau \sim n^3$ (hundreds of μs)

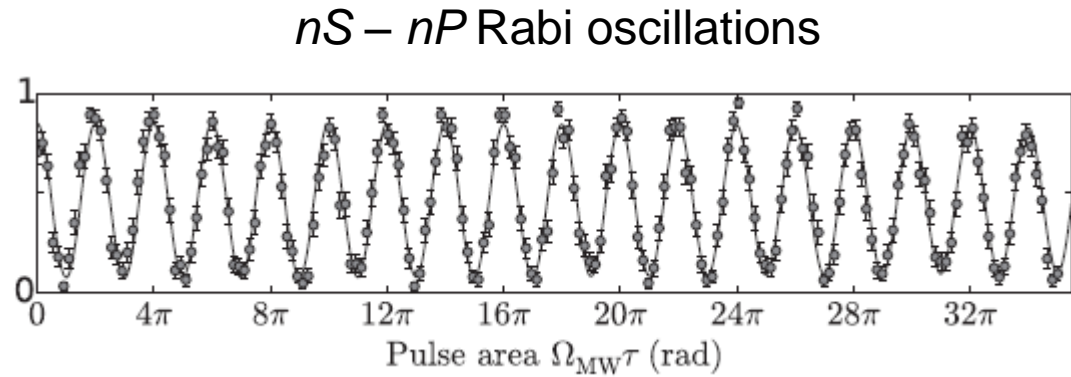
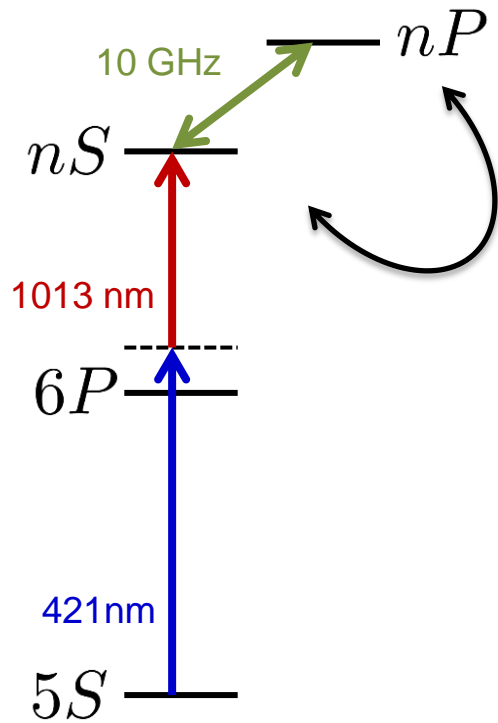
Polarizability $\alpha \sim n^7$

Interactions $V_{\text{dd}} \sim n^4$ $V_{\text{vdW}} \sim n^{11}$

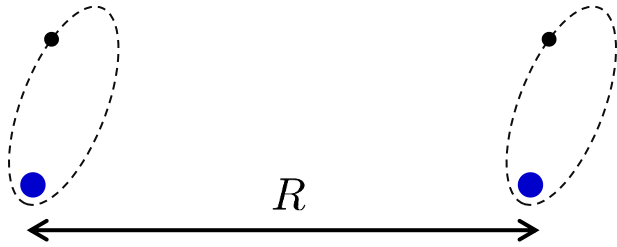
Rydberg excitation



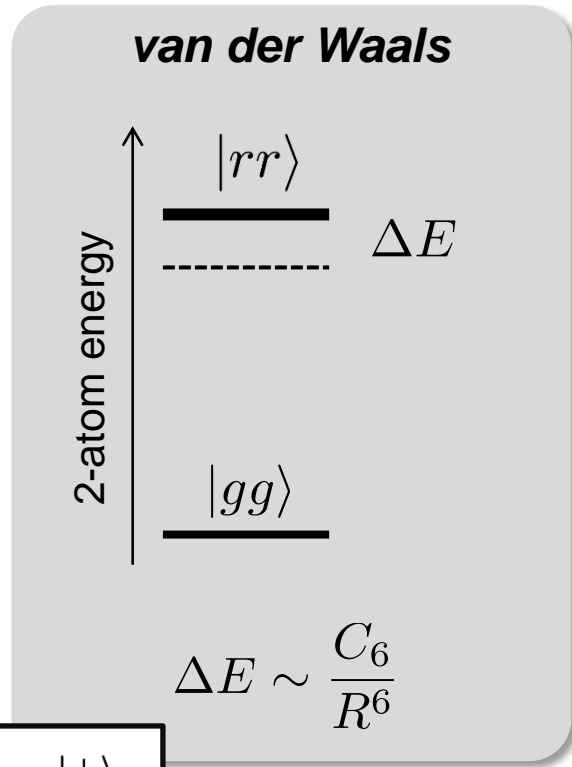
Rydberg atoms: microwave transitions



Interactions between Rydberg states



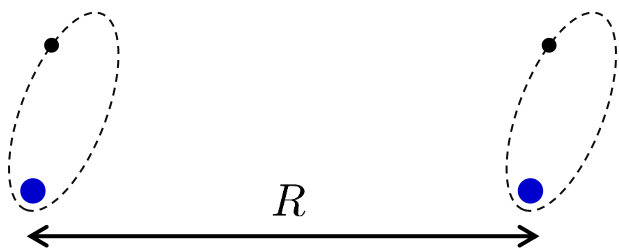
$$\hat{V}_{\text{ddi}} = \frac{1}{4\pi\epsilon_0} \frac{\hat{\mathbf{d}}_1 \cdot \hat{\mathbf{d}}_2 - 3(\hat{\mathbf{d}}_1 \cdot \hat{\mathbf{n}})(\hat{\mathbf{d}}_2 \cdot \hat{\mathbf{n}})}{R^3}$$



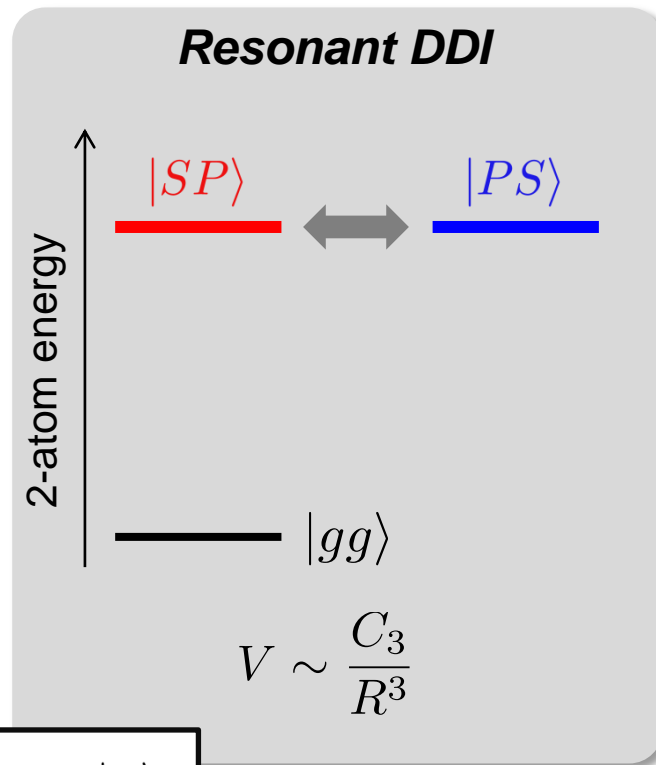
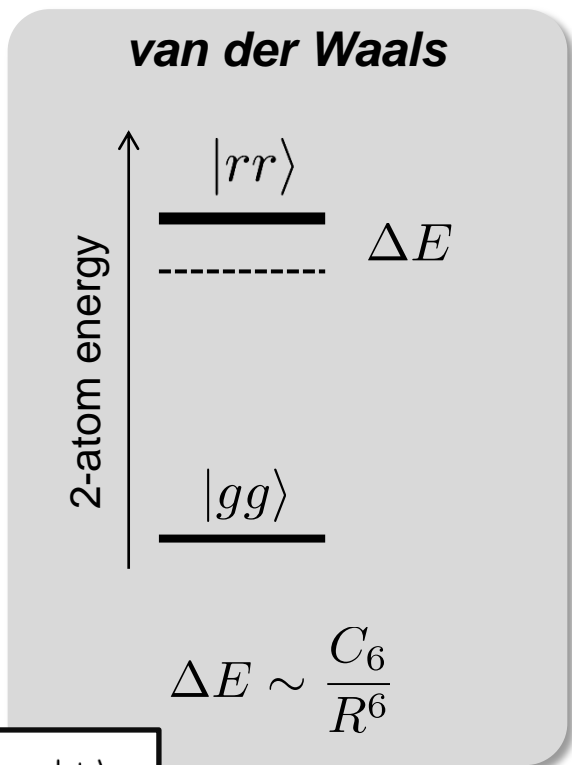
$$\begin{cases} |g\rangle \rightarrow |\downarrow\rangle \\ |r\rangle \rightarrow |\uparrow\rangle \end{cases}$$

Ising-like interaction

Interactions between Rydberg states



$$\hat{V}_{\text{ddi}} = \frac{1}{4\pi\epsilon_0} \frac{\hat{\mathbf{d}}_1 \cdot \hat{\mathbf{d}}_2 - 3(\hat{\mathbf{d}}_1 \cdot \hat{\mathbf{n}})(\hat{\mathbf{d}}_2 \cdot \hat{\mathbf{n}})}{R^3}$$



$$\begin{cases} |g\rangle \rightarrow |\downarrow\rangle \\ |r\rangle \rightarrow |\uparrow\rangle \end{cases}$$

Ising-like interaction

$$\begin{cases} |S\rangle \rightarrow |\downarrow\rangle \\ |P\rangle \rightarrow |\uparrow\rangle \end{cases}$$

XY interaction (flip-flop)

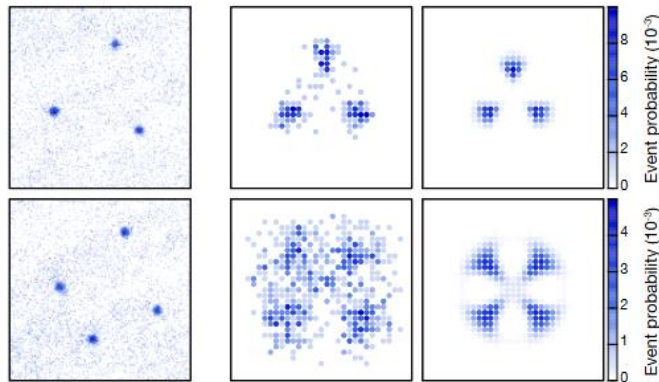
Quantum simulation of the Ising model



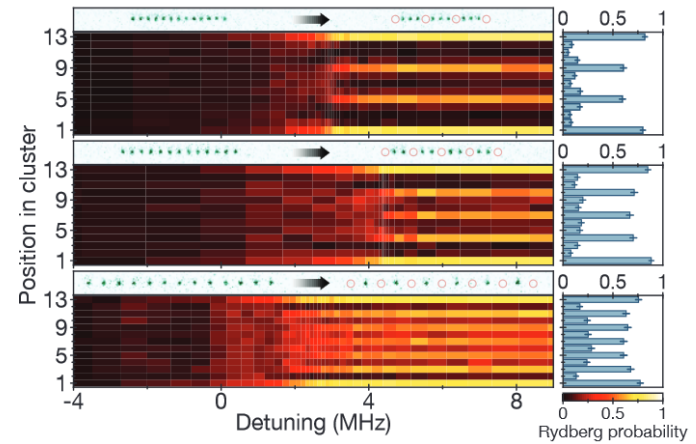
Andreas Läuchli
(PSI & EPFL)

P. Scholl *et al.*, [Nature](#) **595**, 233 (2021)

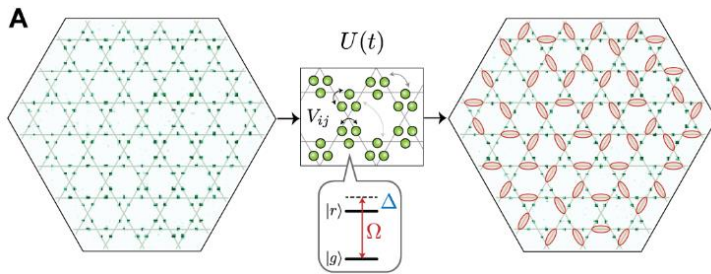
Many experiments using vdW interactions



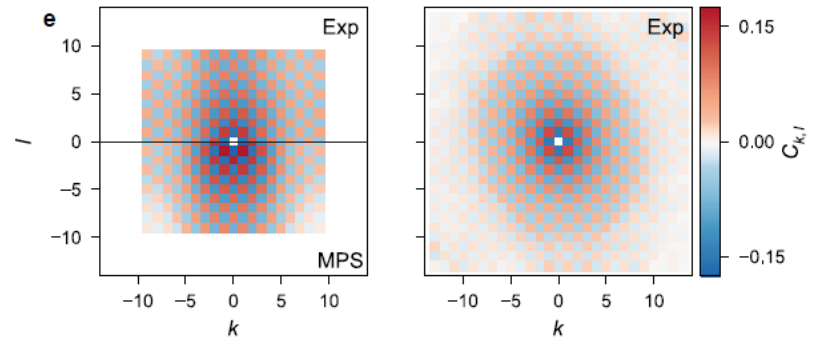
P. Schauss *et al.*, *Nature* **491**, 87 (2012)



Bernien *et al.*, *Nature* **551**, 579 (2017)



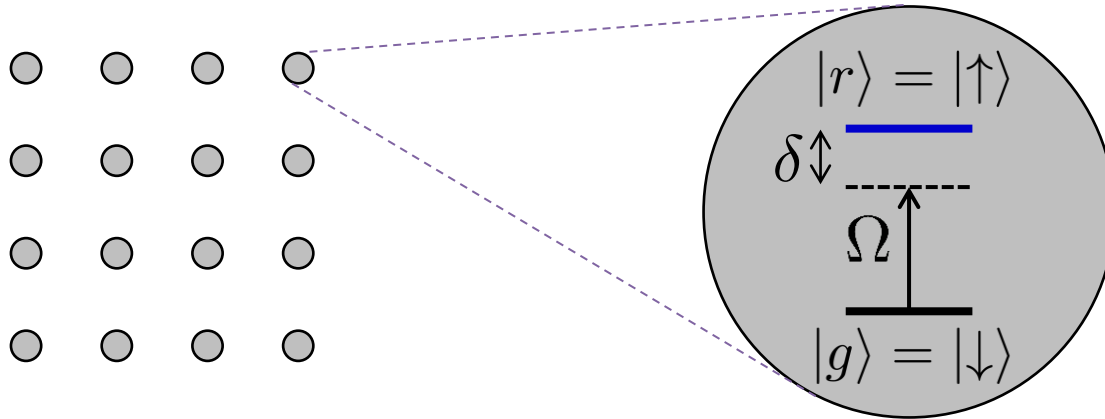
G. Semeghini *et al.*, *Science* **374**, 1242 (2021)



P. Scholl *et al.*, *Nature* **595**, 233 (2021).

And many, many more examples!

