Probing equilibrium and non-equilibrium dynamics of the Hubbard model with ultracold atoms: non-local correlations and full counting statistics

I.B. 13 June 2023

Precision Many-Body Physics CdF/Paris

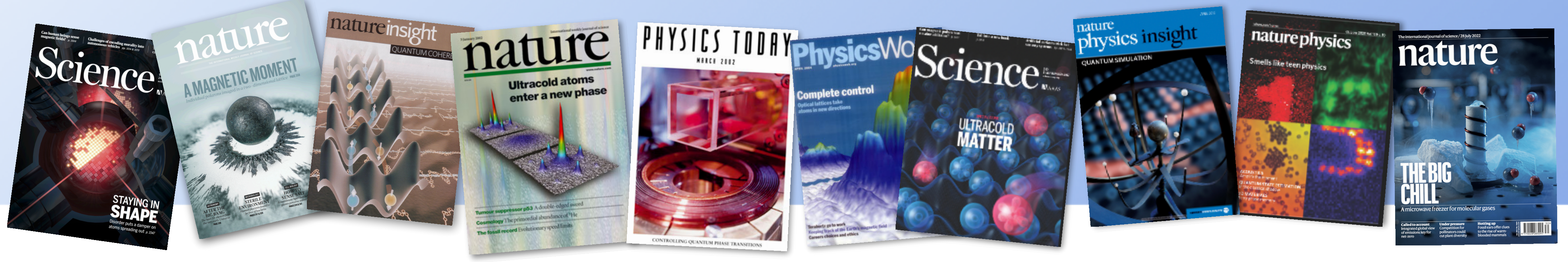
MCQST

Munich Quantum Valley

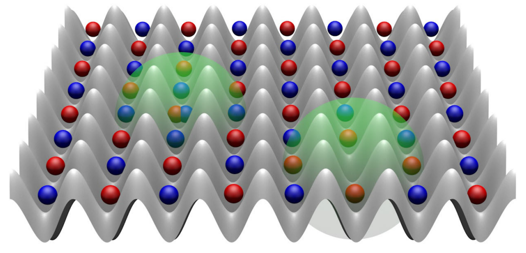
QUANTUM FLAGSHIP

PASQuanS

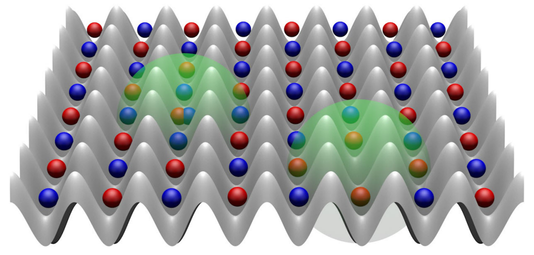
Bundesministerium für Bildung und Forschung



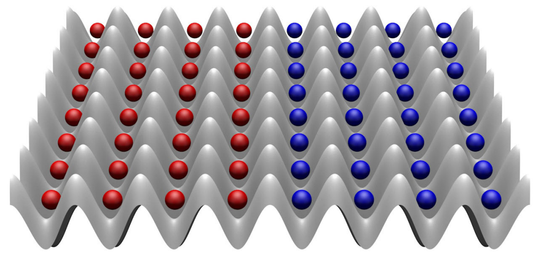
Applications in Many-Body Physics



0 **State Preparation/Engineering and Detection**



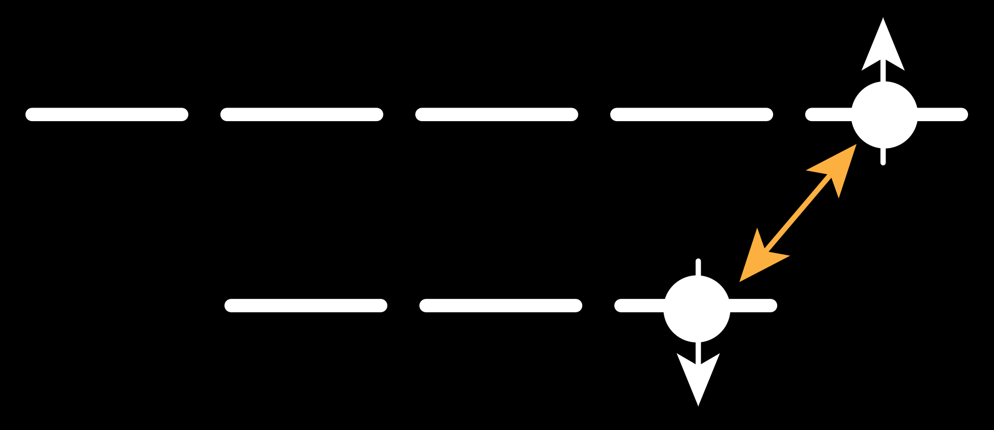
1 **Strongly Correlated Electron Physics**
Spin-Charge Separation, Hidden Order, Incommensurate Magnetism, Polarons, Strange Metals, Hole Pairing,…



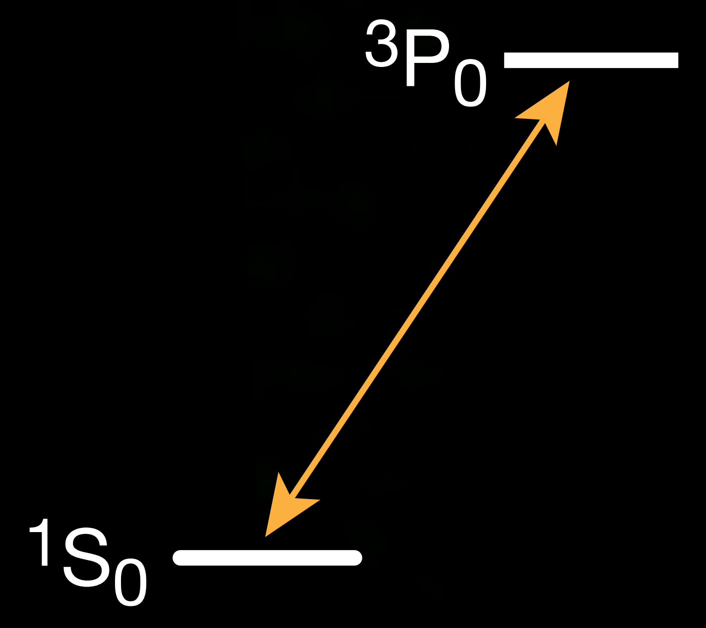
2 **Nonequilibrium Dynamics - Counting Atoms one-by-one**
Fluctuation Hydrodynamics

Particles: Fermions, Bosons, Mixtures

Spin degree of freedom:



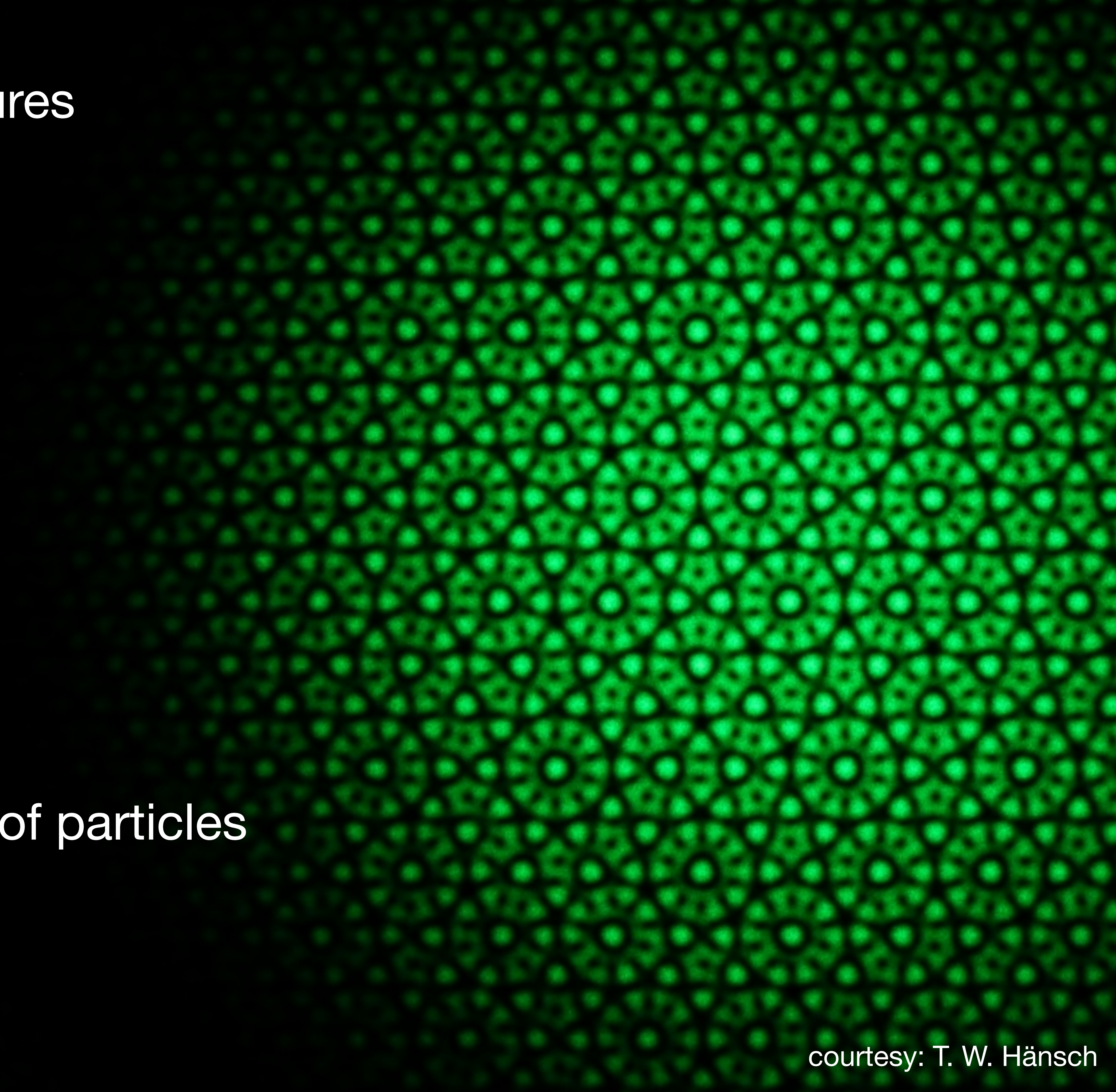
Hyperfine
(Microwave)