Blockade: quantum Ising model



$$n^i = |r_i\rangle \langle r_i|$$

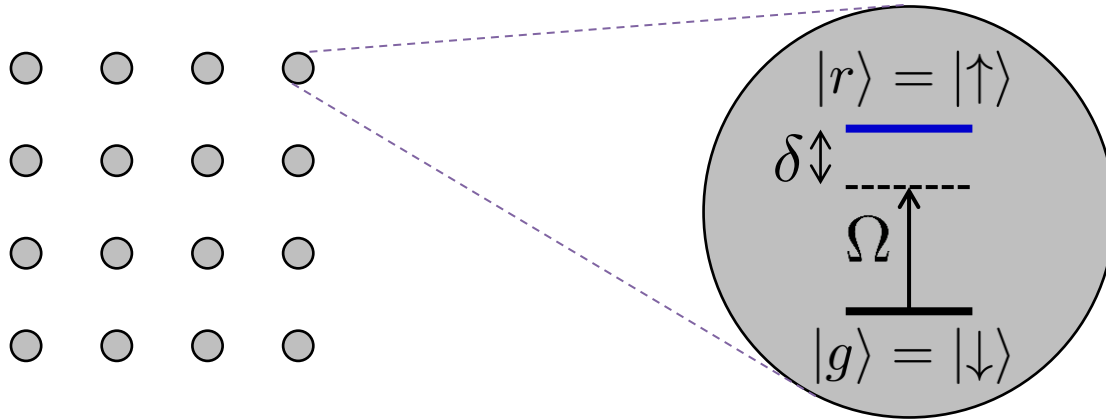
$$H = \sum_i \left(\frac{\hbar\Omega}{2} \sigma_x^i - \hbar\delta n^i \right) + \sum_{i<j} \frac{C_6}{R_{ij}^6} n_i n_j$$

Rabi frequency

Laser detuning

van der Waals interactions

Blockade: quantum Ising model



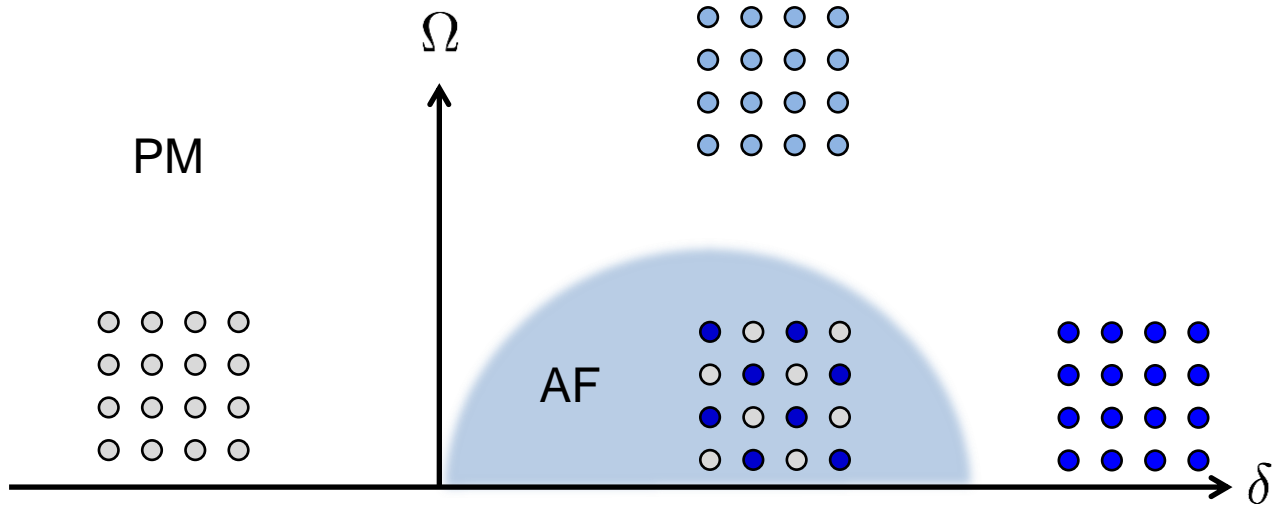
$$n^i = |r_i\rangle \langle r_i| = (1 + \sigma_z^i)/2$$

$$H = \sum_i \left(\frac{\hbar\Omega}{2} \sigma_x^i - \hbar\delta n^i \right) + \sum_{i < j} \frac{C_6}{R_{ij}^6} n_i n_j$$

Transverse B Longitudinal B Ising couplings

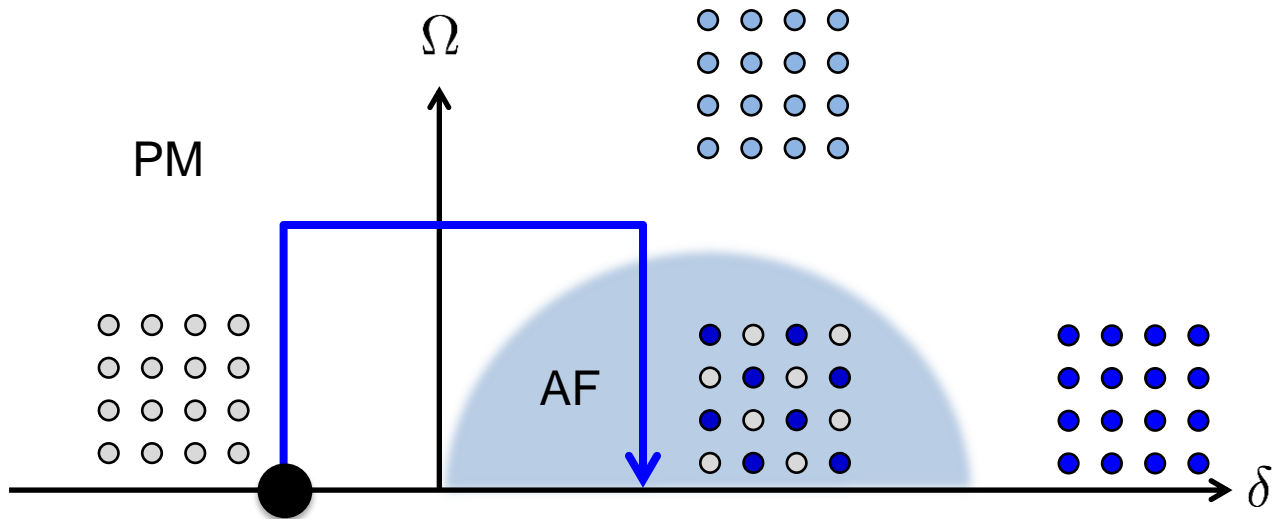
Ising model: adiabatic preparation

Ising AF phase diagram



Ising model: adiabatic preparation

Ising AF phase diagram

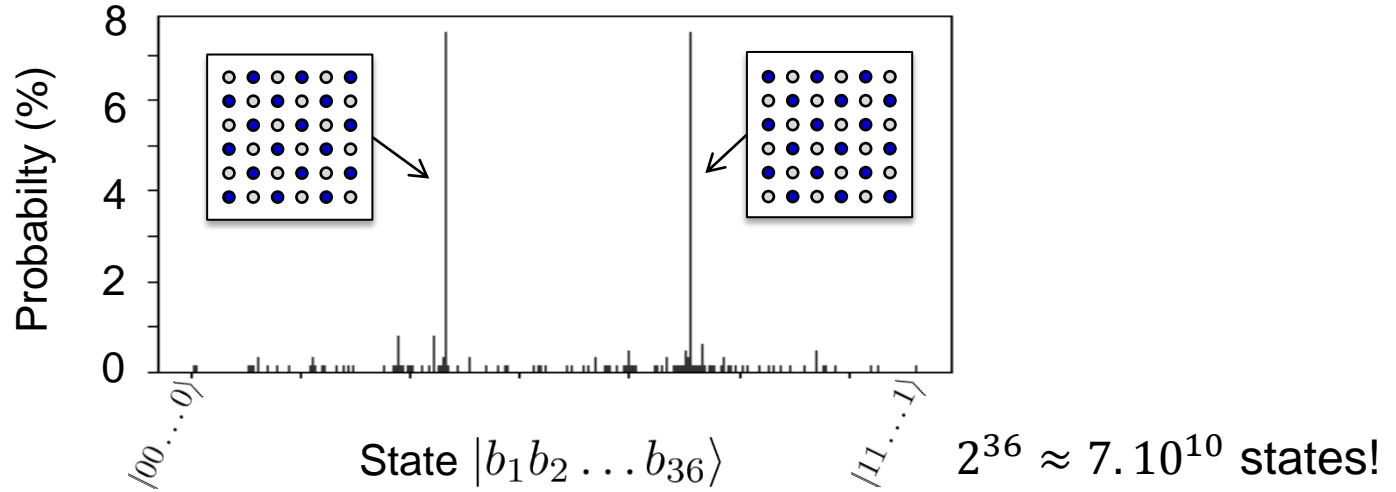
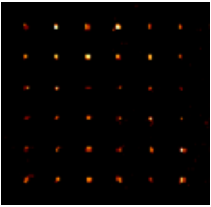


$$H = \sum_i \left(\frac{\hbar\Omega}{2} \sigma_x^i - \hbar\delta n^i \right) + \sum_{i < j} \frac{C_6}{R_{ij}^6} n_i n_j$$

Vary slowly **Rabi frequency** and **detuning** to explore the phase diagram

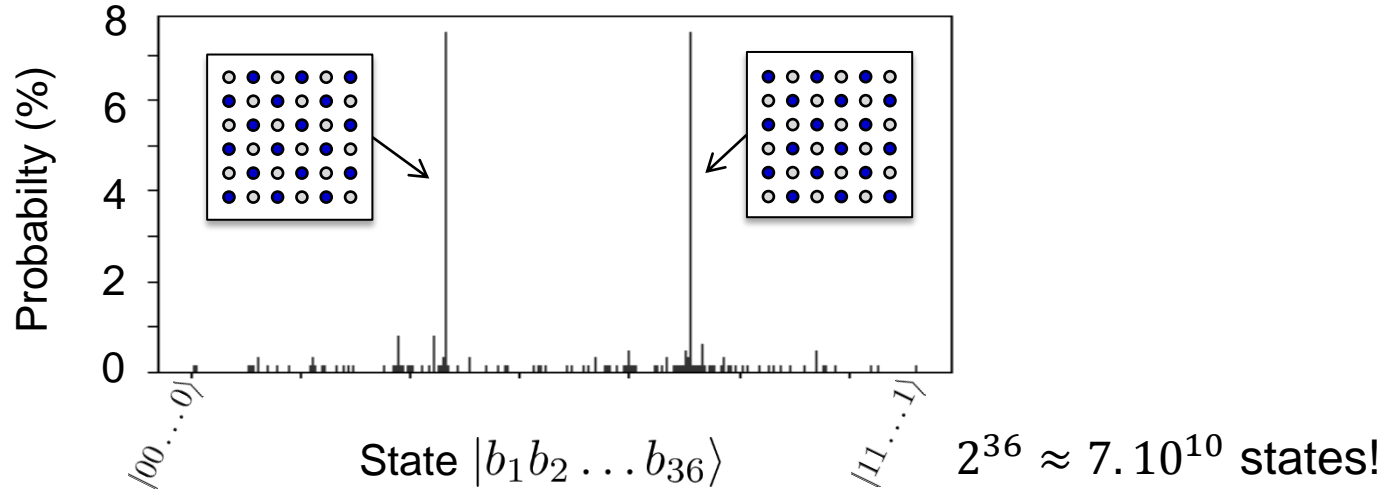
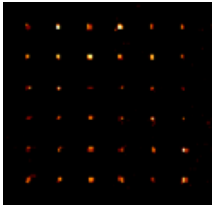
'Adiabatic' preparation on a square array

6×6
square array

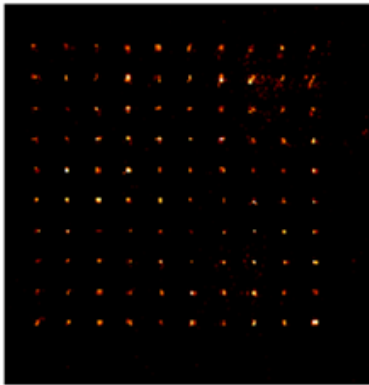


'Adiabatic' preparation on a square array

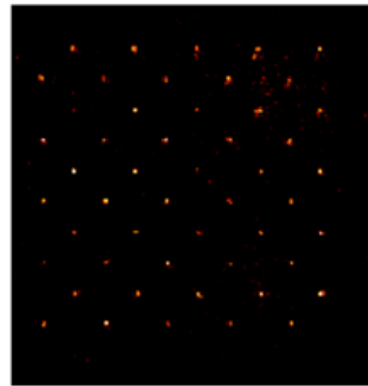
6×6
square array



10×10



sweep



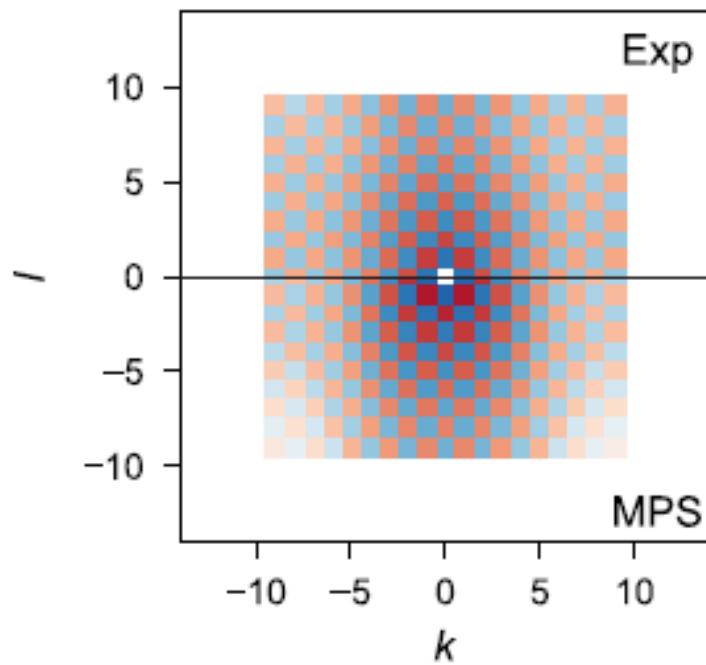
Perfect AF ordering!

(1 shot in 500)

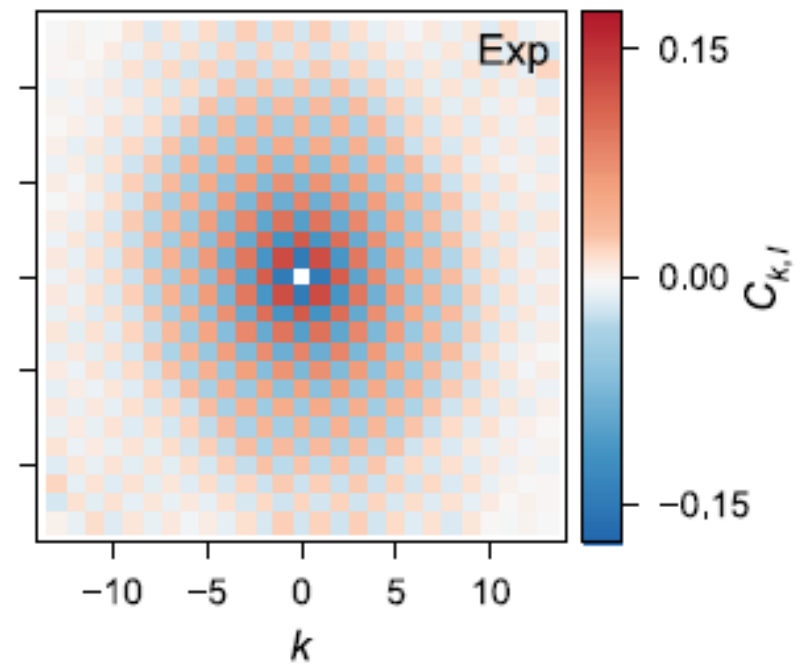
Correlation functions

$$C_{k,l} = \frac{1}{N_{k,l}} \sum_{i,j} \langle n_i n_j \rangle - \langle n_i \rangle \langle n_j \rangle$$

10×10 array



14×14 array



Quantum simulation of the XY model

Quantum simulation of the XY model

Theory support:



N. Yao
(Harvard)

M. Bintz
V. Liu
S. Chatterjee

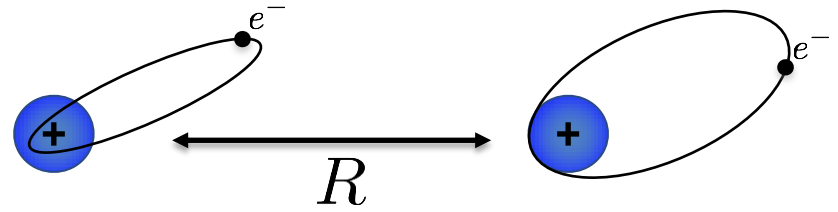
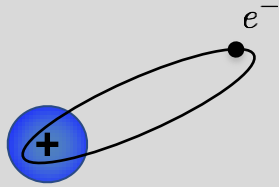


T. Roscilde

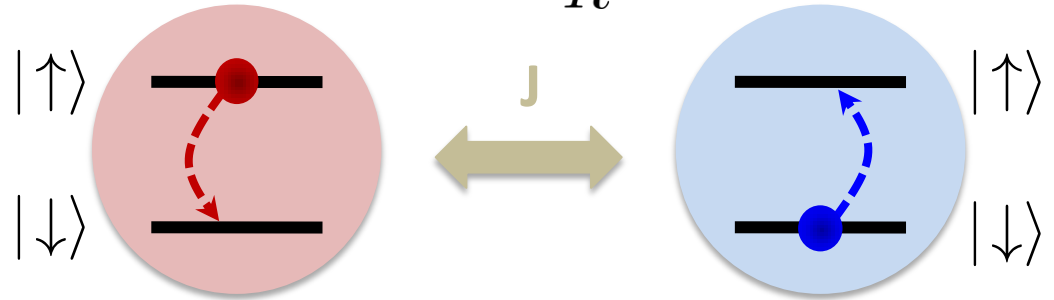
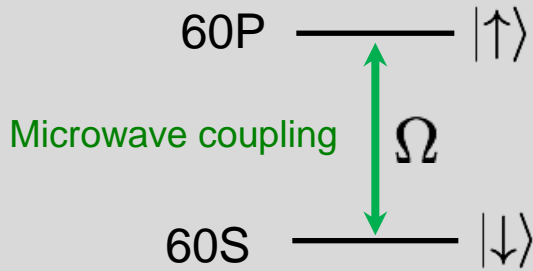