Clock
(Optical)

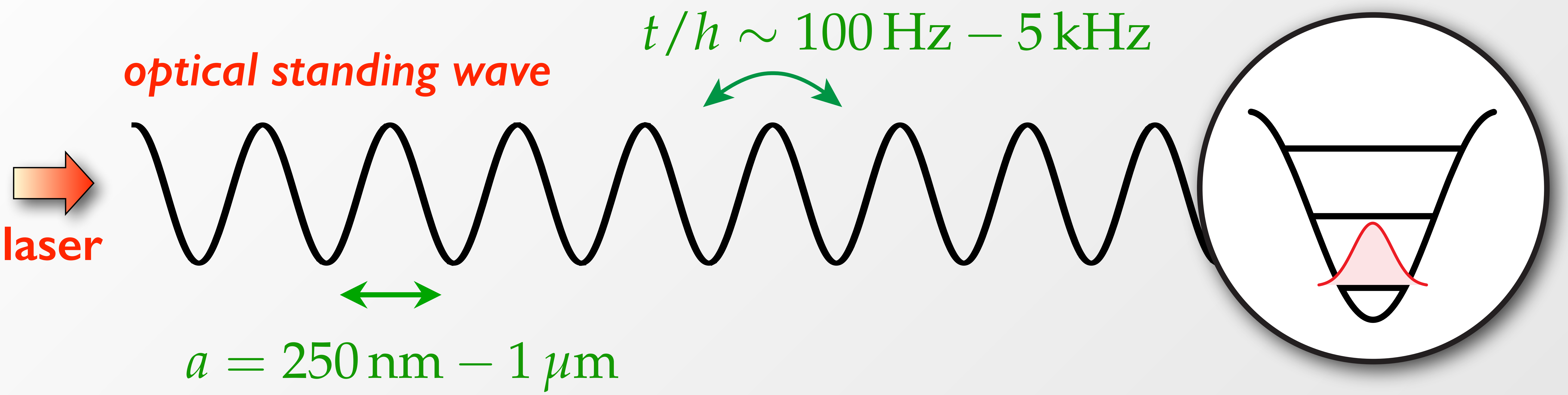
System size: up to few thousands of particles

Mobility: itinerant or static



courtesy: T. W. Hänsch

Primer on Optical Lattices

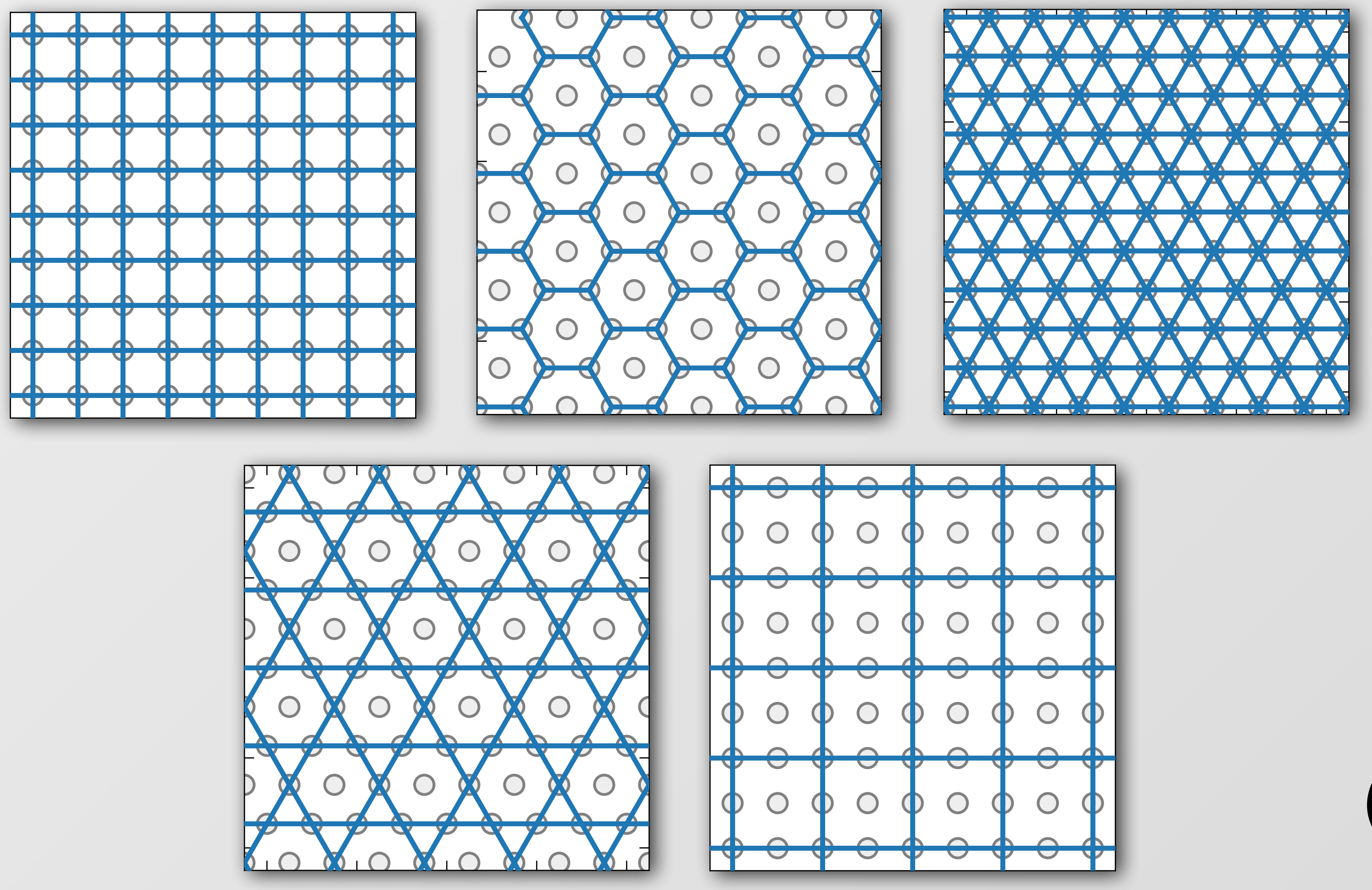


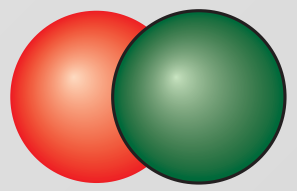
$\omega \sim 2\pi \times (10\,\mathrm{kHz} - 1\mathrm{MHz})$
$\sigma \sim 10\,\mathrm{nm} - 100\,\mathrm{nm}$

Fourier synthesize aribtrary lattices:

- Square
- Hexagonal/Triangular/Brick Wall
- Kagomé
- Superlattices
- *Spin dependent lattices*
- *Flux Lattices*

...