F. Mezzacapo
(Lyon)

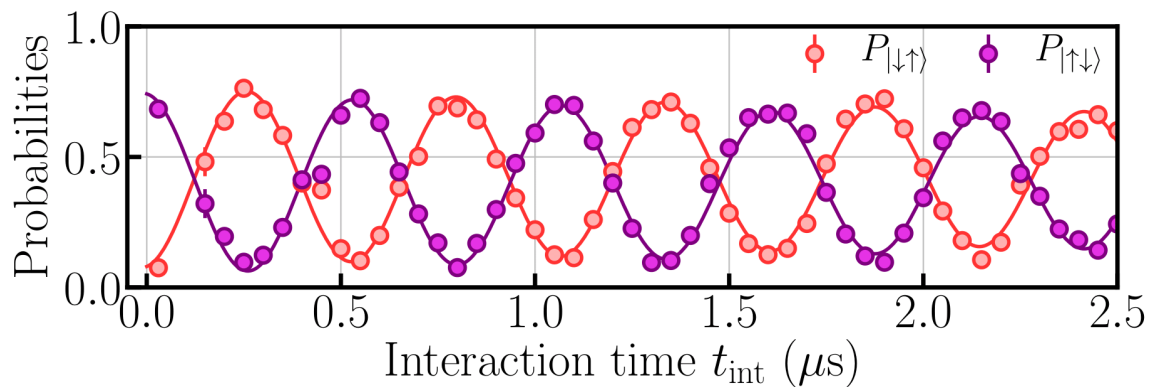
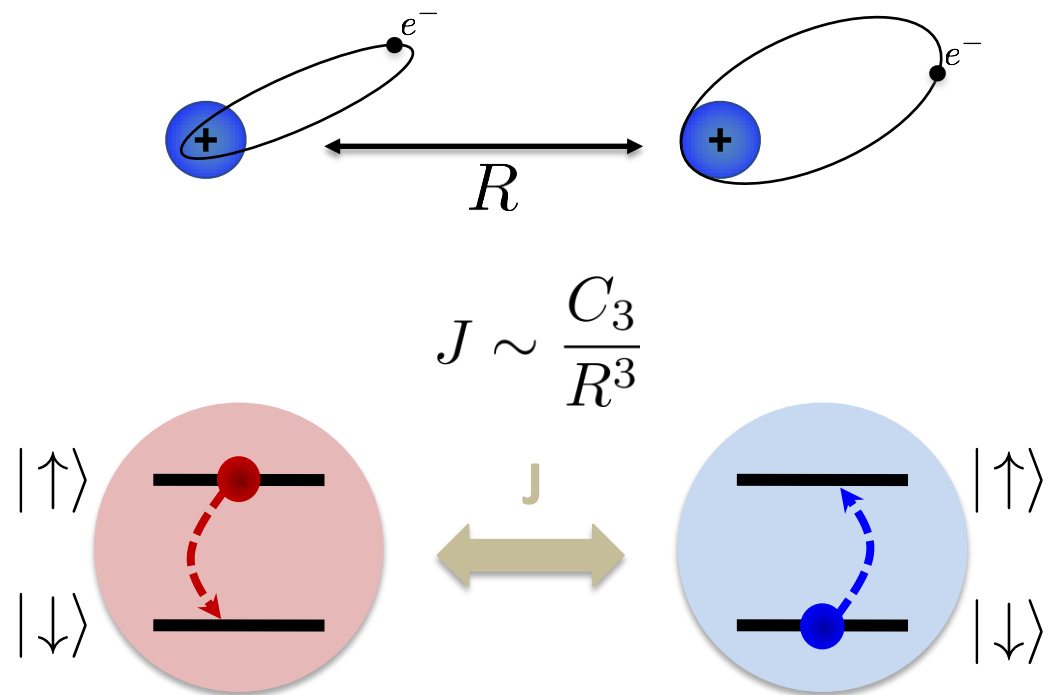
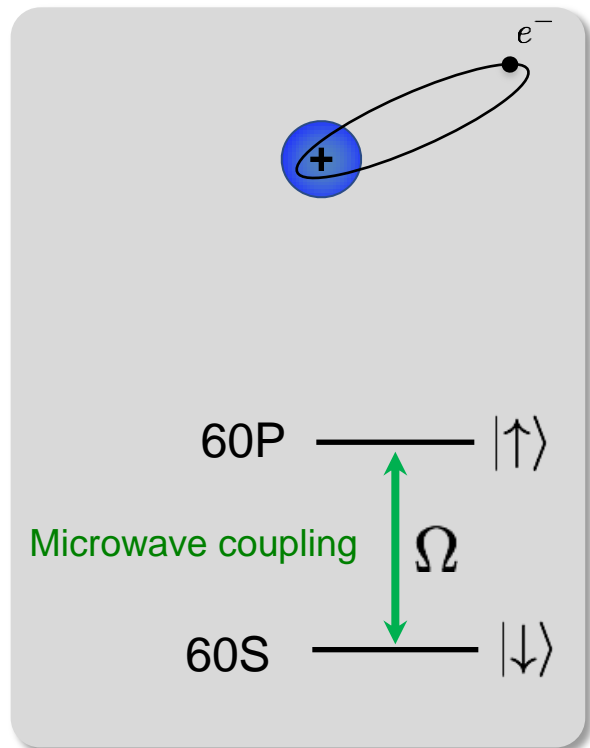
Resonant dipole-dipole interaction



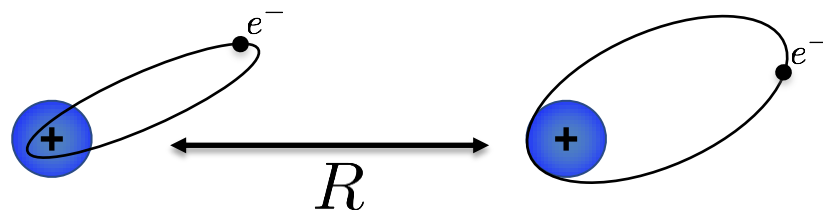
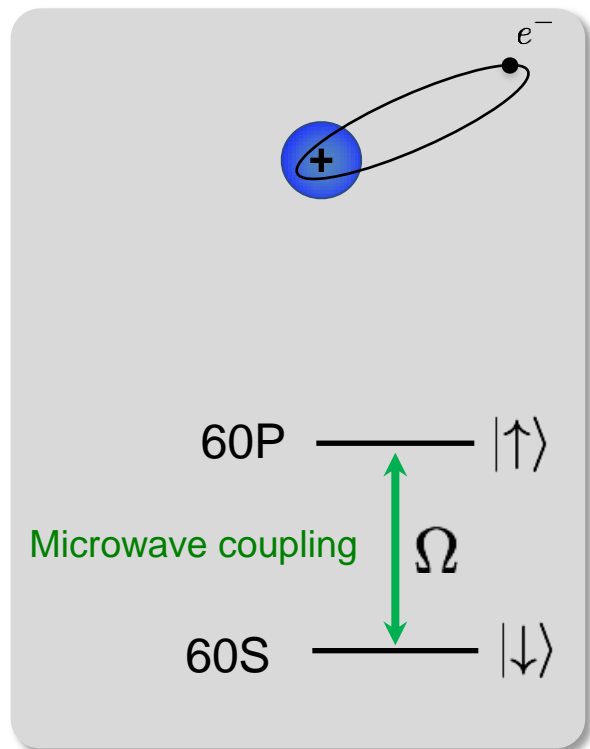
$$J \sim \frac{C_3}{R^3}$$



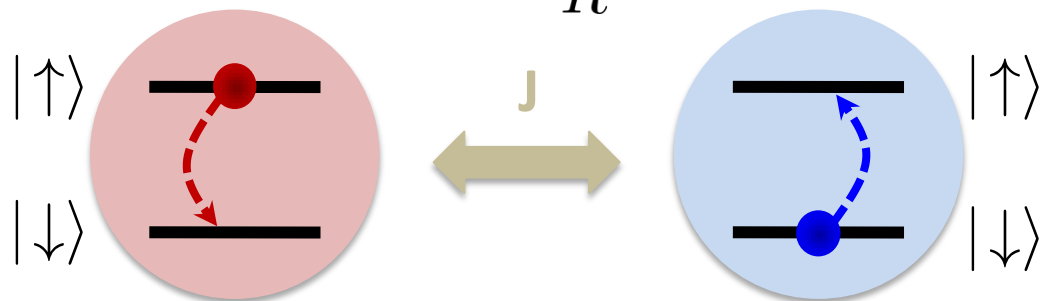
Resonant dipole-dipole interaction



Resonant dipole-dipole interaction

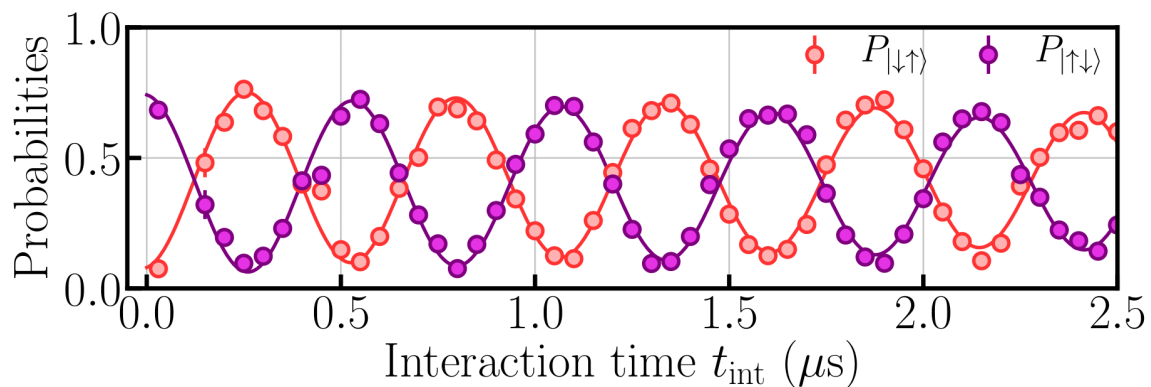


$$J \sim \frac{C_3}{R^3}$$



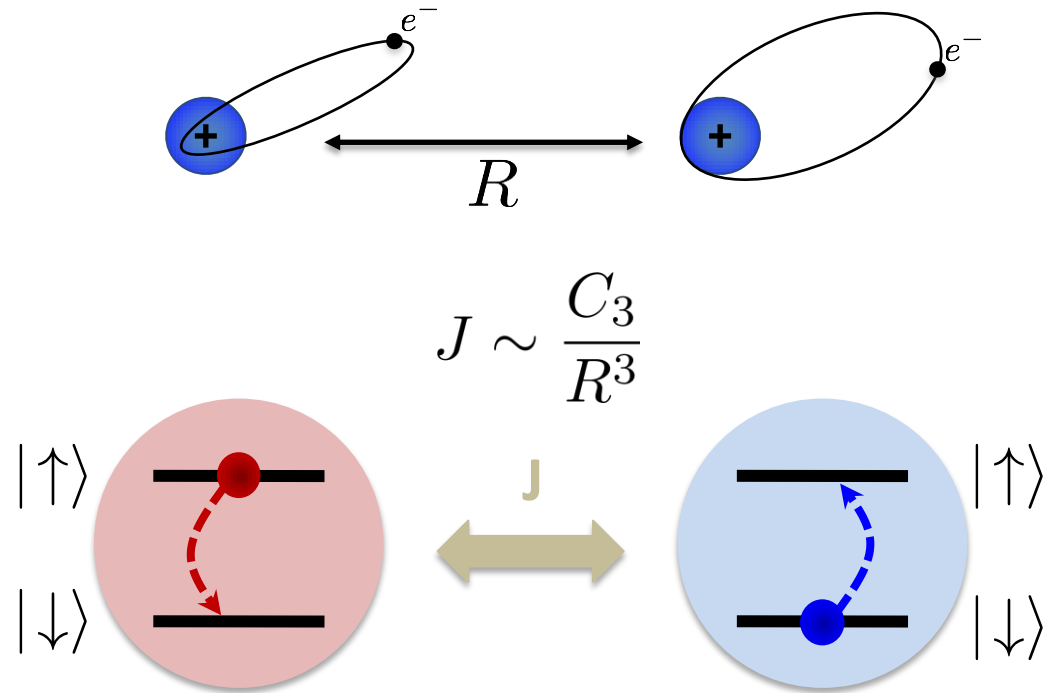
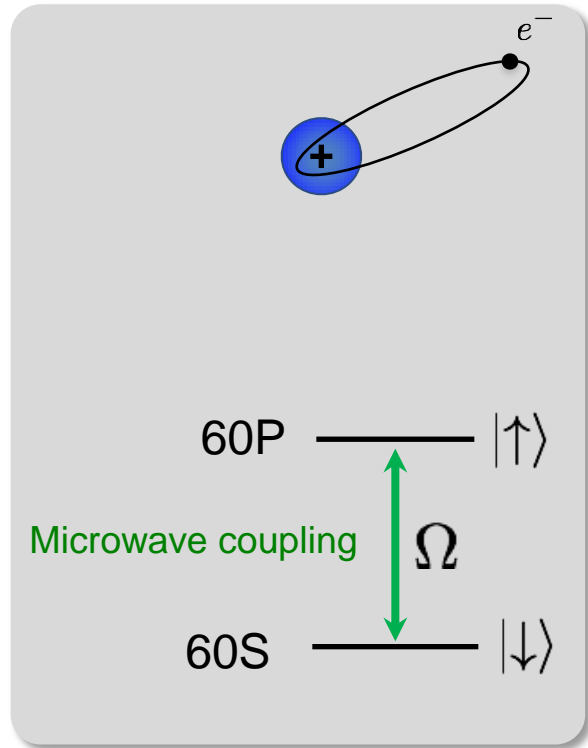
N atoms: XY model

$$H = \sum_{i \neq j} \frac{C_3}{R_{ij}^3} (\sigma_x^i \sigma_x^j + \sigma_y^i \sigma_y^j)$$



Barredo *et al.*, PRL **114**, 113002 (2015)

Resonant dipole-dipole interaction



Studies conducted using the resonant dipole-dipole interaction:

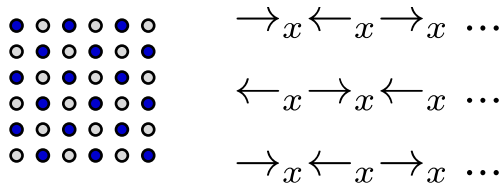
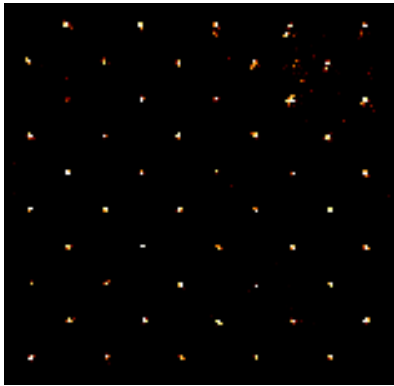
- Preparation of a many-body topological phase de Léséleuc *et al.*, [Science](#) **365**, 775 (2019)
- Implementation of a density-dependent Peierls phase Lienhard *et al.*, [PRX](#) **10**, 021031 (2020)
- Floquet engineering of XXZ Hamiltonians Scholl *et al.*, [PRX Quantum](#) **3**, 02303 (2022)

Ising vs. XY model

Ising model

$$\hat{H} = \sum_{\langle i,j \rangle} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x$$

AFM $J_{ij} < 0$



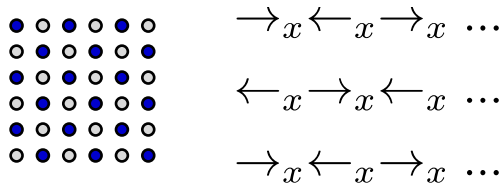
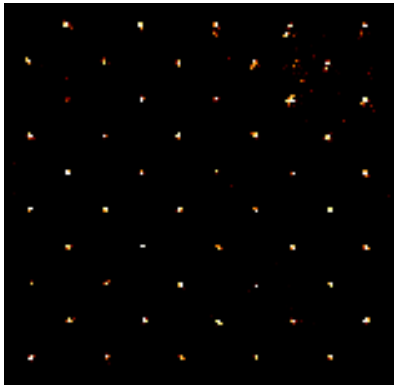
Ground state =
classical Néel configurations

Ising vs. XY model

Ising model

$$\hat{H} = \sum_{\langle i,j \rangle} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x$$

AFM $J_{ij} < 0$



Ground state =

classical Néel configurations

XY model

$$\hat{H} = \sum_{\langle i,j \rangle} J_{ij} (\hat{\sigma}_i^x \hat{\sigma}_j^x + \hat{\sigma}_i^y \hat{\sigma}_j^y)$$

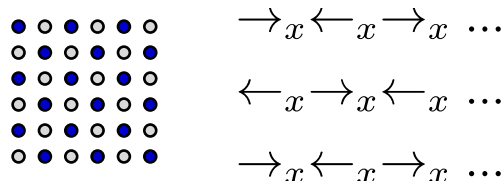
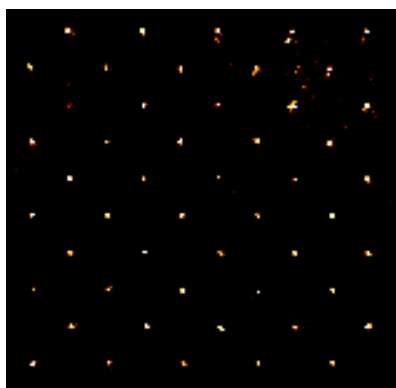
Ising vs. XY model

Ising model

$$\hat{H} = \sum_{\langle i,j \rangle} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x$$

AFM

$$J_{ij} < 0$$



Ground state =

classical Néel configurations

XY model

$$\hat{H} = \sum_{\langle i,j \rangle} J_{ij} (\hat{\sigma}_i^x \hat{\sigma}_j^x + \hat{\sigma}_i^y \hat{\sigma}_j^y)$$



Competing order along x / along y

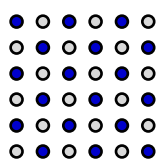
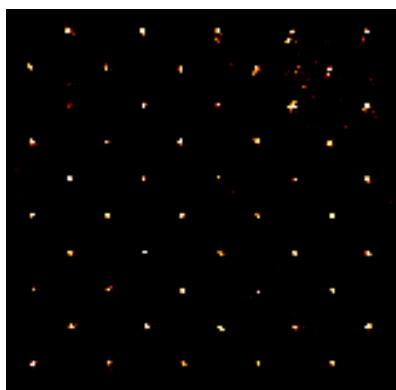
Ising vs. XY model

Ising model

$$\hat{H} = \sum_{\langle i,j \rangle} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x$$

AFM

$$J_{ij} < 0$$



$\rightarrow x \leftarrow x \rightarrow x \dots$

$\leftarrow x \rightarrow x \leftarrow x \dots$

$\rightarrow x \leftarrow x \rightarrow x \dots$

XY model

$$\hat{H} = \sum_{\langle i,j \rangle} J_{ij} (\hat{\sigma}_i^x \hat{\sigma}_j^x + \hat{\sigma}_i^y \hat{\sigma}_j^y)$$

$$= \sum_{\langle i,j \rangle} \frac{J_{ij}}{2} (\hat{\sigma}_i^+ \hat{\sigma}_j^- + \hat{\sigma}_i^- \hat{\sigma}_j^+)$$

Ground state =
classical Néel configurations

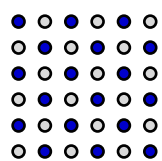
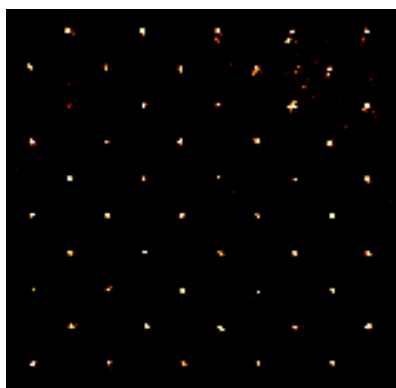
Ising vs. XY model

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$$\hat{H} = \sum_{\langle i,j \rangle} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x$$

AFM

$$J_{ij} < 0$$



$\rightarrow x \leftarrow x \rightarrow x \dots$

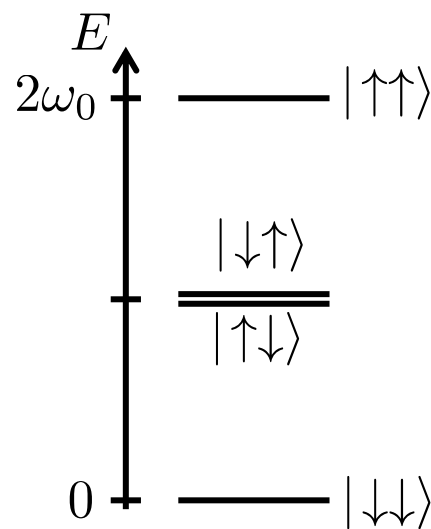
$\leftarrow x \rightarrow x \leftarrow x \dots$

$\rightarrow x \leftarrow x \rightarrow x \dots$

XY model

$$\hat{H} = \sum_{\langle i,j \rangle} J_{ij} (\hat{\sigma}_i^x \hat{\sigma}_j^x + \hat{\sigma}_i^y \hat{\sigma}_j^y)$$

$$= \sum_{\langle i,j \rangle} \frac{J_{ij}}{2} (\hat{\sigma}_i^+ \hat{\sigma}_j^- + \hat{\sigma}_i^- \hat{\sigma}_j^+)$$



Ground state =
classical Néel configurations

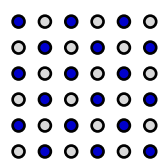
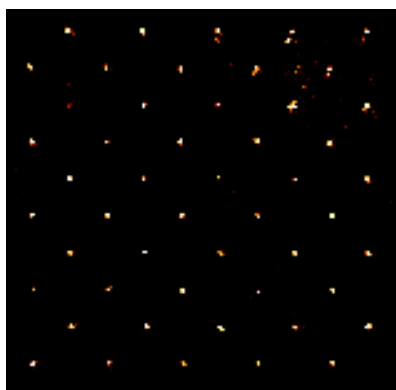
Ising vs. XY model

Ising model

$$\hat{H} = \sum_{\langle i,j \rangle} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x$$

AFM

$$J_{ij} < 0$$



$\rightarrow x \leftarrow x \rightarrow x \dots$

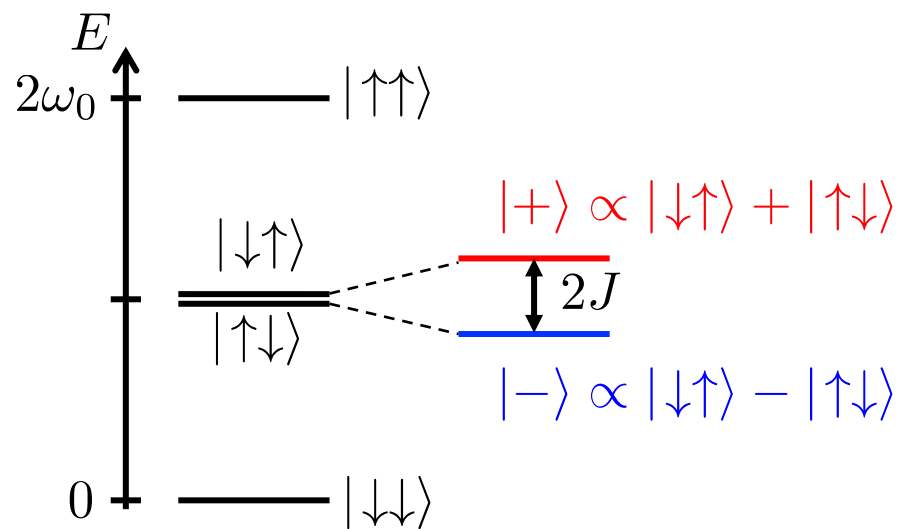
$\leftarrow x \rightarrow x \leftarrow x \dots$

$\rightarrow x \leftarrow x \rightarrow x \dots$

XY model

$$\hat{H} = \sum_{\langle i,j \rangle} J_{ij} (\hat{\sigma}_i^x \hat{\sigma}_j^x + \hat{\sigma}_i^y \hat{\sigma}_j^y)$$

$$= \sum_{\langle i,j \rangle} \frac{J_{ij}}{2} (\hat{\sigma}_i^+ \hat{\sigma}_j^- + \hat{\sigma}_i^- \hat{\sigma}_j^+)$$



Ground state =

classical Néel configurations

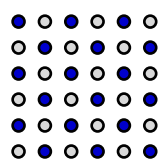
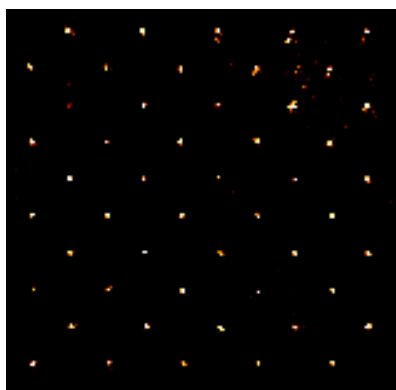
Ising vs. XY model

Ising model

$$\hat{H} = \sum_{\langle i,j \rangle} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x$$

AFM

$$J_{ij} < 0$$



$\rightarrow x \leftarrow x \rightarrow x \dots$

$\leftarrow x \rightarrow x \leftarrow x \dots$

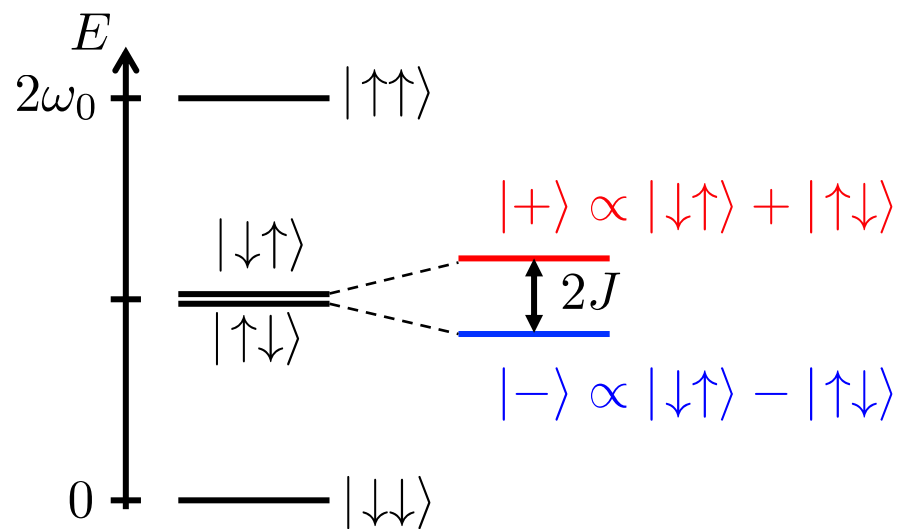
$\rightarrow x \leftarrow x \rightarrow x \dots$

Ground state =
classical Néel configurations

XY model

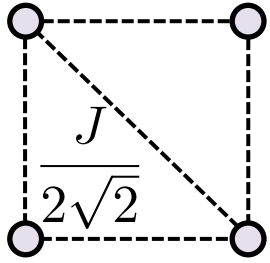
$$\hat{H} = \sum_{\langle i,j \rangle} J_{ij} (\hat{\sigma}_i^x \hat{\sigma}_j^x + \hat{\sigma}_i^y \hat{\sigma}_j^y)$$

$$= \sum_{\langle i,j \rangle} \frac{J_{ij}}{2} (\hat{\sigma}_i^+ \hat{\sigma}_j^- + \hat{\sigma}_i^- \hat{\sigma}_j^+)$$



Ground state =
non-classical entangled state

XY on square lattice (1/2 filling)

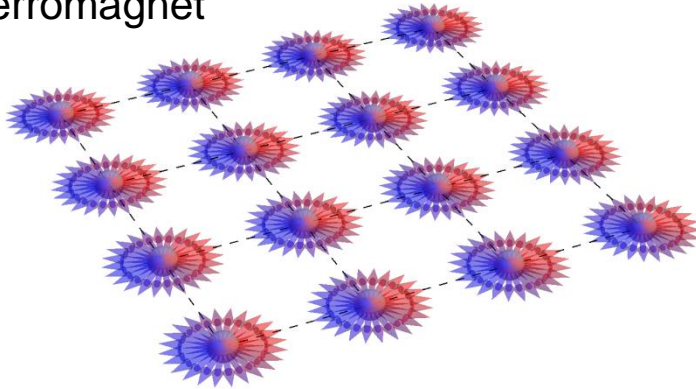


Ansätze wavefunctions

continuous $U(1)$ symmetry

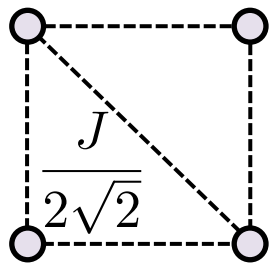
$$M^z = \sum_i \sigma_i^z \quad \text{conserved}$$

XY ferromagnet



$$|\text{FM}\rangle_{\text{XY}} \propto \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-i\phi S_z} |\text{FM}\rangle_{\text{X}}$$

XY on square lattice (1/2 filling)

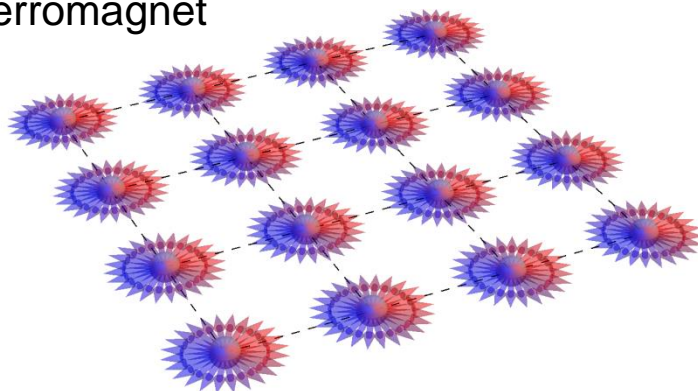


Ansätze wavefunctions

continuous $U(1)$ symmetry

$$M^z = \sum_i \sigma_i^z \quad \text{conserved}$$

XY ferromagnet



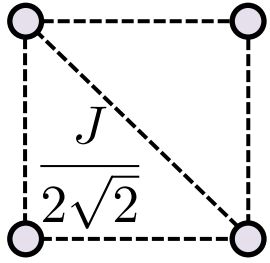
$$|\text{FM}\rangle_{\text{XY}} \propto \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-i\phi S_z} |\text{FM}\rangle_{\text{X}}$$

Expect: $\langle \hat{X} \rangle = 0$

$$\langle \hat{X} \hat{X} \rangle_{NN}^F > 0$$

$$\langle \hat{X} \hat{X} \rangle_{NNN}^F > 0$$

XY on square lattice (1/2 filling)

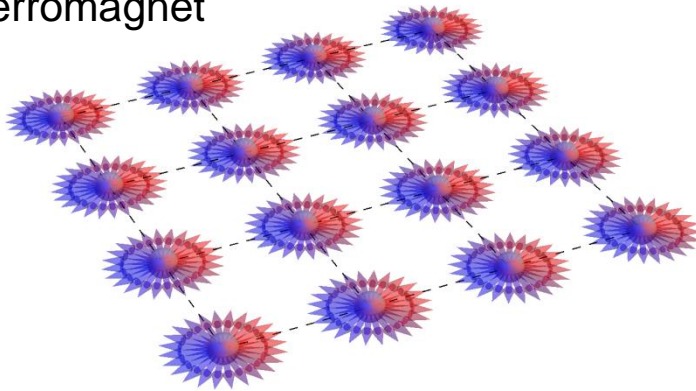


Ansätze wavefunctions

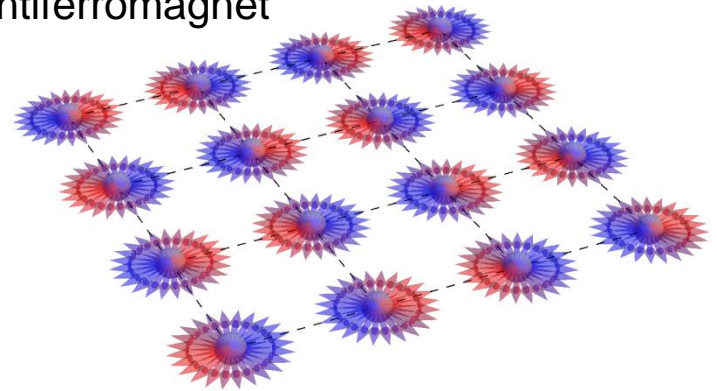
continuous $U(1)$ symmetry

$$M^z = \sum_i \sigma_i^z \quad \text{conserved}$$

XY ferromagnet



XY antiferromagnet



$$|\text{FM}\rangle_{\text{XY}} \propto \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-i\phi S_z} |\text{FM}\rangle_{\text{X}}$$

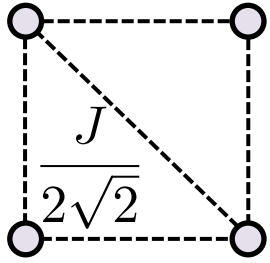
$$|\text{AFM}\rangle_{\text{XY}} \propto \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-i\phi S_z} |\text{AFM}\rangle_{\text{X}}$$

Expect: $\langle \hat{X} \rangle = 0$

$$\langle \hat{X} \hat{X} \rangle_{NN}^F > 0$$

$$\langle \hat{X} \hat{X} \rangle_{NNN}^F > 0$$

XY on square lattice (1/2 filling)

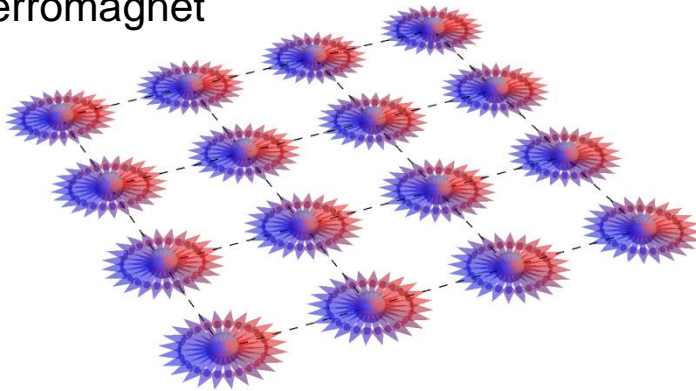


Ansätze wavefunctions

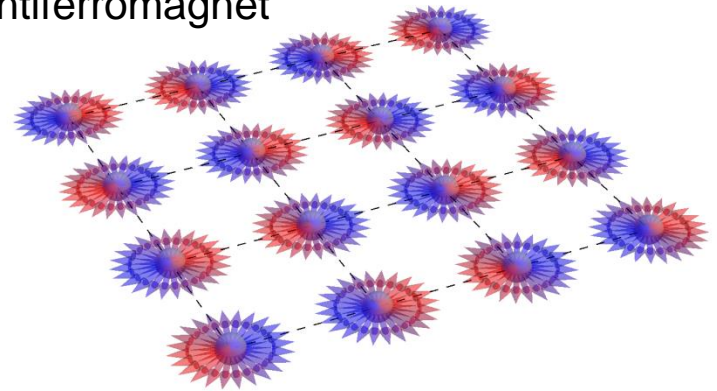
continuous $U(1)$ symmetry

$$M^z = \sum_i \sigma_i^z \quad \text{conserved}$$

XY ferromagnet



XY antiferromagnet



$$|\text{FM}\rangle_{\text{XY}} \propto \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-i\phi S_z} |\text{FM}\rangle_{\text{X}}$$

$$|\text{AFM}\rangle_{\text{XY}} \propto \int_0^{2\pi} \frac{d\phi}{2\pi} e^{-i\phi S_z} |\text{AFM}\rangle_{\text{X}}$$