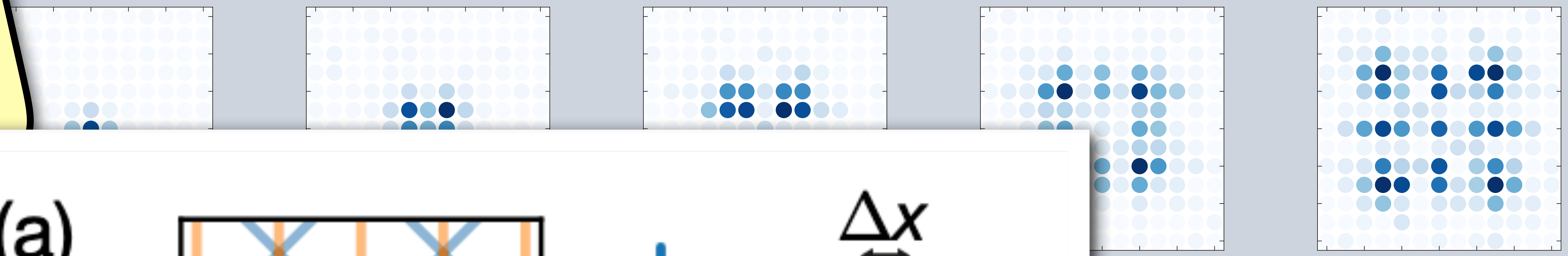


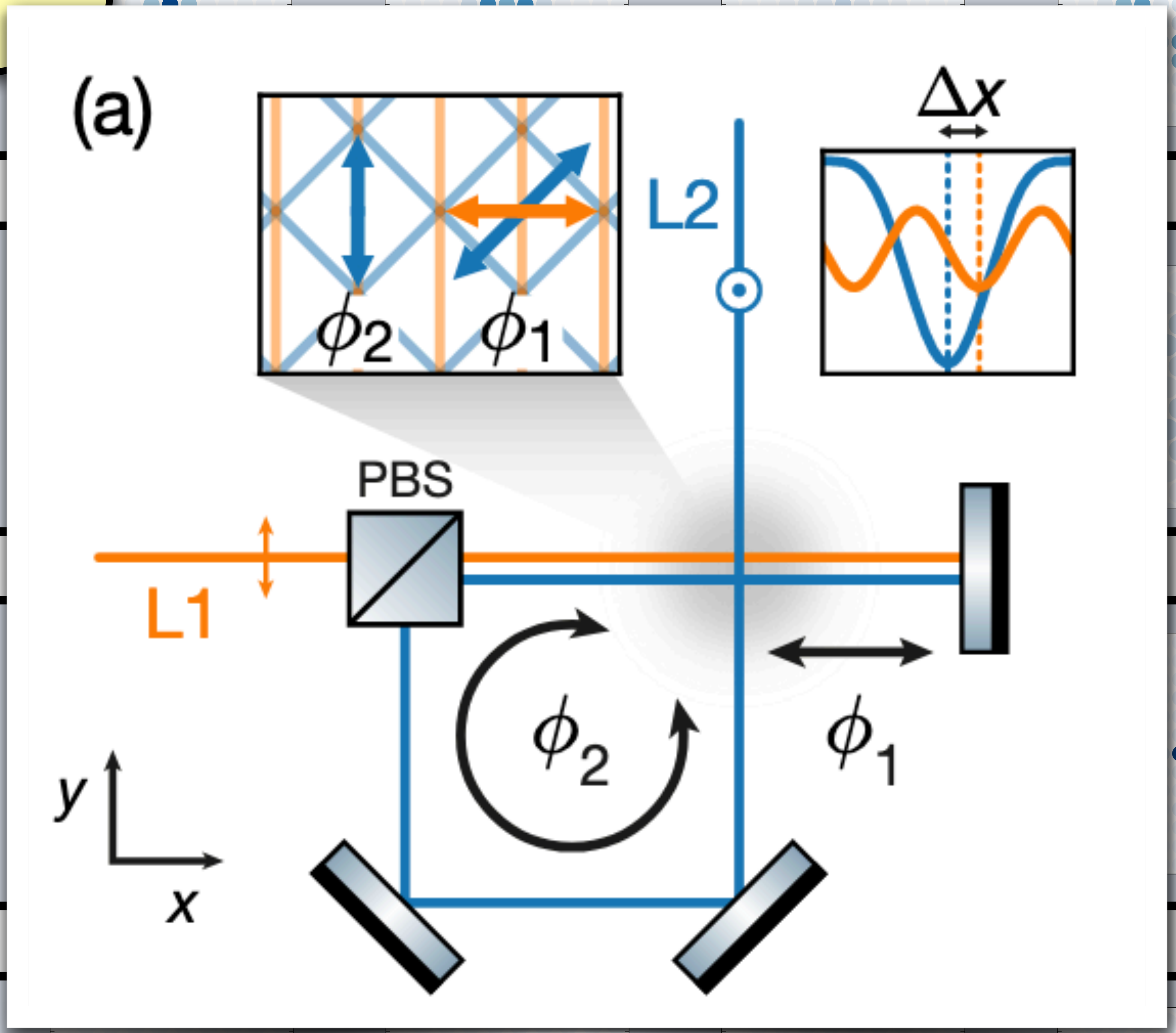
Collisional
(onsite few kHz)

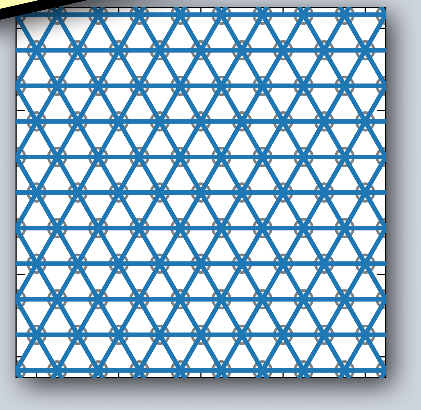
Full **dynamical** control over **lattice depth, geometry, dimensionality!**

2D Quantum Walks in Optical Lattices

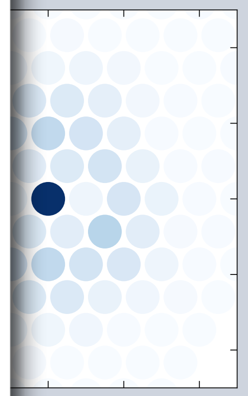
Derived from a single optical lattice setup!
(with passive phase stability)

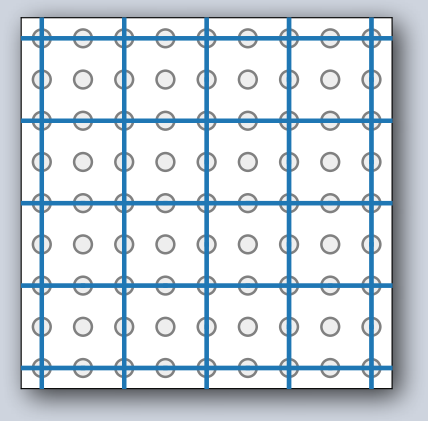




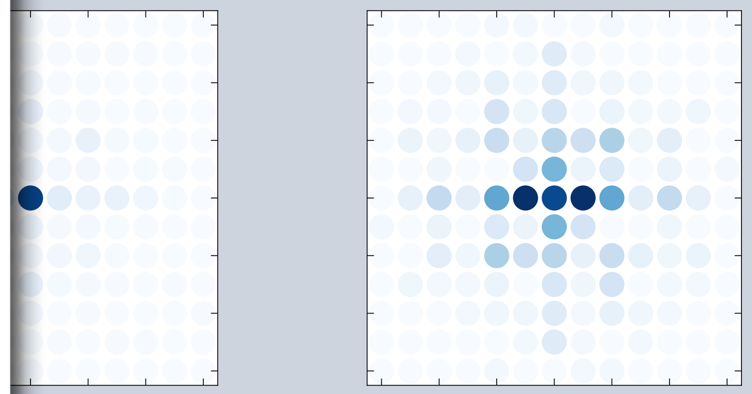


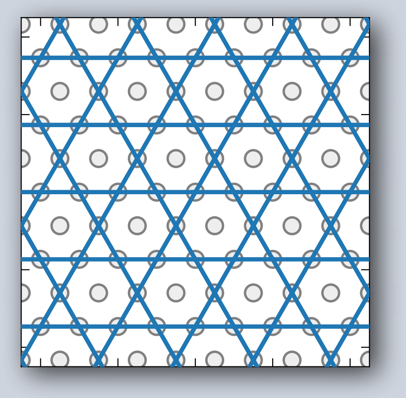
Triangular



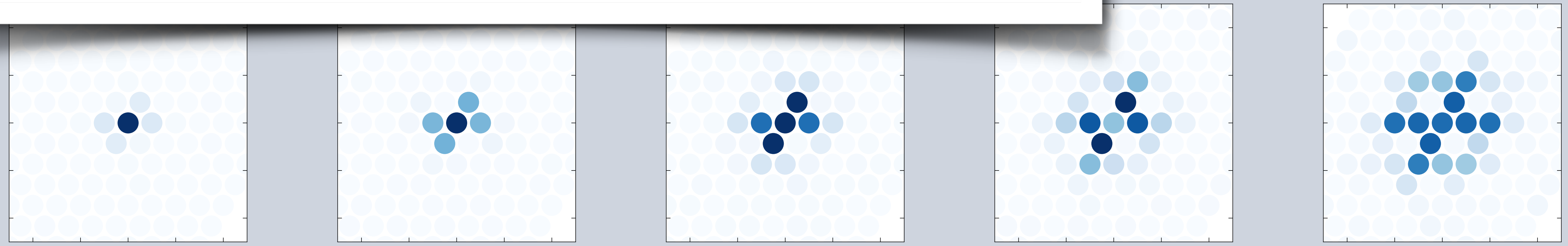


Lieb





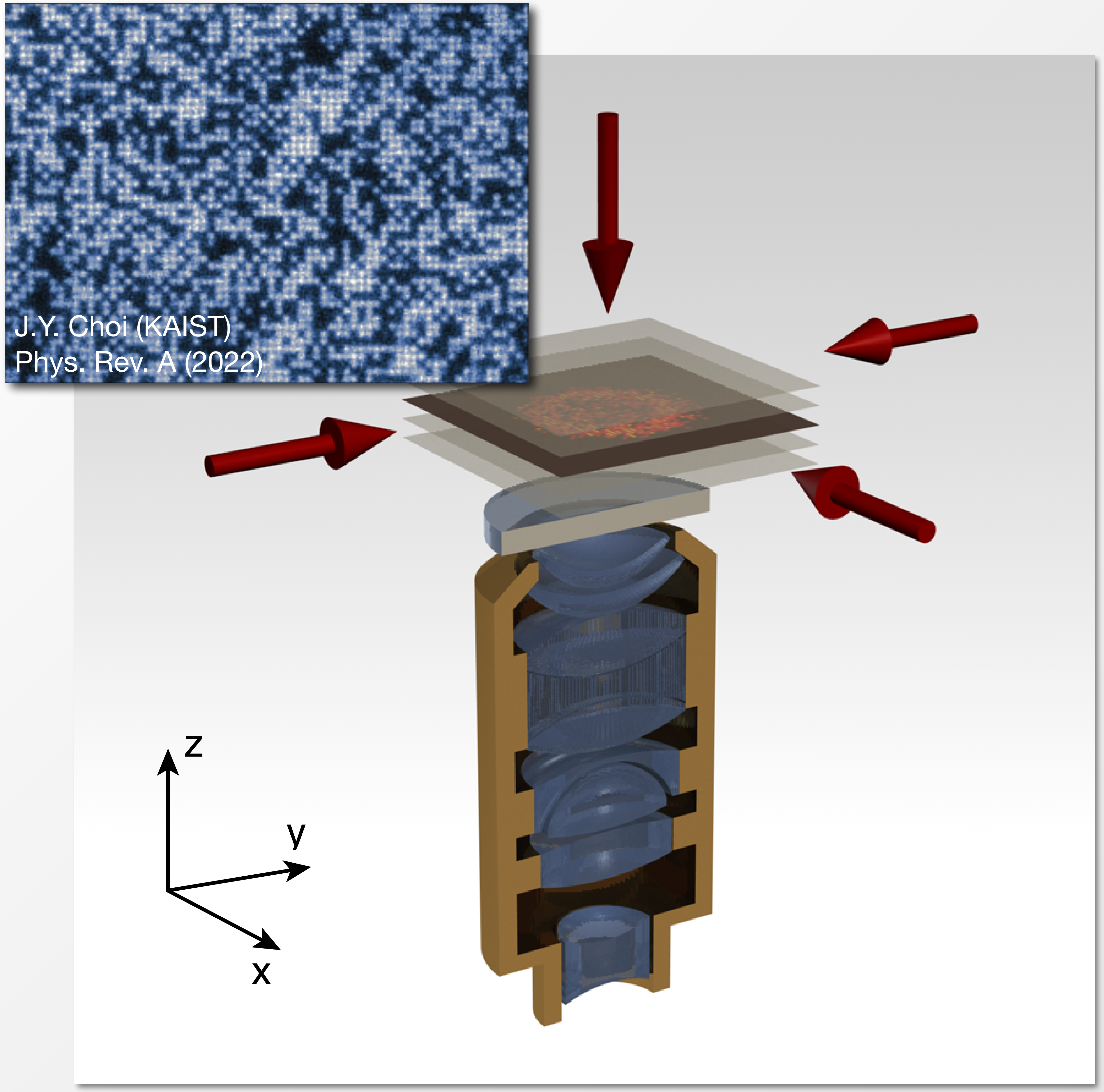
Kagomé



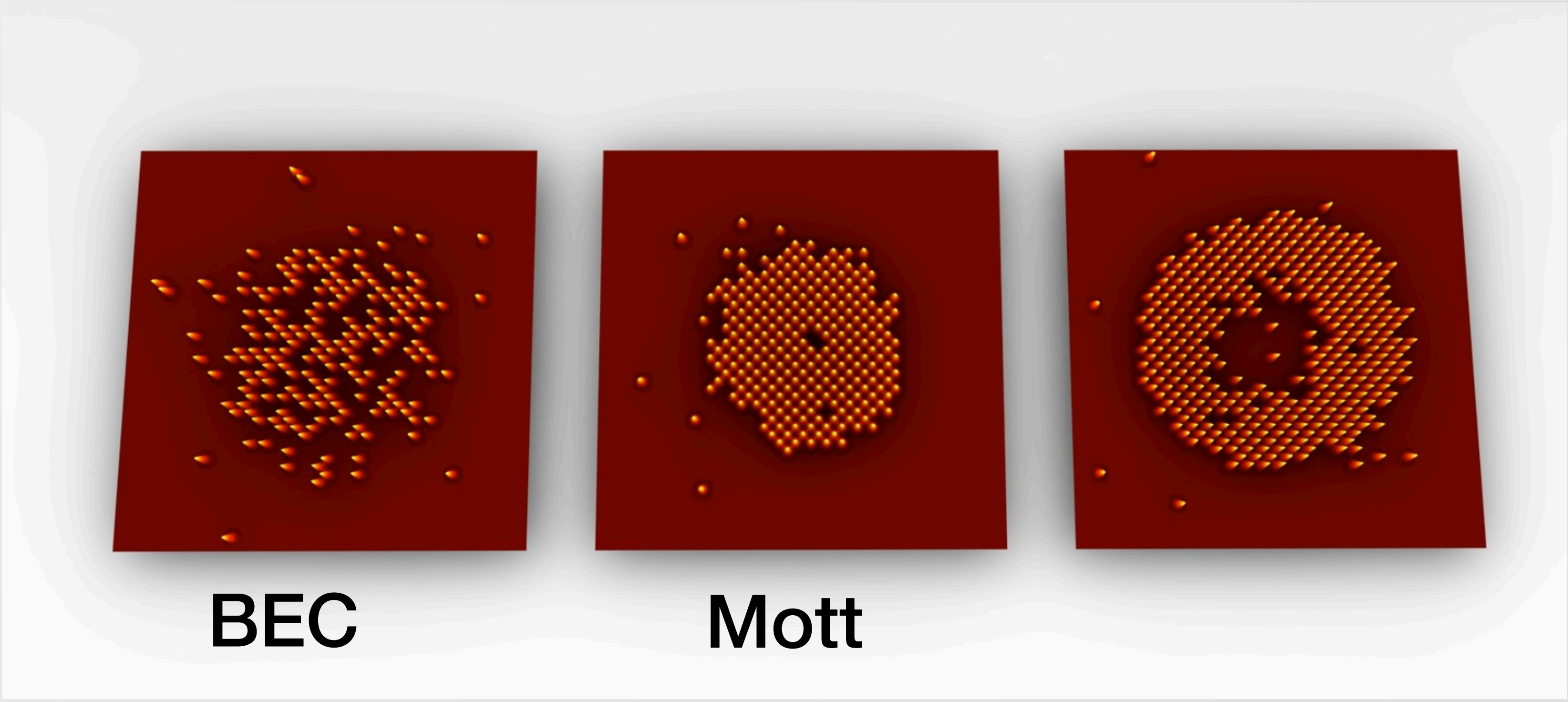
D. Wei *et al.* arXiv:2301.11869 (2023)
recently used: Bakr group (Princeton)

related: D. Greif *et al.* Nature **535** 217 (2014), M. Xu *et al.* arXiv:2212.13983