Expect: $\langle \hat{X} \rangle = 0$

$$\langle \hat{X} \hat{X} \rangle_{NN}^F > 0$$

$$\langle \hat{X} \hat{X} \rangle_{NNN}^F > 0$$

$$\langle \hat{X} \rangle = 0$$

$$\langle \hat{X} \hat{X} \rangle_{NN}^{AF} < 0$$


$$\langle \hat{X} \hat{X} \rangle_{NNN}^{AF} > 0$$

Preparing FM and AFM XY magnets

C. Chen *et al.*, Nature **616**, 691 (2023)

Preparing XY ferro- and antiferromagnets

Start from:
$$H_{XY} = -J \sum_{i < j} \frac{a^3}{r_{ij}^3} (\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y) + \hbar \sum_i \delta_i \sigma_i^z$$

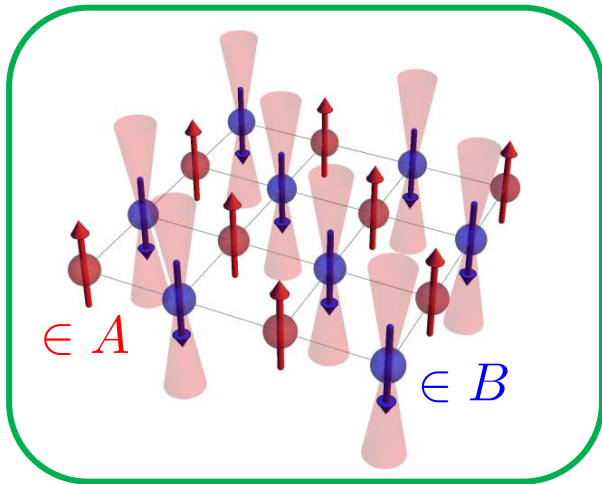
 **staggered**

Preparing XY ferro- and antiferromagnets

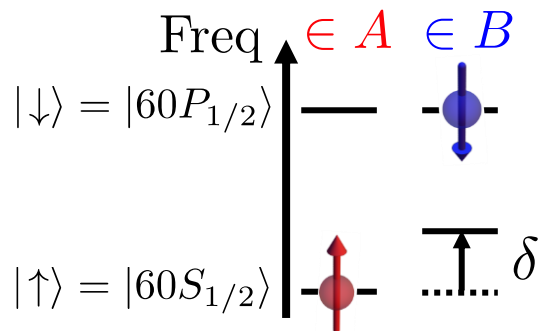
Start from:
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← staggered

1. Prepare a **classical Néel state** along z: checkerboard pattern



apply local light-shift
(2nd SLM)
+
microwaves

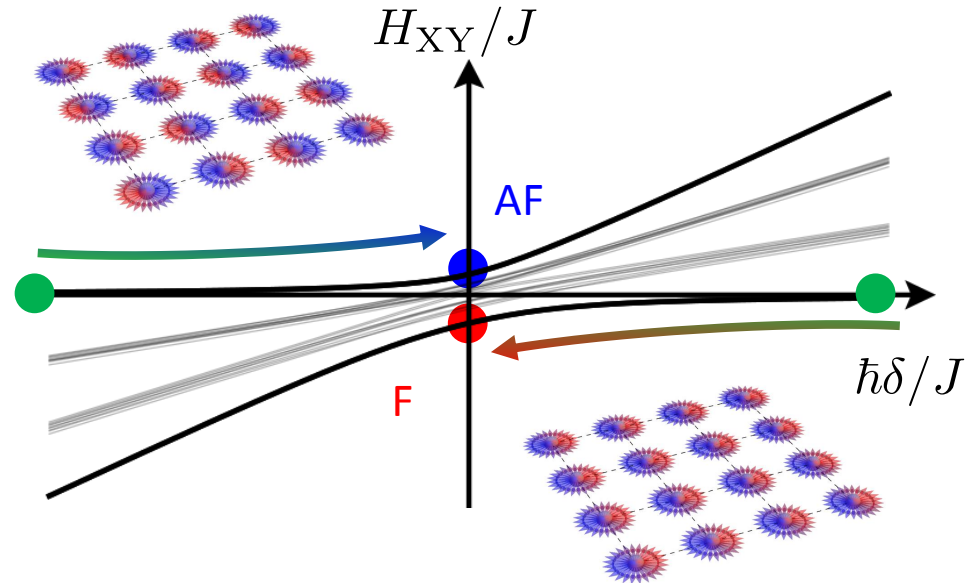
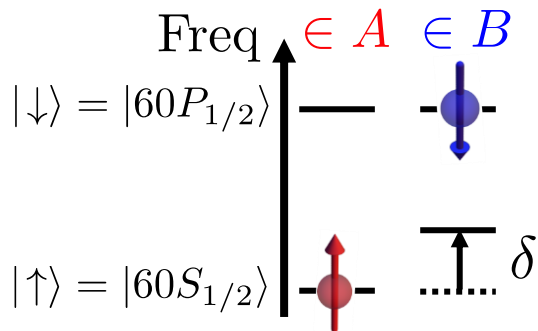
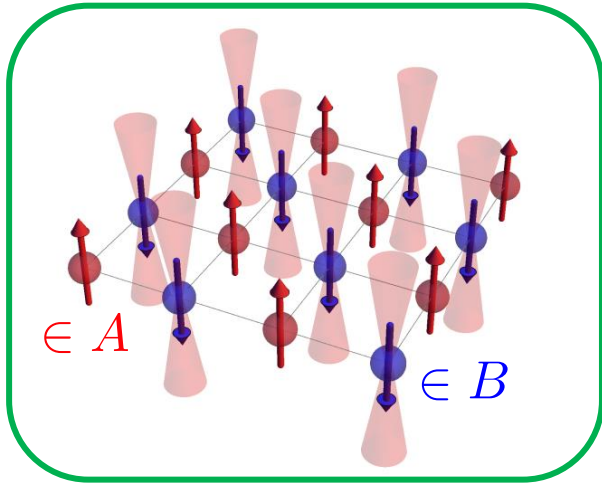


Preparing XY ferro- and antiferromagnets

Start from:
$$H_{XY} = -J \sum_{i < j} \frac{a^3}{r_{ij}^3} (\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y) + \hbar \sum_i \delta_i \sigma_i^z$$

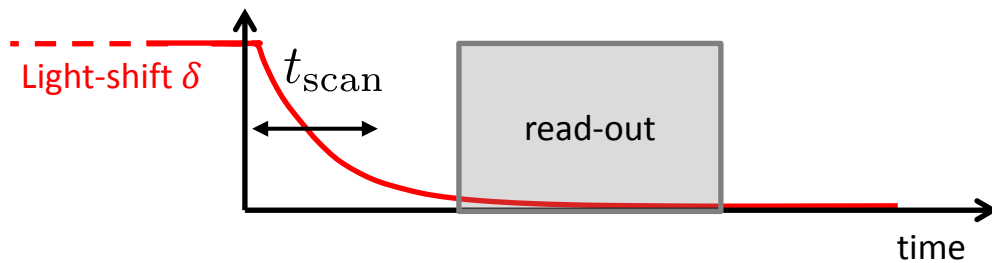
← staggered

2. Adiabatically decrease δ to “melt” into XY AF/F

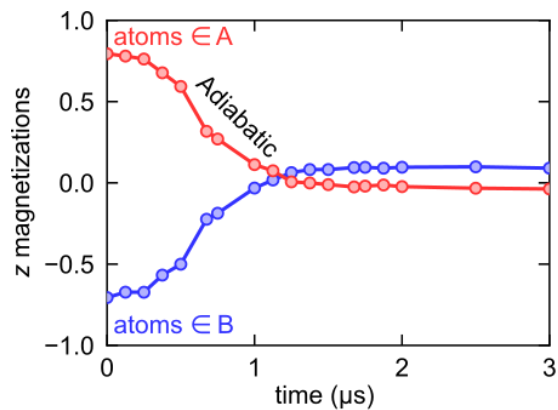


Preparing XY ferro- and antiferromagnets

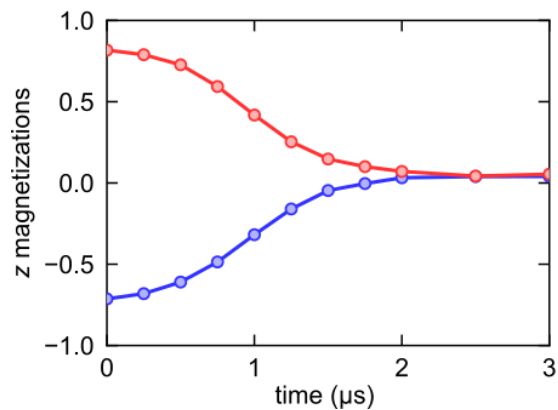
42 atoms



Ferromagnet

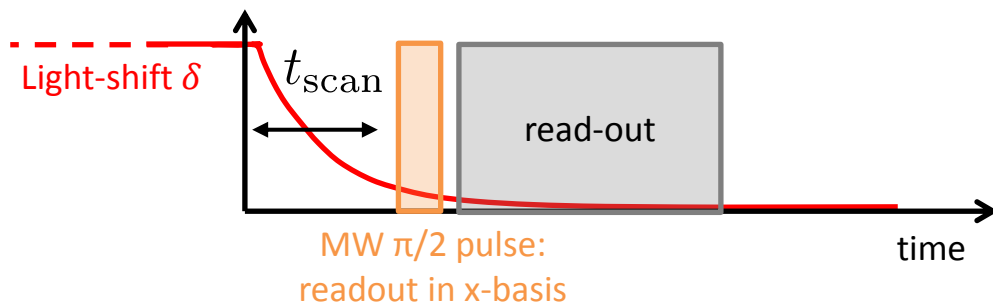


Antiferromagnet

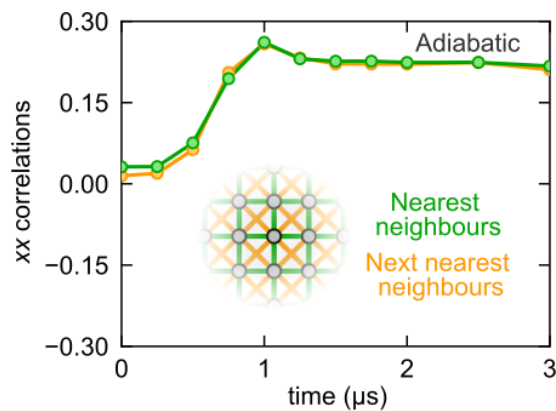
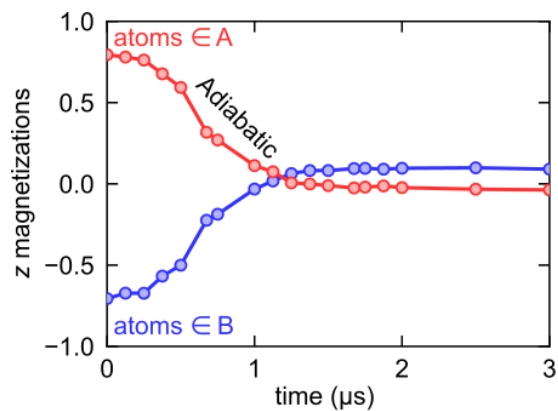


Preparing XY ferro- and antiferromagnets

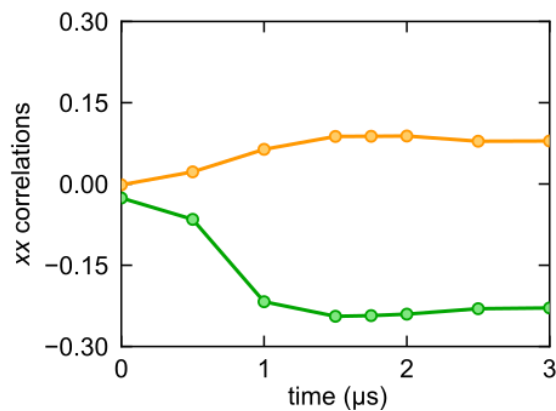
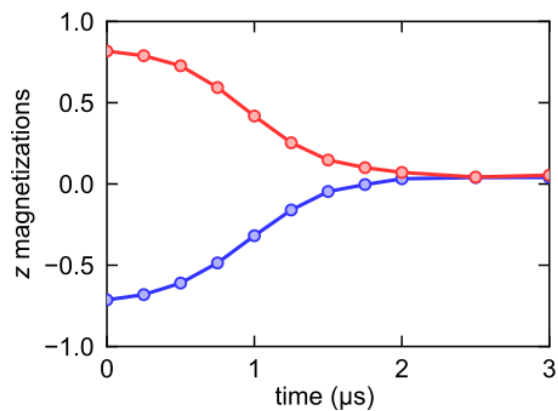
42 atoms



Ferromagnet

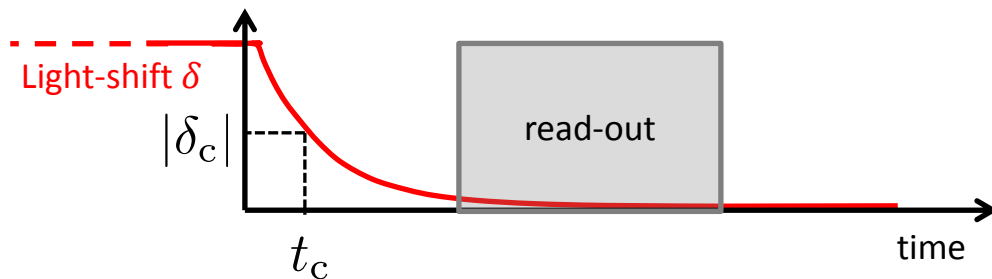


Antiferromagnet

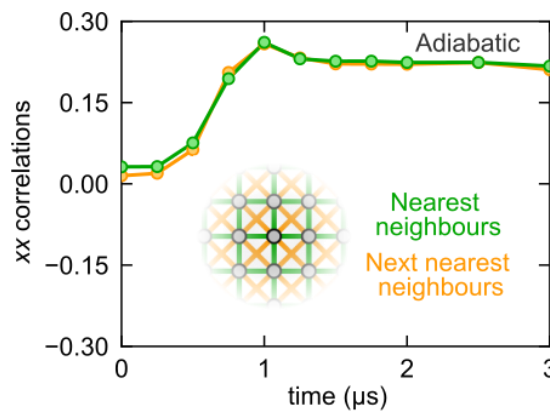
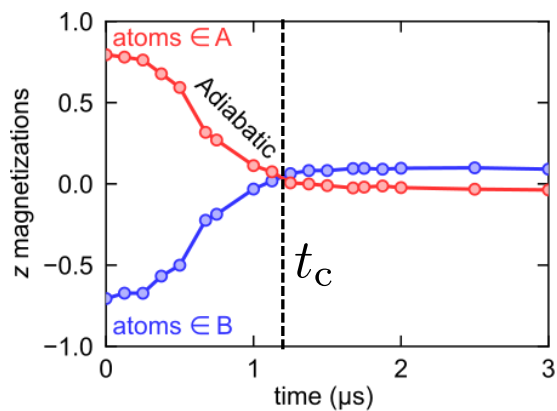


Preparing XY ferro- and antiferromagnets

42 atoms



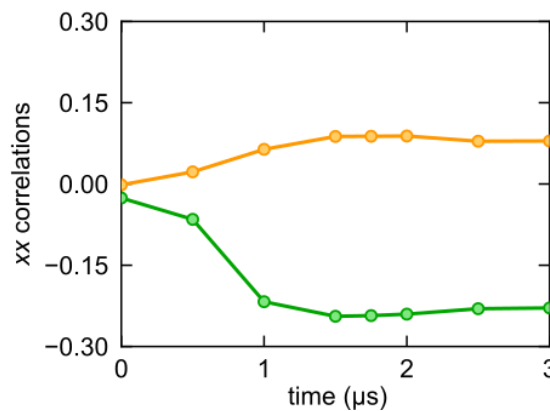
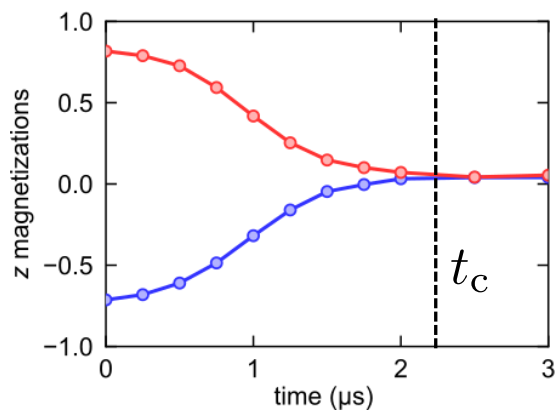
Ferromagnet



If only NN interactions:

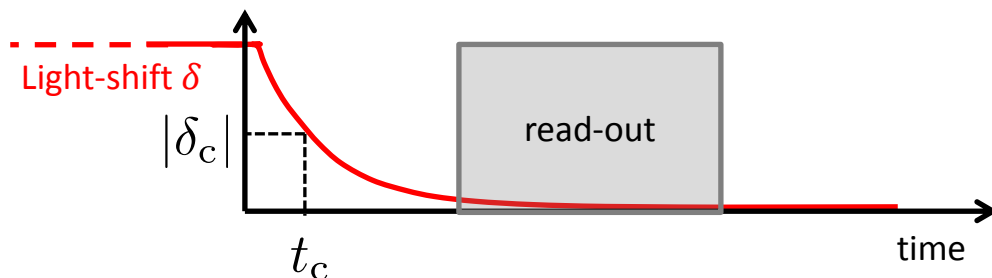
$$|\delta_c^{\text{AFM}}| = |\delta_c^{\text{FM}}|$$