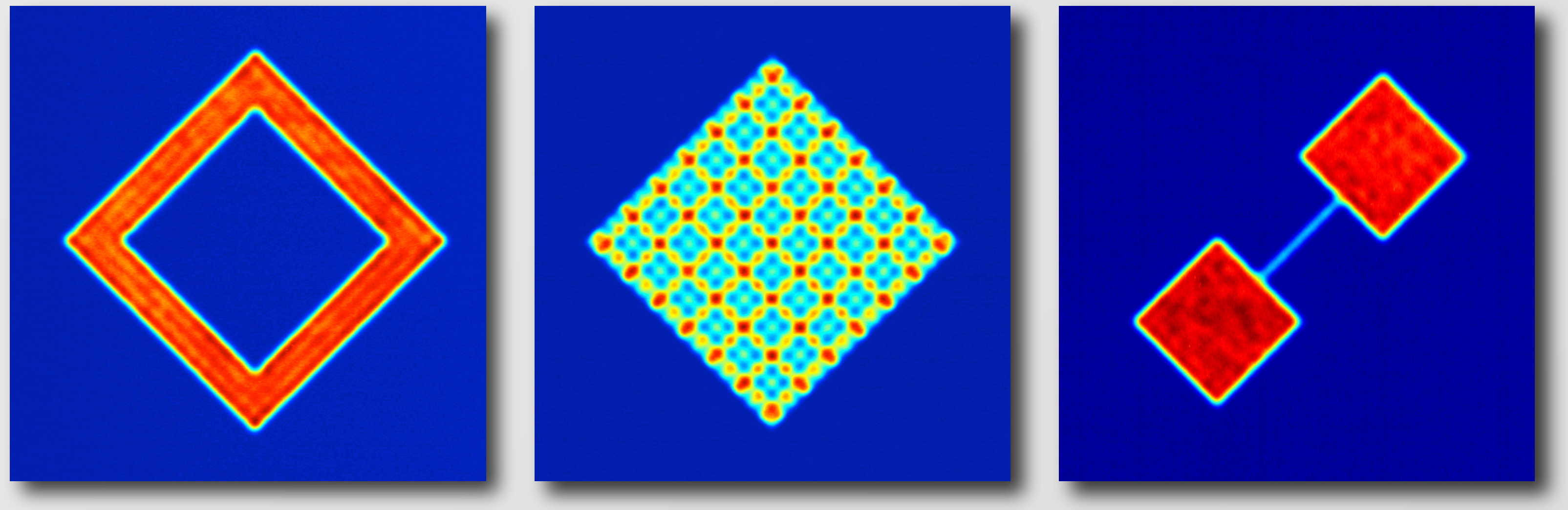
Quantum Gas Microscopy



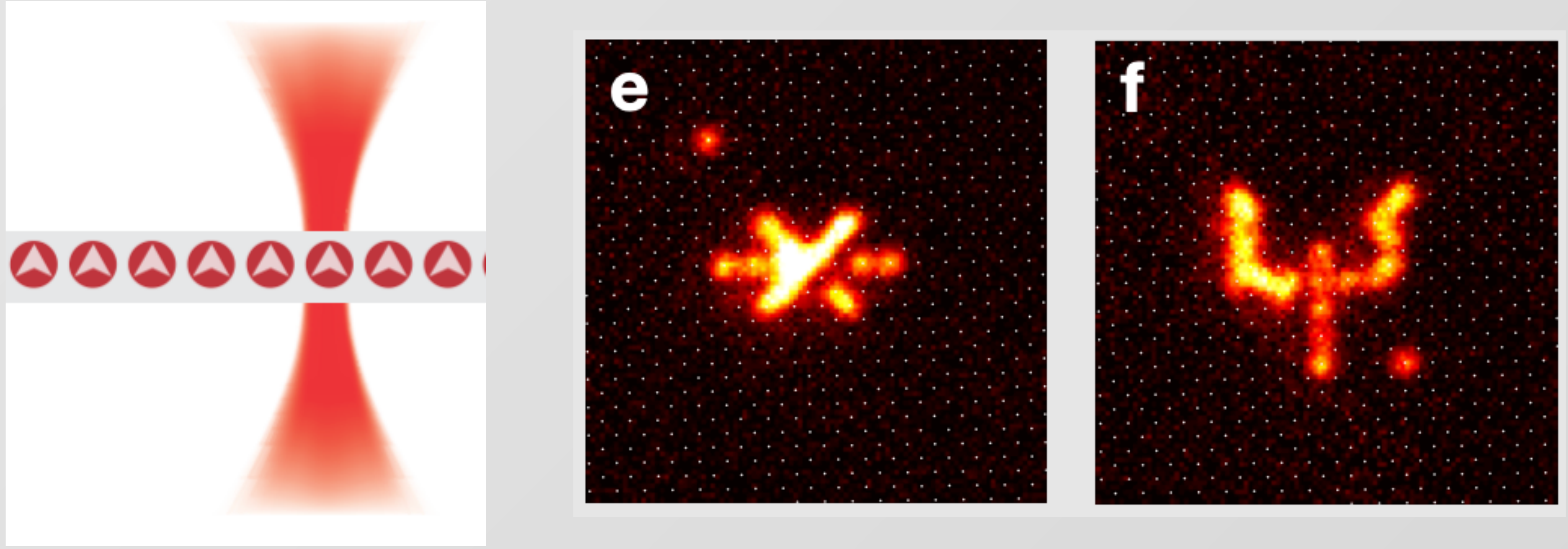
Single atom detection



Potential Engineering



Single Atom Adressing

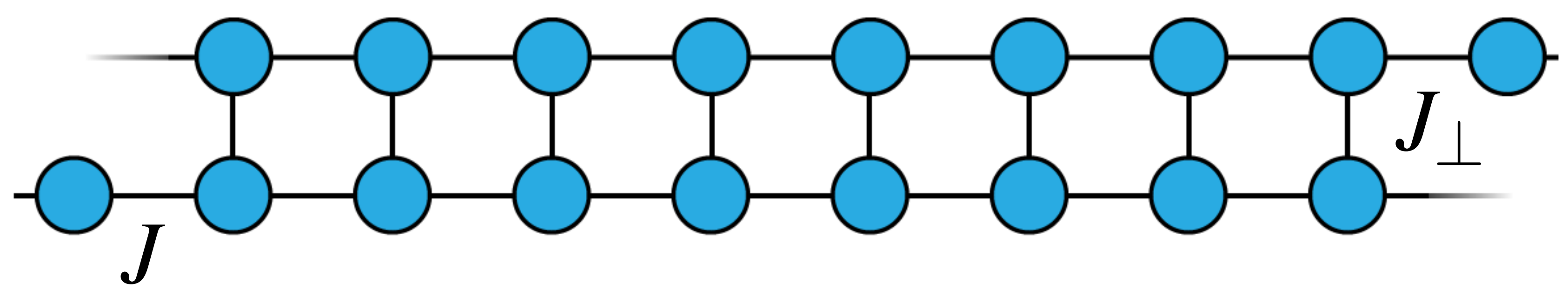


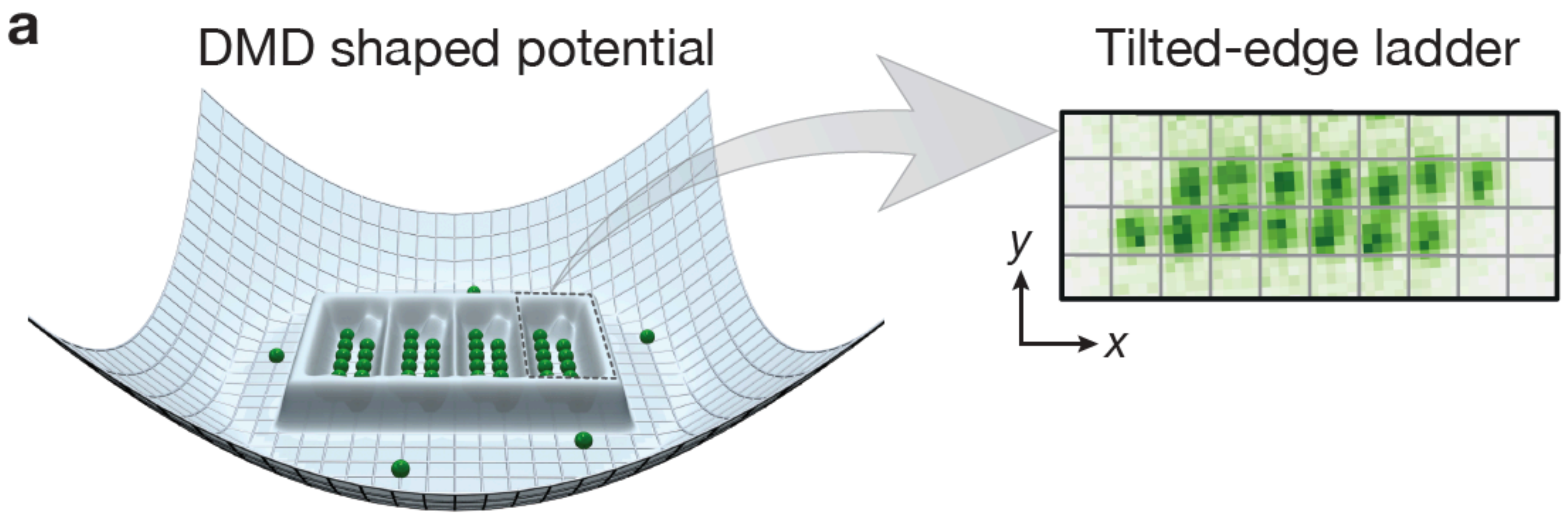
W. Bakr et al., Science (2010) & J. Sherson et al., Nature (2010)
Addressing: C. Weitenberg et al., Nature (2011)

Flexible Geometries and Large Sizes

Quantum Ladders with flexible edge geometries

(SPT Spin-1 Haldane Phase)





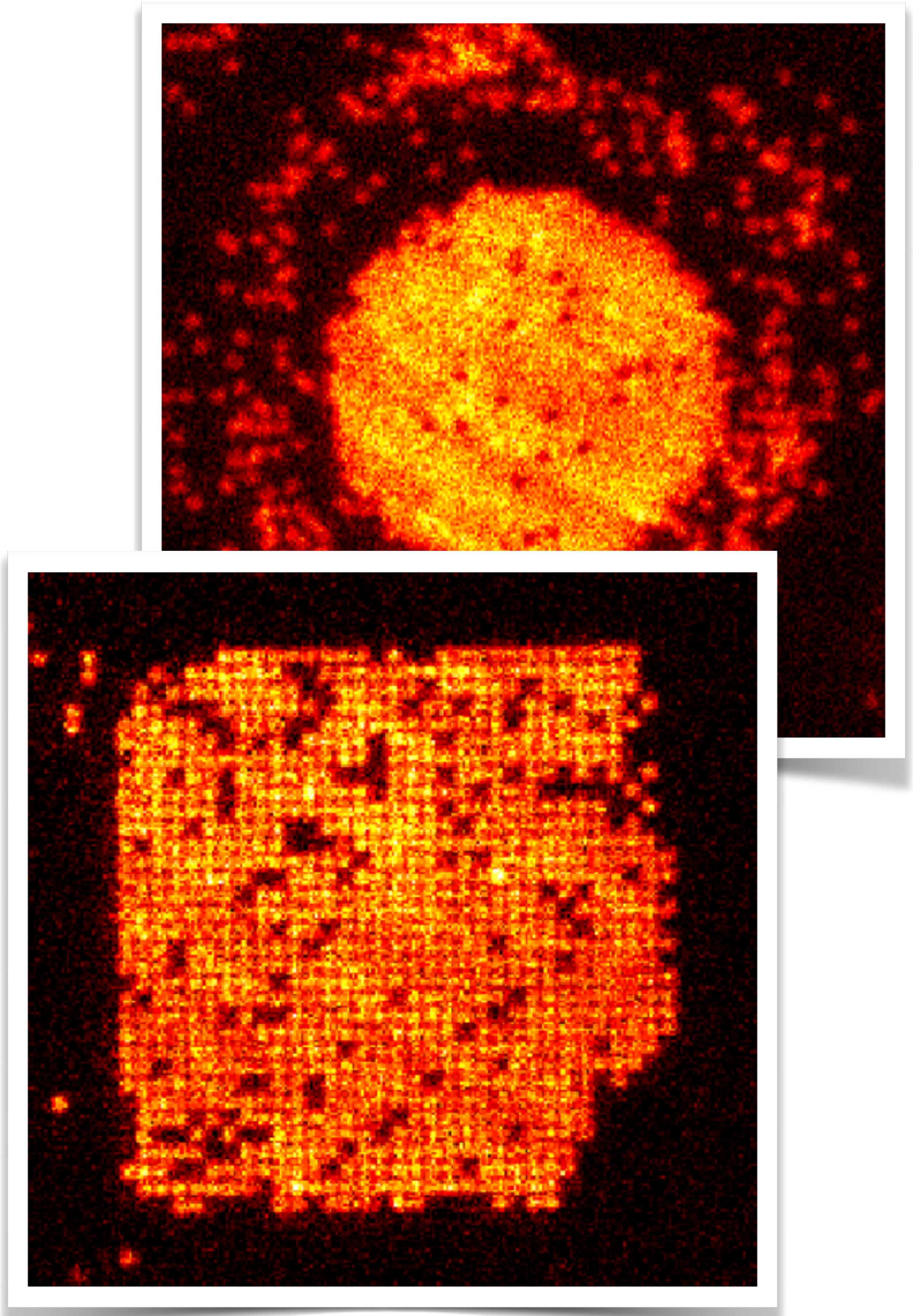
Fully tuneable coupling strengths +dimensionality +flux +frustration

P. Sompet *et al.* Nature **606,** 484 (2022)
Tweezer SPT: Léséluc *et al.* Science **365**, 6455 (2019)

Large Homogeneous 2D Systems

(2000-5000 atoms, filling 95-98%)