Antiferromagnet

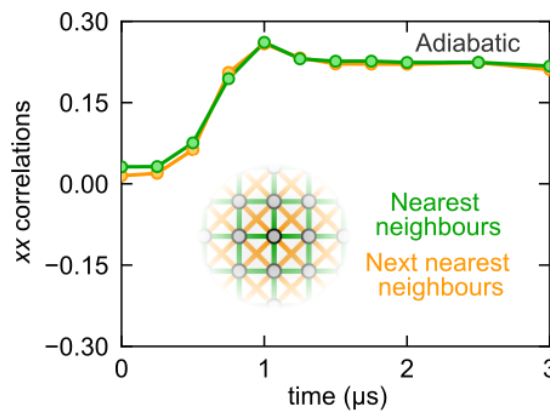
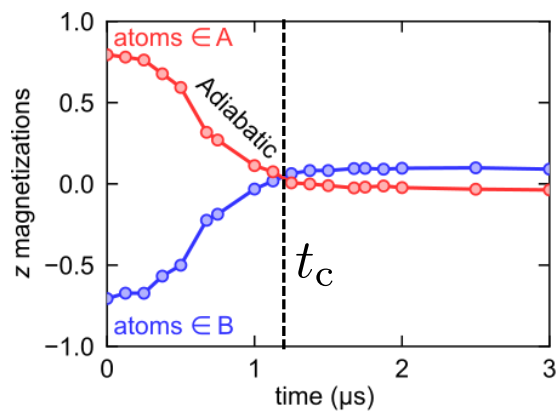


Preparing XY ferro- and antiferromagnets

42 atoms



Ferromagnet



If only NN interactions:

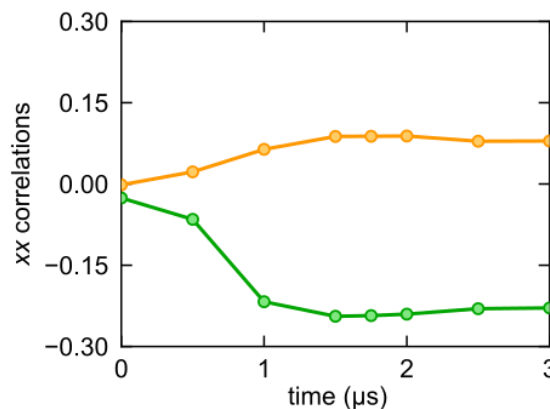
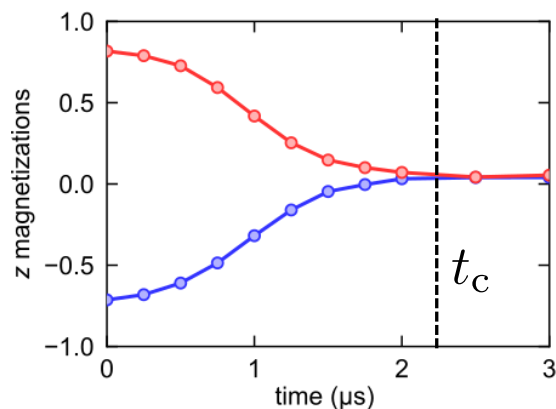
$$|\delta_c^{\text{AFM}}| = |\delta_c^{\text{FM}}|$$

Long-range dipolar interactions:

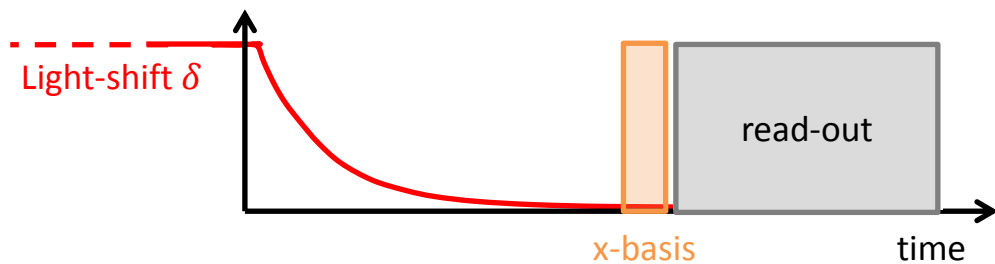
$$|\delta_c^{\text{AFM}}| < |\delta_c^{\text{FM}}|$$

AFM weakly frustrated interactions

Antiferromagnet

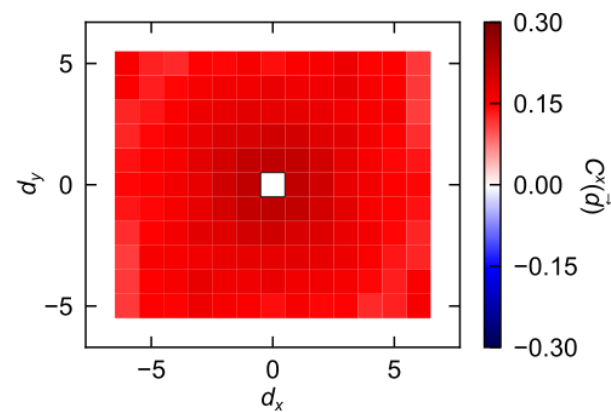
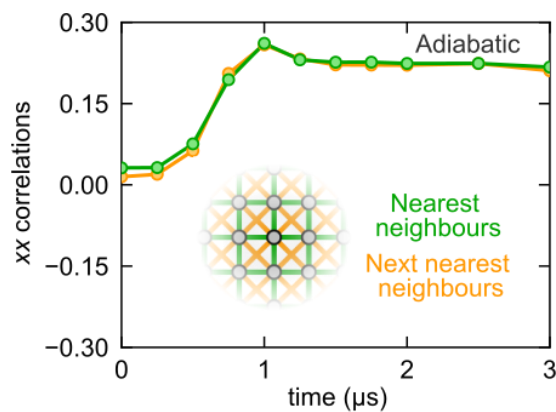
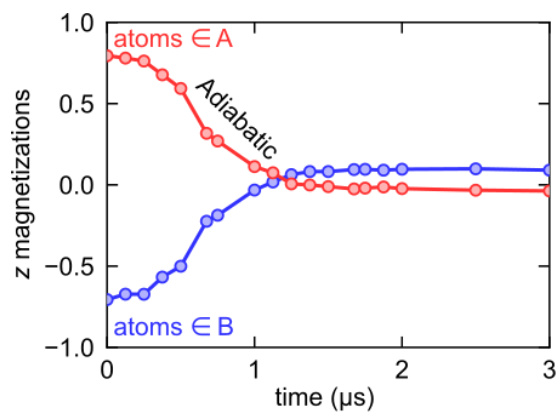


Preparing XY ferro- and antiferromagnets

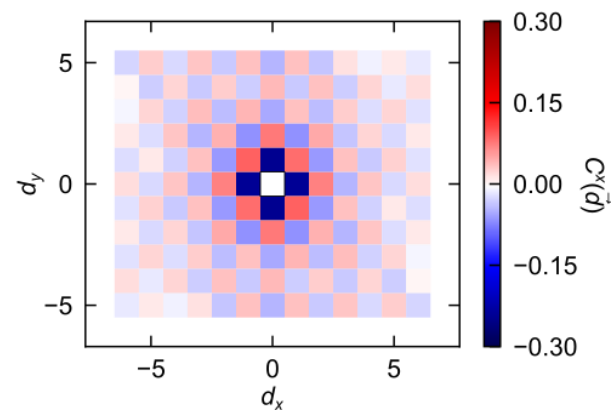
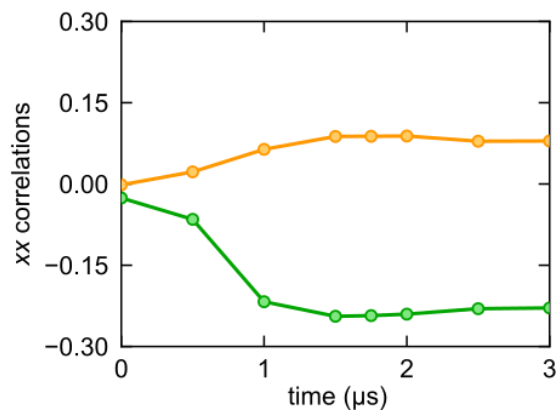
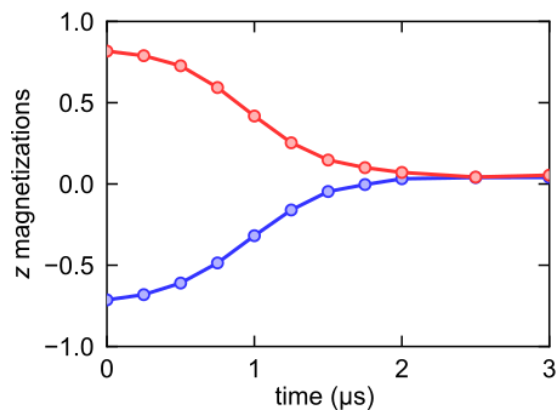


42 atoms

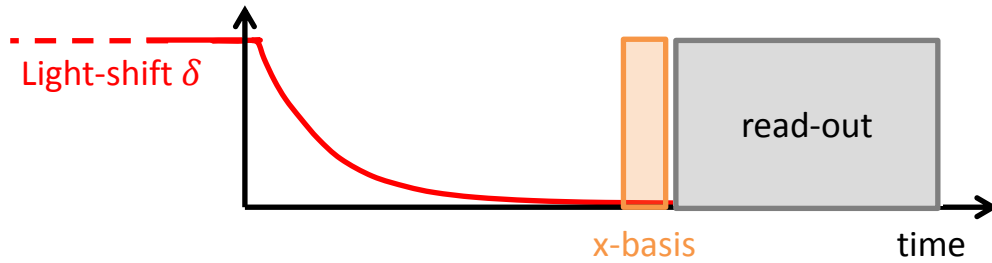
Ferromagnet



Antiferromagnet

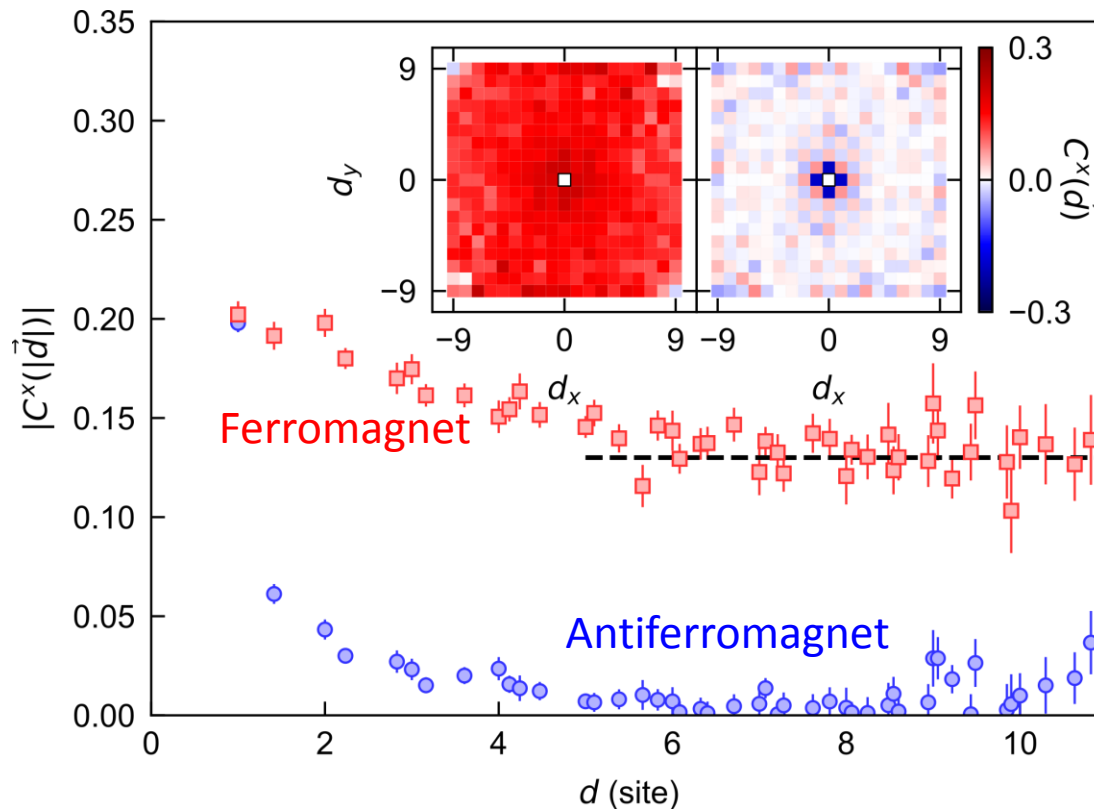


LRO for the FM case



100 atoms

$$C^x(\vec{d}) \equiv \langle C_{\vec{r}, \vec{r} + \vec{d}}^x \rangle_{\vec{r}}$$



Ferromagnet:

Long-range order

Antiferromagnet:

Correlations decay to 0

Crucial role of

$1/r^3$ interactions

Scalable spin squeezing in the dipolar XY model

G. Bornet *et al.*, [arXiv:2303.08053](https://arxiv.org/abs/2303.08053)

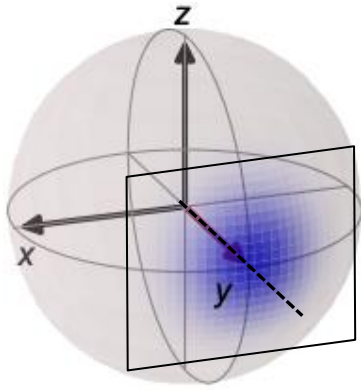
Scalable spin squeezing in the dipolar XY model

G. Bornet *et al.*, [arXiv:2303.08053](https://arxiv.org/abs/2303.08053)

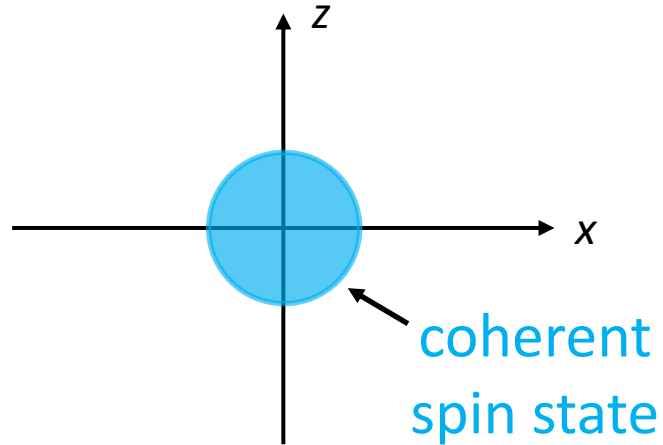
Similar results

- Trapped ions: [arXiv:2303.10688](https://arxiv.org/abs/2303.10688) (C. Roos)
- Dressed Rydberg atoms: [arXiv:2303.08078](https://arxiv.org/abs/2303.08078) (A. Kaufman), [arXiv:2303.08805](https://arxiv.org/abs/2303.08805) (M. Schleier-Smith)

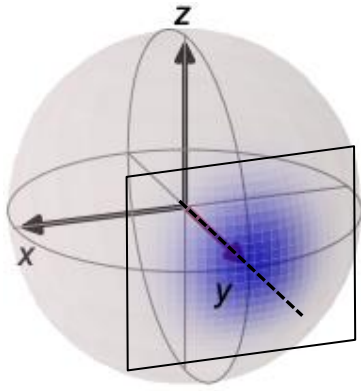
Spin squeezing



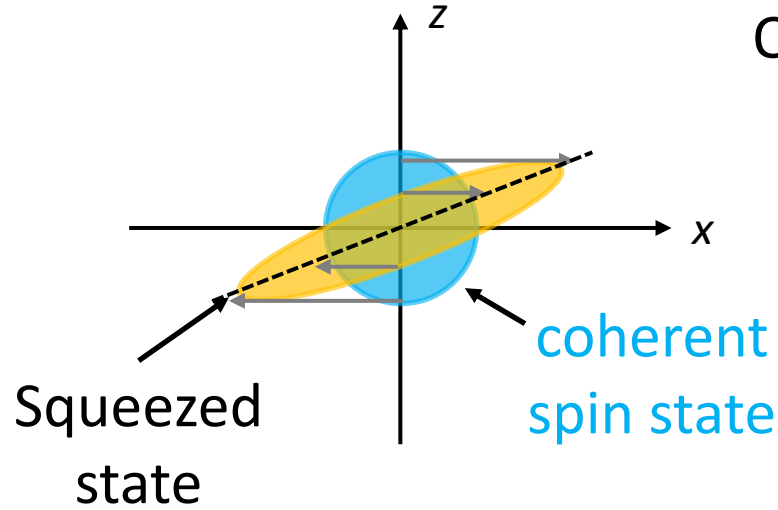
$$\Delta J_x \Delta J_z \geq \frac{|\langle J_y \rangle|}{2}$$