News Cs experiment in collaboration with M. Aidelsburger



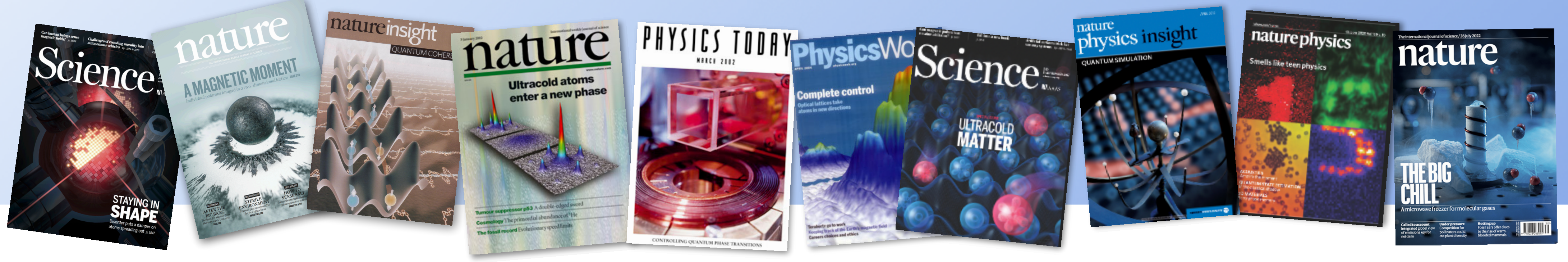
Rb Quantum Gas Microscope

Cs Quantum Gas Microscope

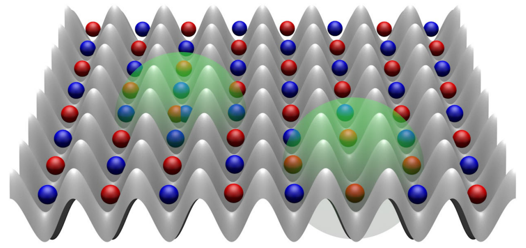
see also: C. Chiu *et al.* Phys. Rev. Lett. **120**, 243201 (2018)
Idea: J.-S. Bernier *et al.* Phys. Rev. A **79**, 061601 (2009)
T.-L. Ho & Q. Zhou arXiv:0911.5506

Full Spin & Density Resolved Detection

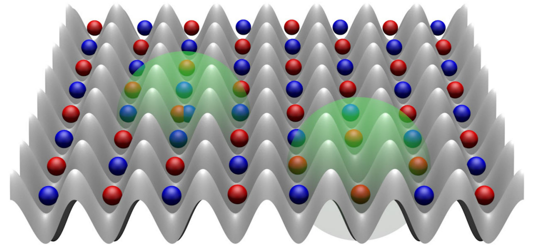
Density and spin readout: M. Boll et al. Science **353**, 1257 (2016), J. Koepsell *et al.* Phys. Rev. Lett. **125**, 010403 (2020),
see also: Harvard (Greiner), Princeton (Bakr) Phys. Rev. Lett. **129**, 123201 (2022), MIT (Zwierlein) arXiv:2208.05948



Applications in Many-Body Physics



0 **State Preparation/Engineering and Detection**



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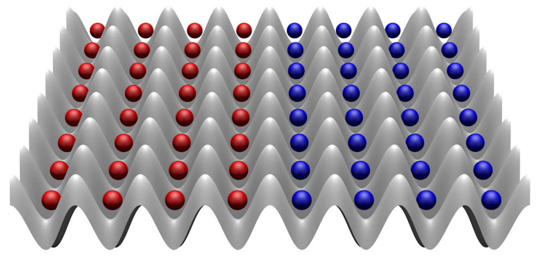
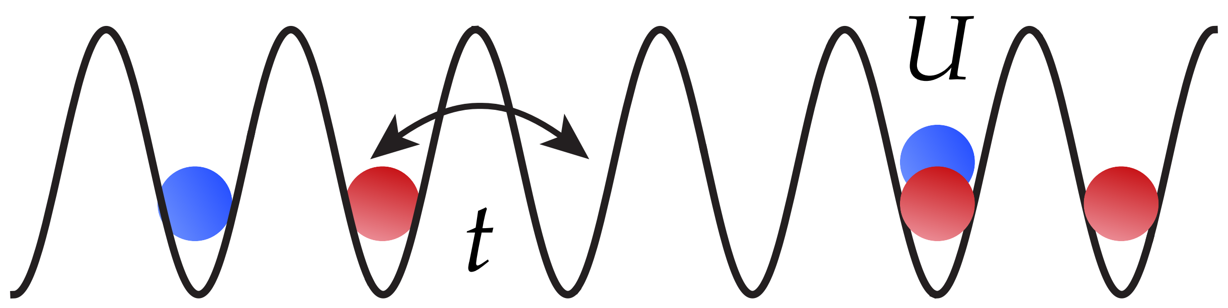
Spin-Charge Separation, Hidden Order, Incommensurate Magnetism, Polarons, Strange Metals, Hole Pairing,…



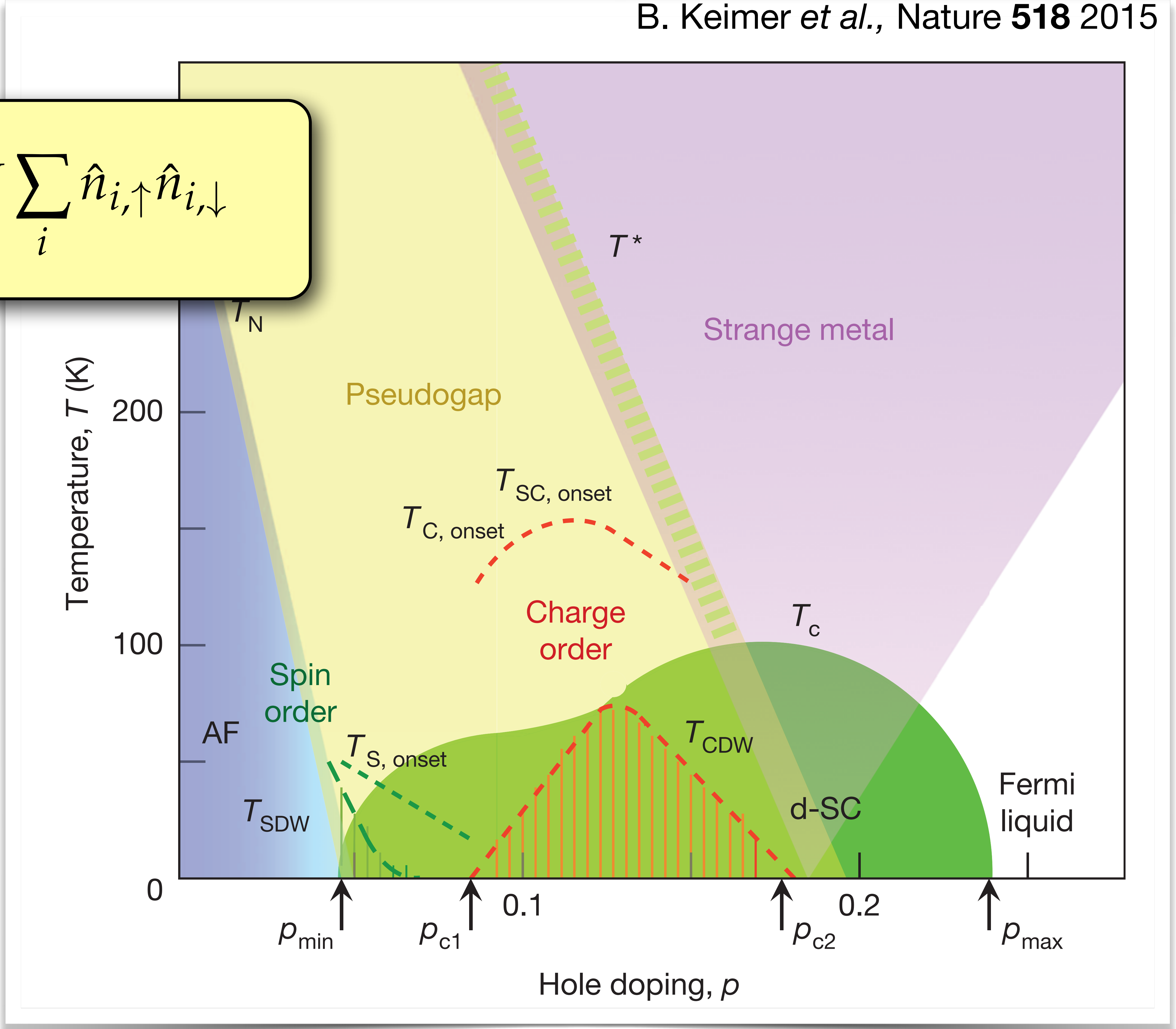
2 **Nonequilibrium Dynamics - Counting Atoms one-by-one**
Fluctuation Hydrodynamics

Fermi Hubbard Model (FHM)

Fermi-Hubbard Model

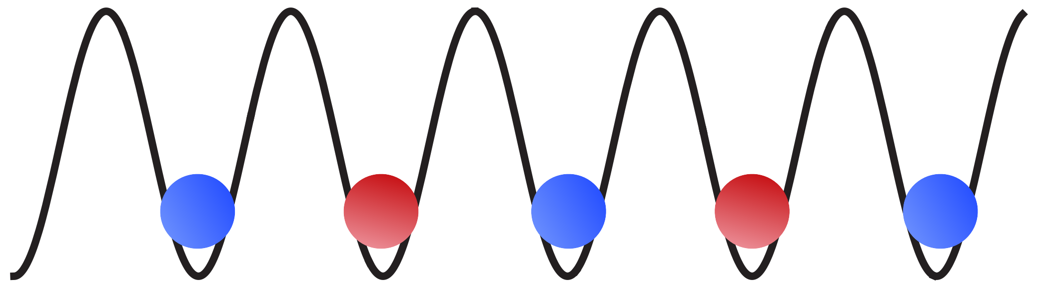


$$\hat{H} = -t \sum_{\langle i,j\rangle,\sigma} \hat{c}^{\dagger}_{i,\sigma}\hat{c}_{j,\sigma} + U \sum_{i} \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$



AFM Heisenberg Model

Half filling & strong interaction

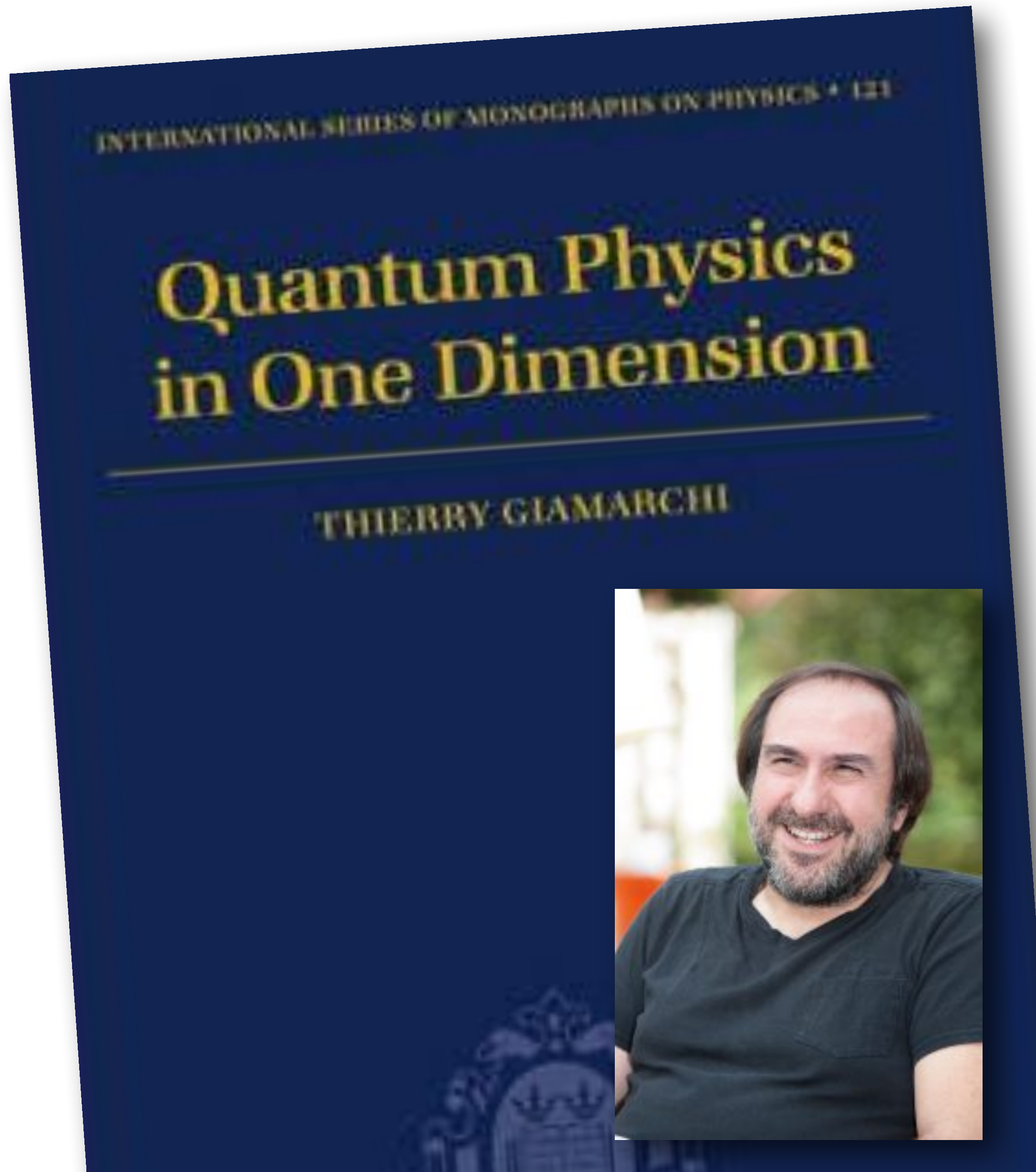


$$H = J \sum_{\langle i,j\rangle} \mathbf{S}_i \cdot \mathbf{S}_j \qquad J = \frac{4t^2}{U}$$

Away from half filling: ***t-J* model**
competition between

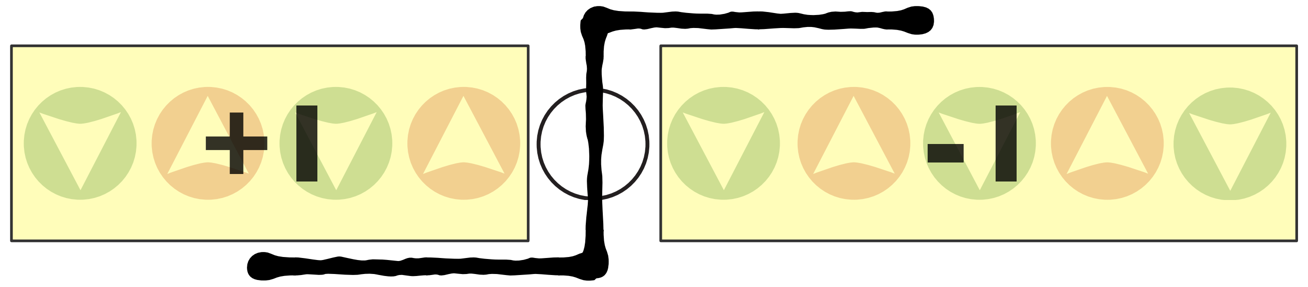
hole delocalization ⟷ ***magnetic order***