Spin squeezing



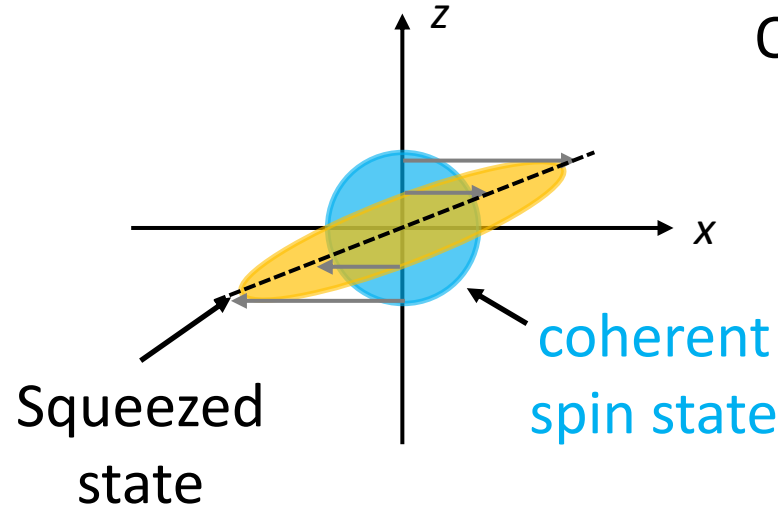
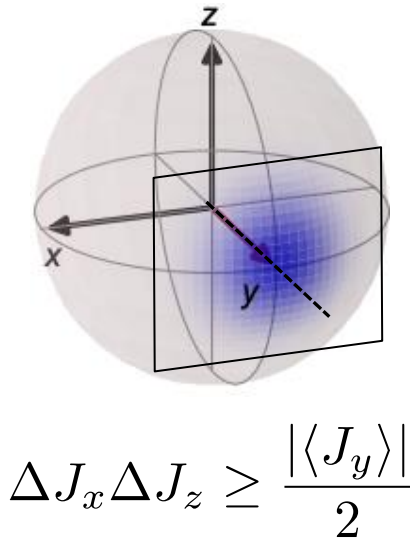
$$\Delta J_x \Delta J_z \geq \frac{|\langle J_y \rangle|}{2}$$



One-axis twisting model

$$\begin{aligned} H_{\text{OAT}} &= \chi J_z^2 \\ &= \chi \sum_{i,j} \hat{\sigma}_i^z \hat{\sigma}_j^z \end{aligned}$$

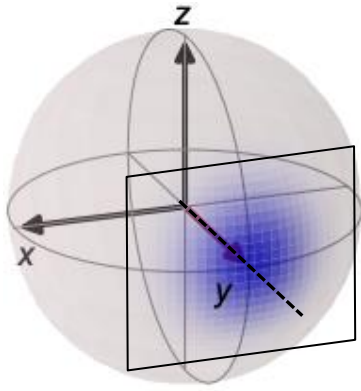
Spin squeezing



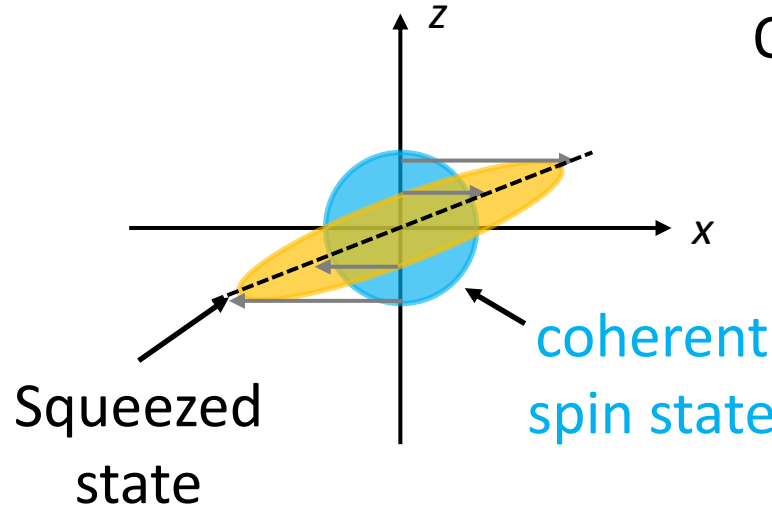
One-axis twisting model

$$\begin{aligned}
 H_{\text{OAT}} &= \chi J_z^2 \\
 &= \chi \sum_{i,j} \hat{\sigma}_i^z \hat{\sigma}_j^z \\
 \xi_R^2 &= N \frac{\min_{\perp} (\Delta J_{\perp}^2)}{\langle \mathbf{J} \rangle^2}
 \end{aligned}$$

Spin squeezing



$$\Delta J_x \Delta J_z \geq \frac{|\langle J_y \rangle|}{2}$$



One-axis twisting model

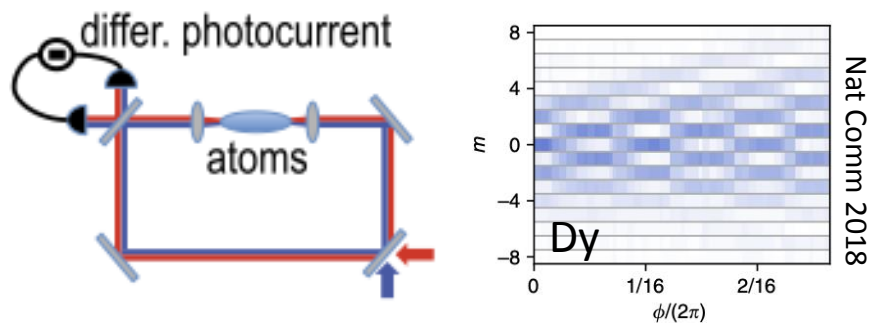
$$\begin{aligned} H_{\text{OAT}} &= \chi J_z^2 \\ &= \chi \sum_{i,j} \hat{\sigma}_i^z \hat{\sigma}_j^z \\ \xi_R^2 &= N \frac{\min_{\perp} (\Delta J_{\perp}^2)}{\langle \mathbf{J} \rangle^2} \end{aligned}$$

Metrological gain in Ramsey interf.: $\delta\theta_{\text{sq}} = \xi_R^2 \delta\theta_{\text{SQL}}$ Wineland, PRA 1994

Experimental observations of spin squeezing

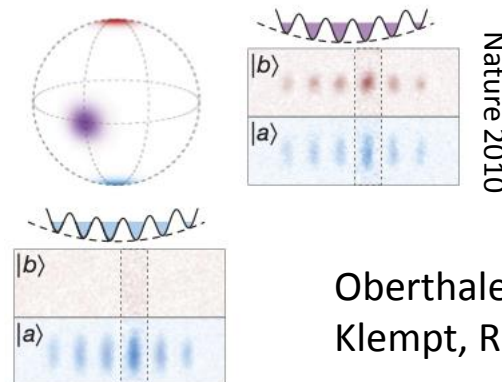
Pezzé *et al.*, RMP 2018

Hot / cold atomic vapors



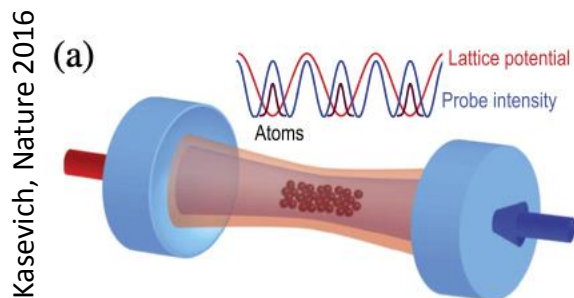
Polzik (1999), Giacobino, Mitchell, Nascimbene...

Bose-Einstein condensate (OAT)



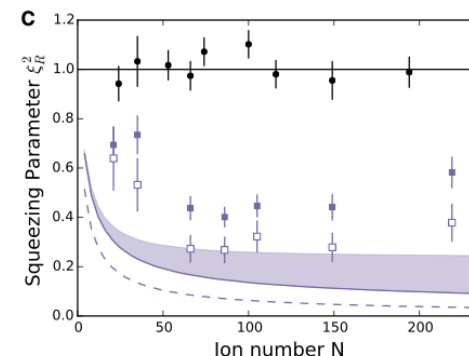
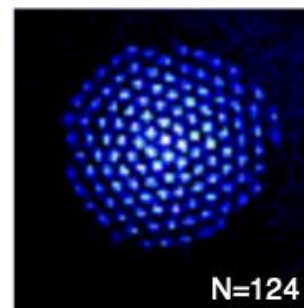
Oberthaler, Treutlein, Klempt, Reichel, You...

Cavity QED + cold atoms (OAT)



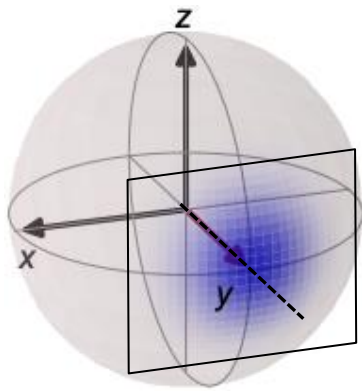
Vuletic, Kasevich, Thompson (JILA), Je, Schleier-Smith...

Ion crystal (~OAT)

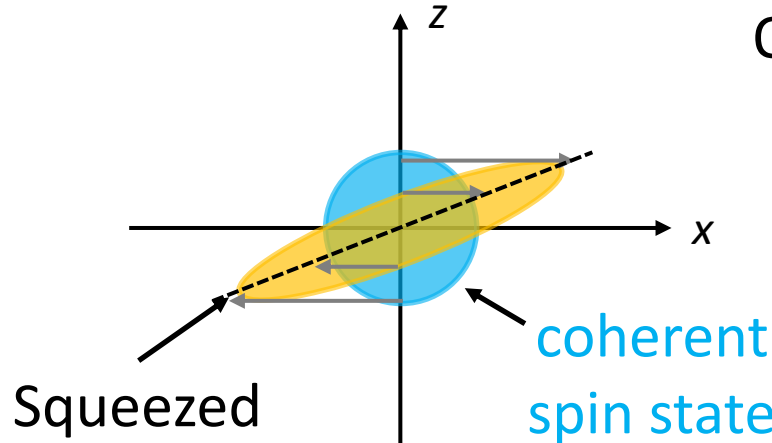


Bollinger, Science 2016

Spin squeezing in OAT and dipolar XY



$$\Delta J_x \Delta J_z \geq \frac{|\langle J_y \rangle|}{2}$$



One-axis twisting model

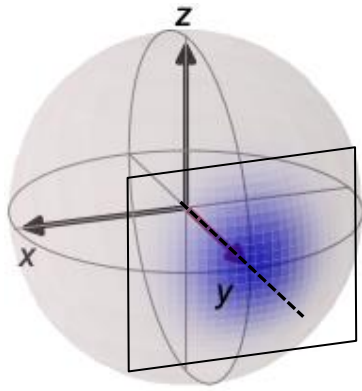
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Metrological gain in Ramsey interf.: $\delta\theta_{\text{sq}} = \xi_R^2 \delta\theta_{\text{SQL}}$ Wineland, PRA 1994

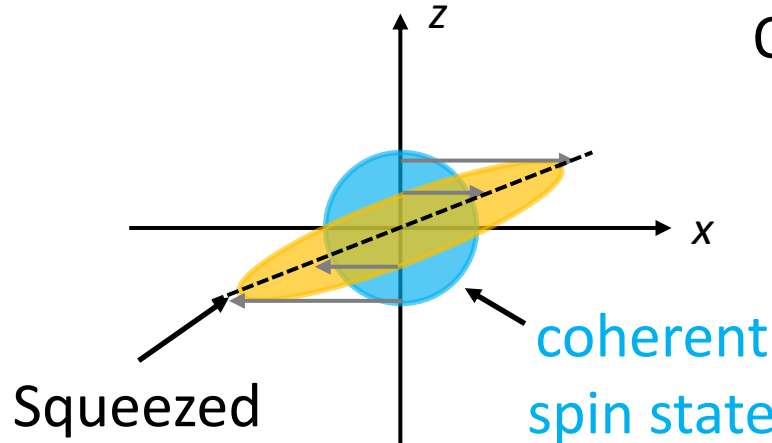
Dipolar XY: "same" structure $H_{\text{XY}} = J \sum_{i < j} \frac{a^3}{r_{ij}^3} (\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y)$

Is $1/r^3$ long-range enough to generate squeezing?

Spin squeezing in OAT and dipolar XY



$$\Delta J_x \Delta J_z \geq \frac{|\langle J_y \rangle|}{2}$$



One-axis twisting model

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Is $1/r^3$ long-range enough to generate squeezing?

Prediction: yes! M.P.A Jones & T. Pohl, PRL (2014)

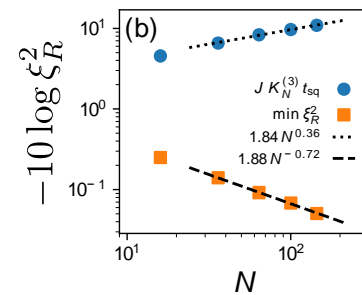
A-M. Rey, PRL (2020)

T. Roscilde, PRL **129**, 150503 (2022)

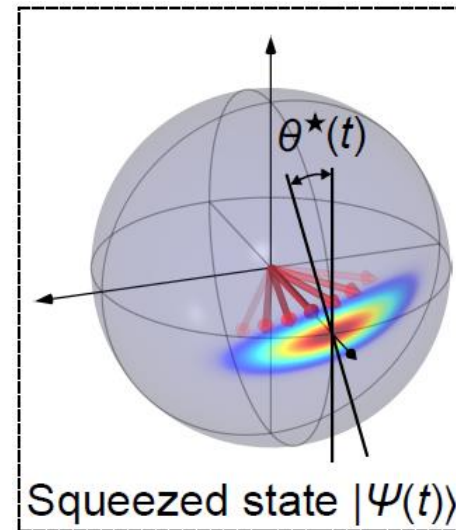
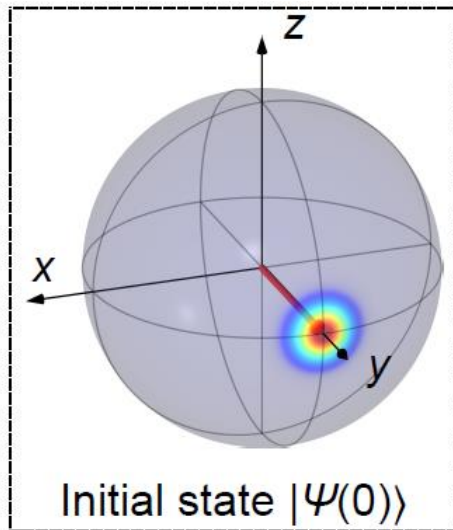
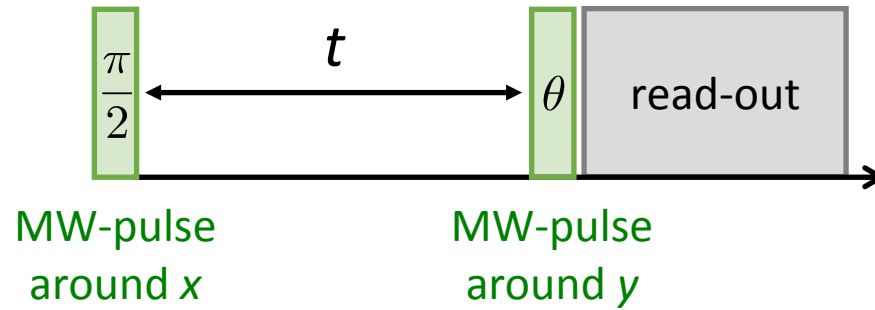
N. Yao, arXiv:2301.09636

And should scale:

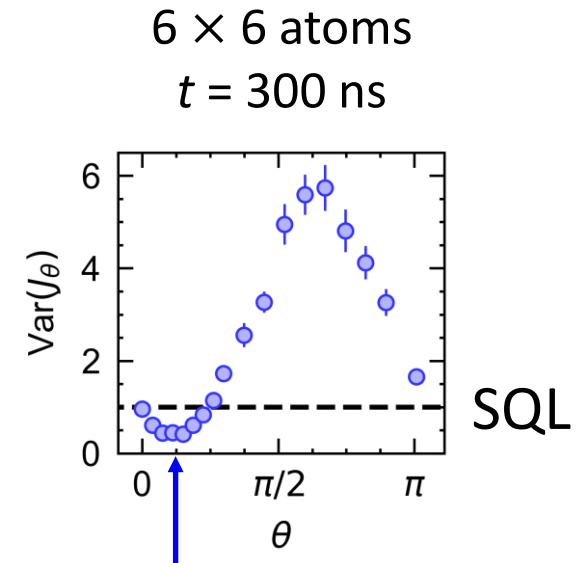
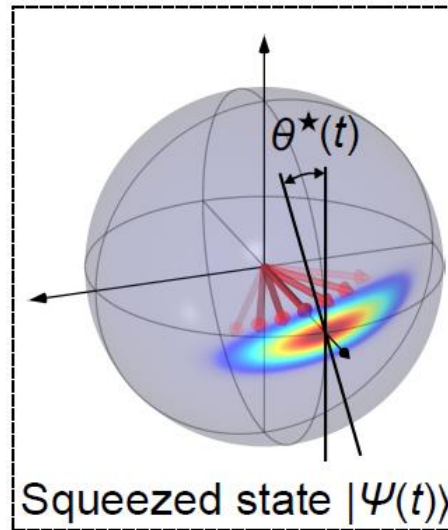
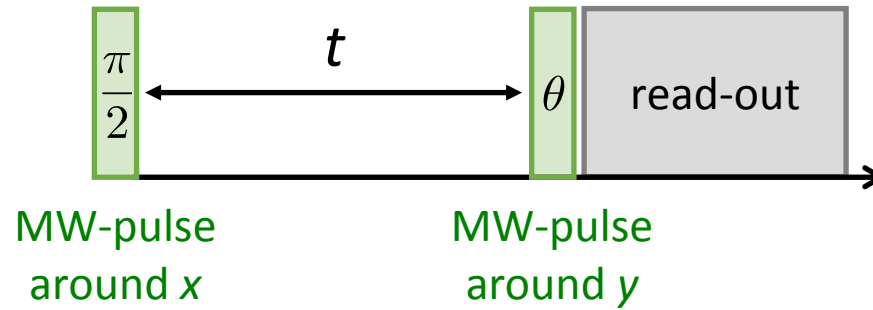
Roscilde
PRL (2022)



Dipolar spin squeezing with Rydberg atoms

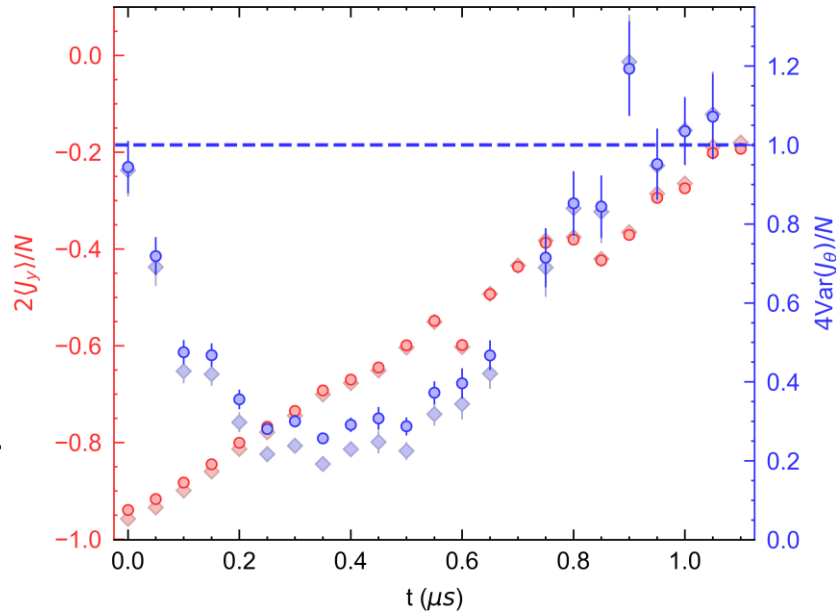
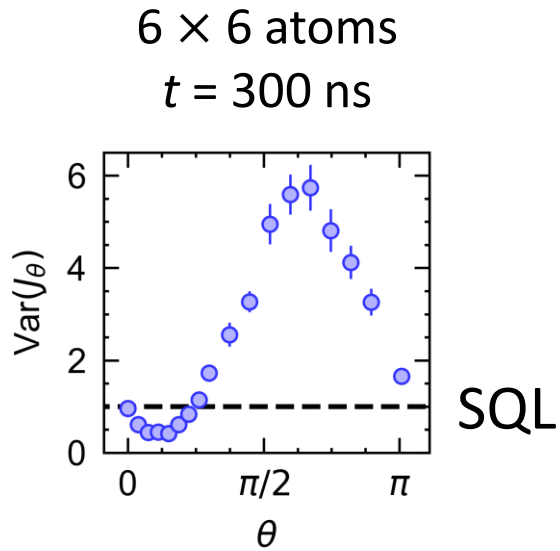
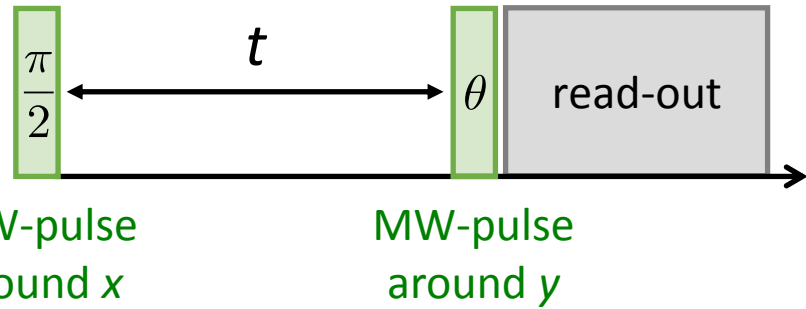


Dipolar spin squeezing with Rydberg atoms

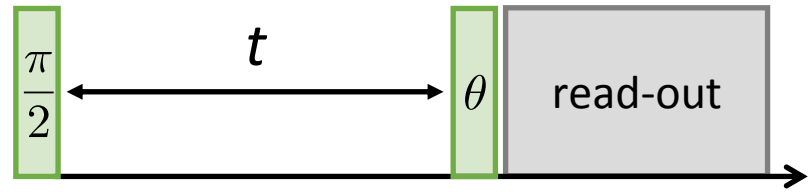


Squeezing !

Dipolar spin squeezing with Rydberg atoms



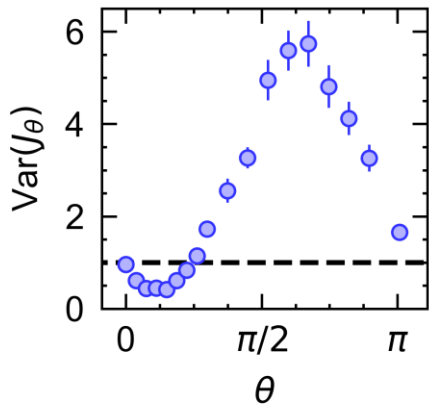
Dipolar spin squeezing with Rydberg atoms



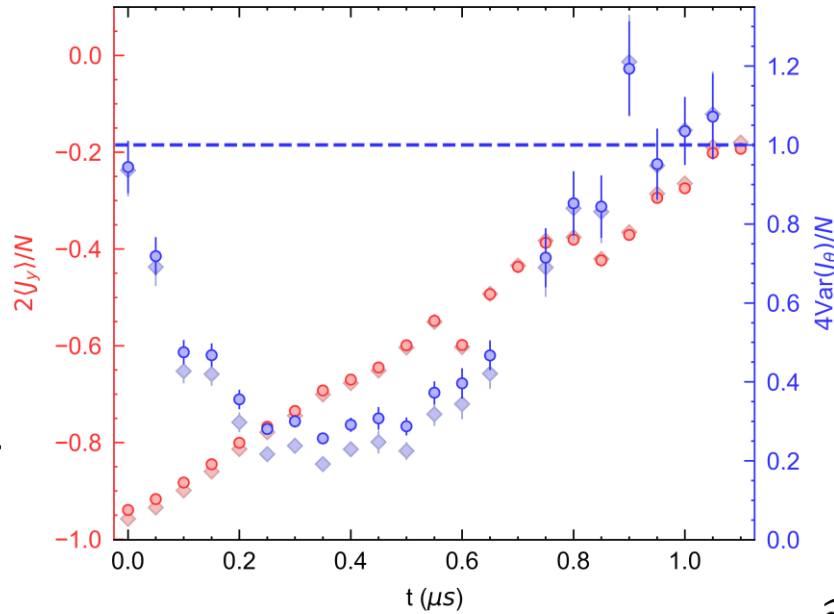
MW-pulse
around x

MW-pulse
around y

6 × 6 atoms
t = 300 ns

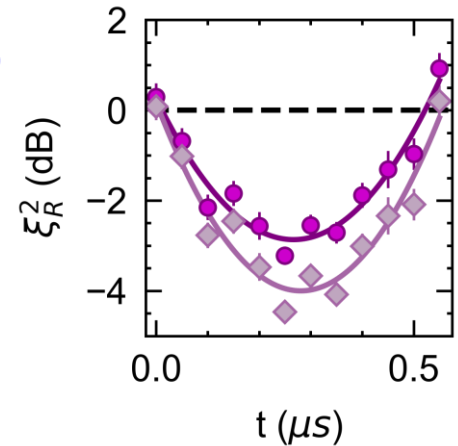


SQL

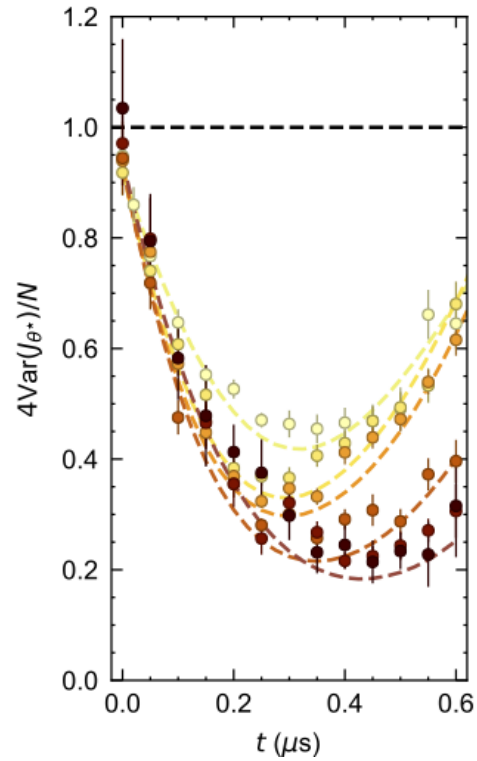
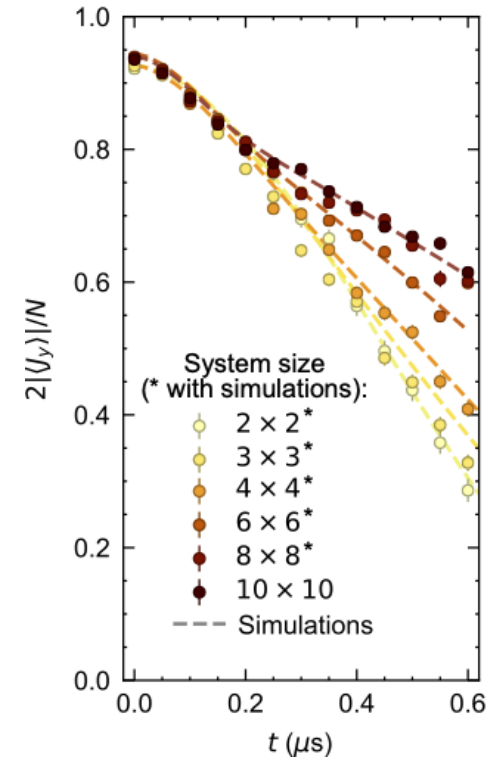


4Var(J_θ)/N

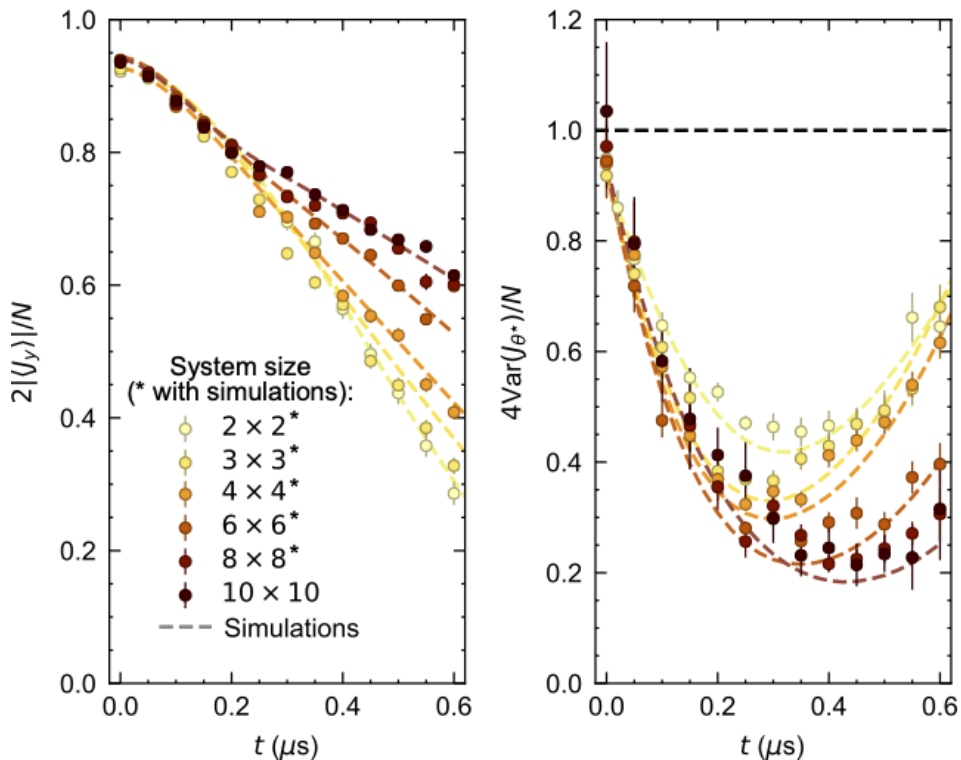
$$\xi_R^2 = N \frac{\min_{\perp} (\Delta J_{\perp}^2)}{\langle \mathbf{J} \rangle^2}$$



Scaling of the squeezing with the atom number



Scaling of the squeezing with the atom number

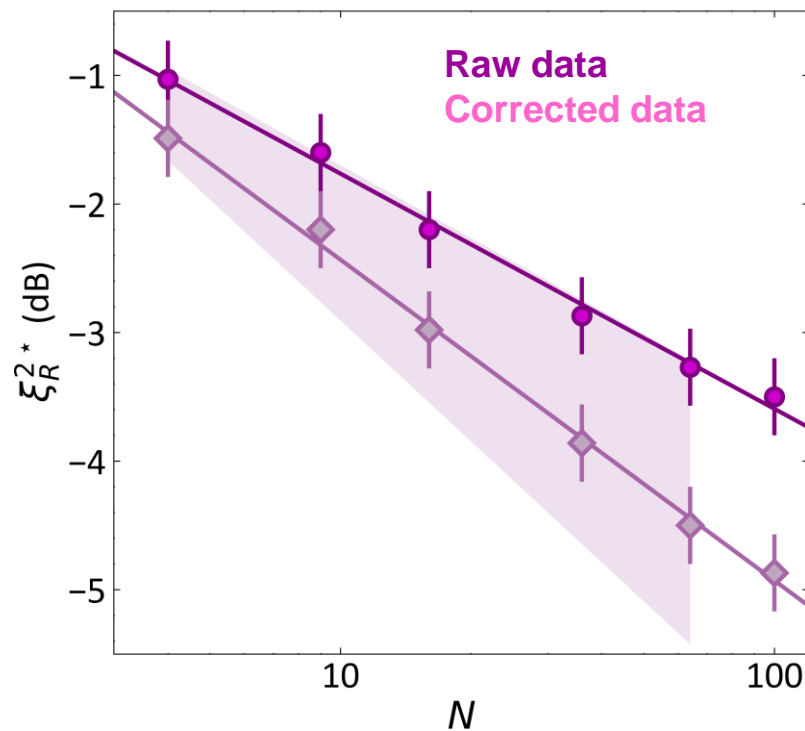


Comparin *et al.*, PRL **129**, 150503 (2022)

Block *et al.*, arXiv:2301.09636

Roscilde *et al.*, arXiv:2303.00380

Scalable squeezing!!

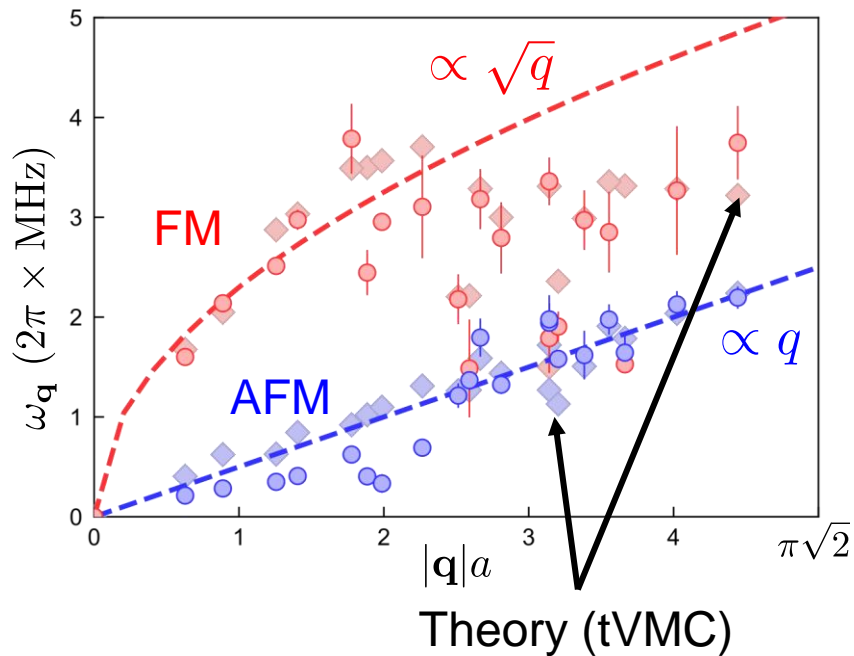


Conclusion and future directions

Future directions with RDDI

XY models:

- “Quench spectroscopy”: elementary excitations of FM and AFM



WORK IN PROGRESS

Future directions with RDDI

XY models:

- “Quench spectroscopy”: elementary excitations of FM and AFM
- On Kagome arrays: Spin liquids (Dirac, Chiral)
- Spin transport

Beyond XY:

- Topological matter with RDDI
Weber *et al.*, [PRX Quantum](#) **3**, 030302 (2022)
- Floquet engineering of exotic spin models (DM interaction...)

Thanks for your attention!