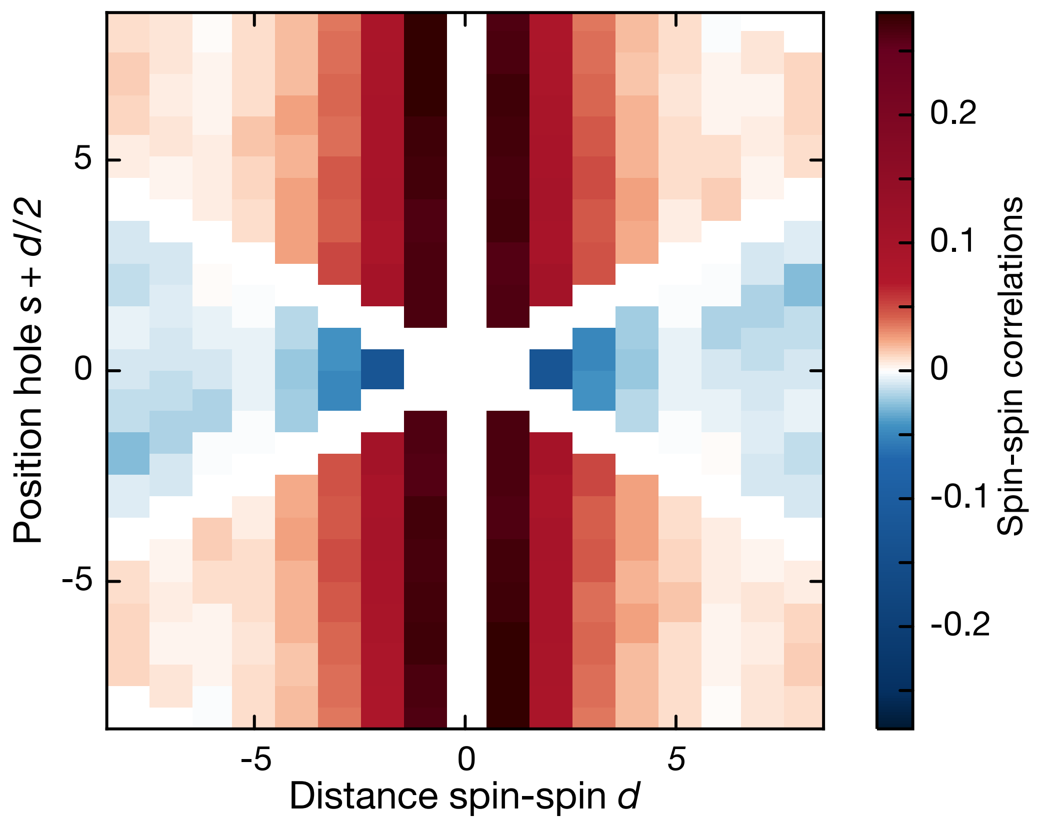
Doping in 1D Fermi Hubbard Model

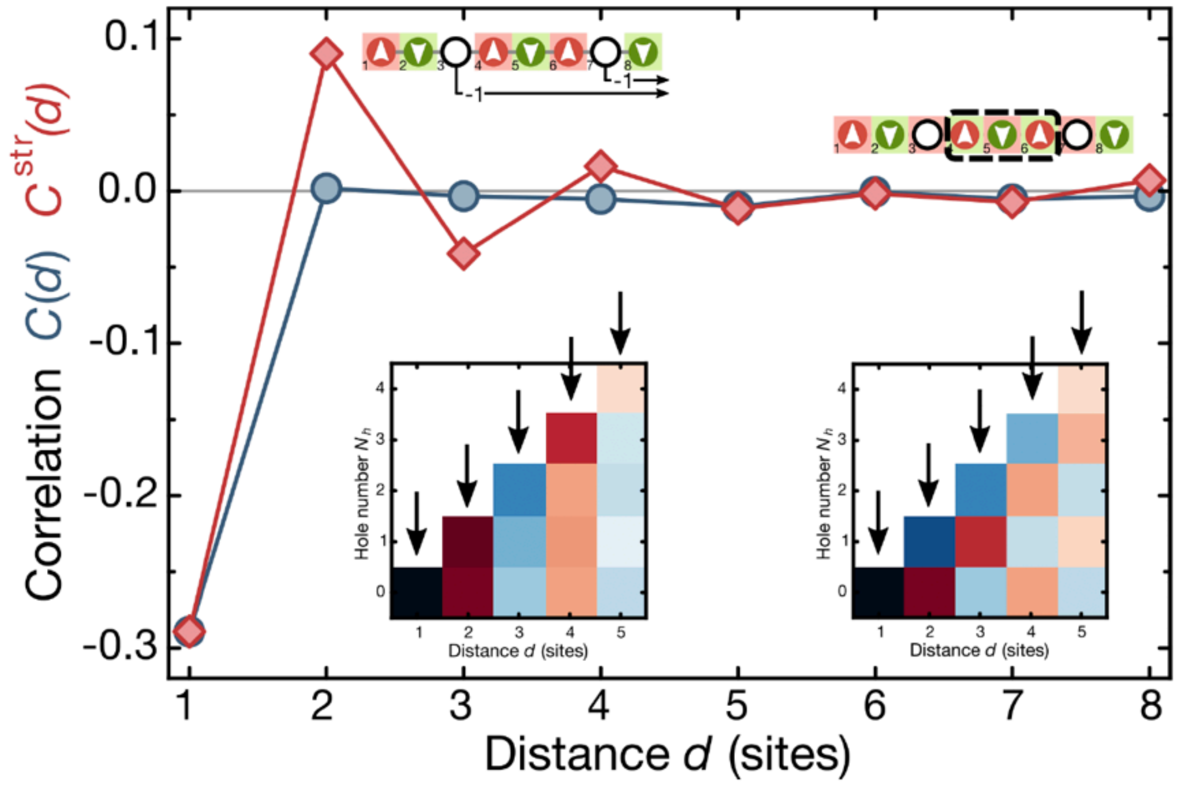


Doping in the 1D Fermi Hubbard Model

1 Ground State Spin-Charge Separation







T. Hilker et al. Science **357**, 484 (2017)

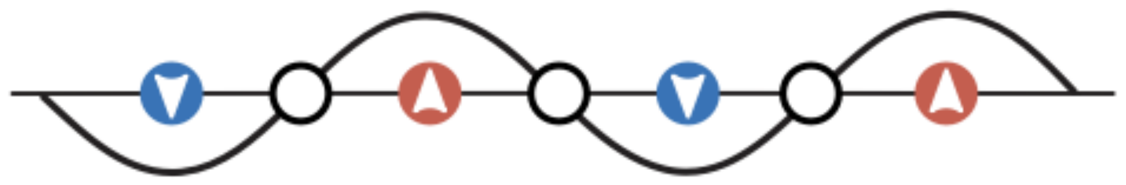
Holes form topological excitations & Hidden Antiferromagnetic Correlations (Non-local string correlations)

F. Woynarovich J. Phys. C (1982)
M. Ogata & H. Shiba Phys. Rev. B (1990)

Incommensurate AFM

F.D.M. Haldane J. Phys. C: Solid State Phys. (1981)

Doping, n



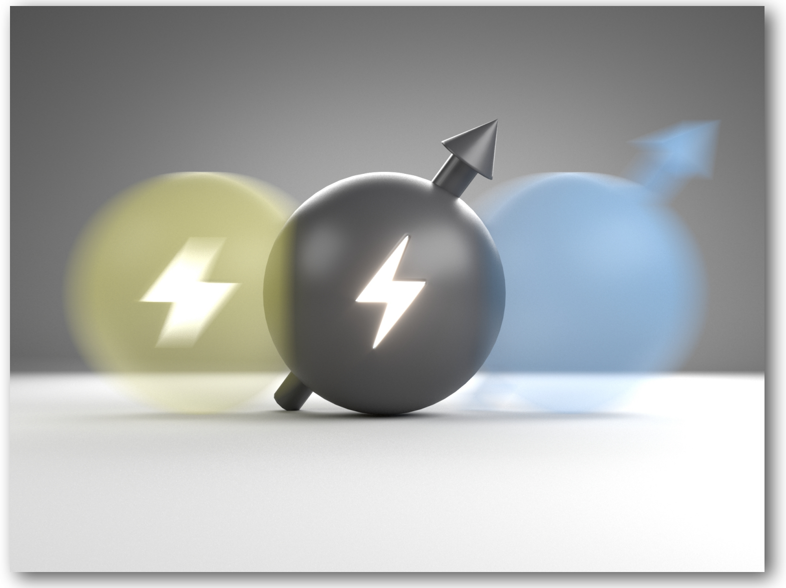
$$\langle \hat{S}^z_i \hat{S}^z_{i+x} \rangle \approx \cos(\pi n x)$$

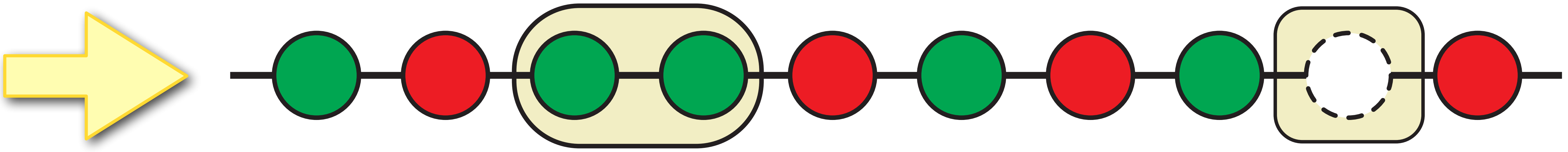
Polarization, m

$$\langle \hat{S}^z_i \hat{S}^z_{i+x} \rangle \approx \cos[\pi(1-2m)x]$$

G. Salomon et al. Nature **565,** 56 (2019)

2 Dynamical Spin-Charge Fractionalization



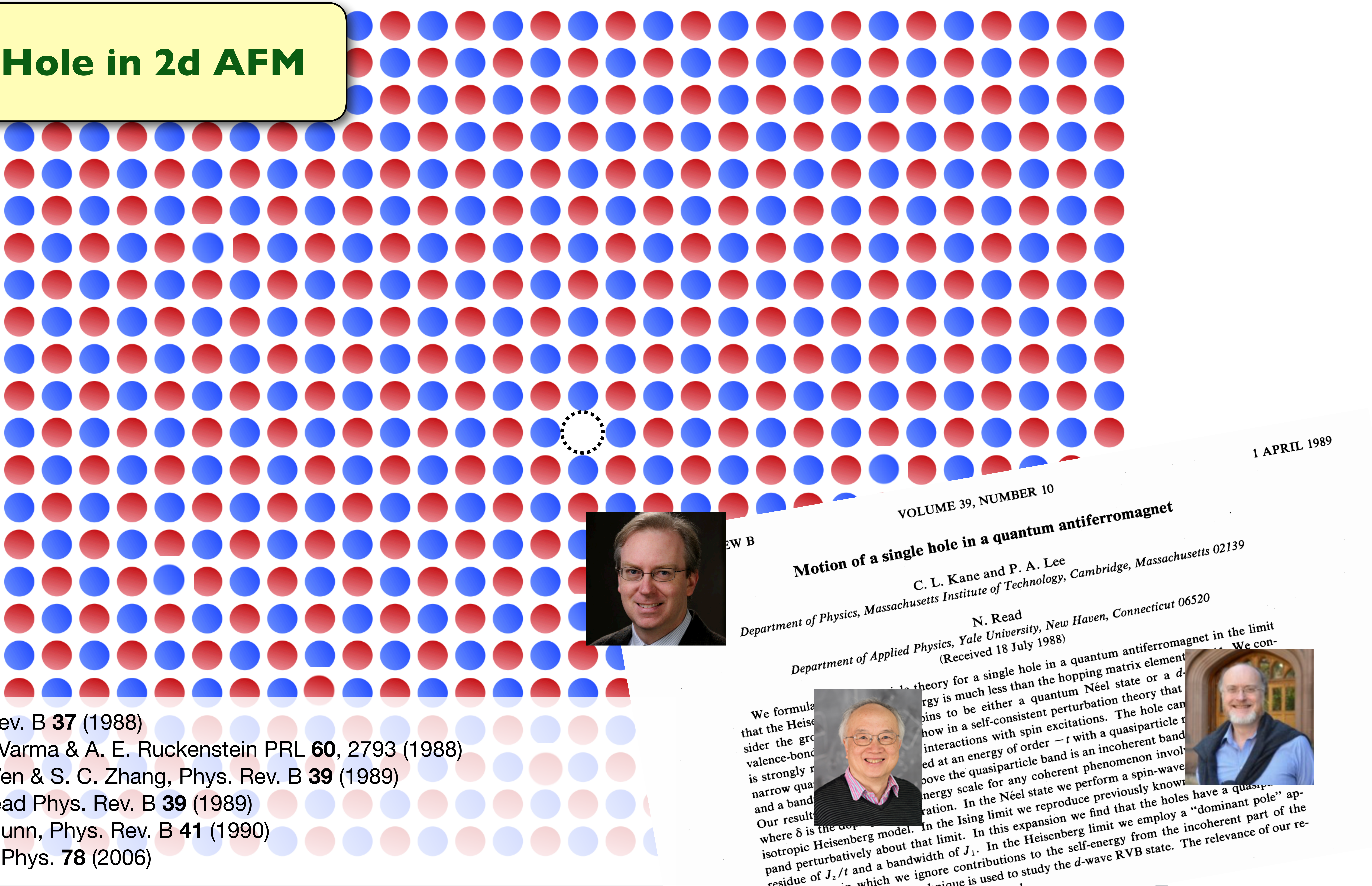


Spin 1/2 Spinon

Chargon/ Holon

J. Vijayan et al. Science **367**, 186–189 (2020)

Single Hole in 2d AFM



1 APRIL 1989

VOLUME 39, NUMBER 10

Motion of a single hole in a quantum antiferromagnet

C. L. Kane and P. A. Lee

Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

N. Read

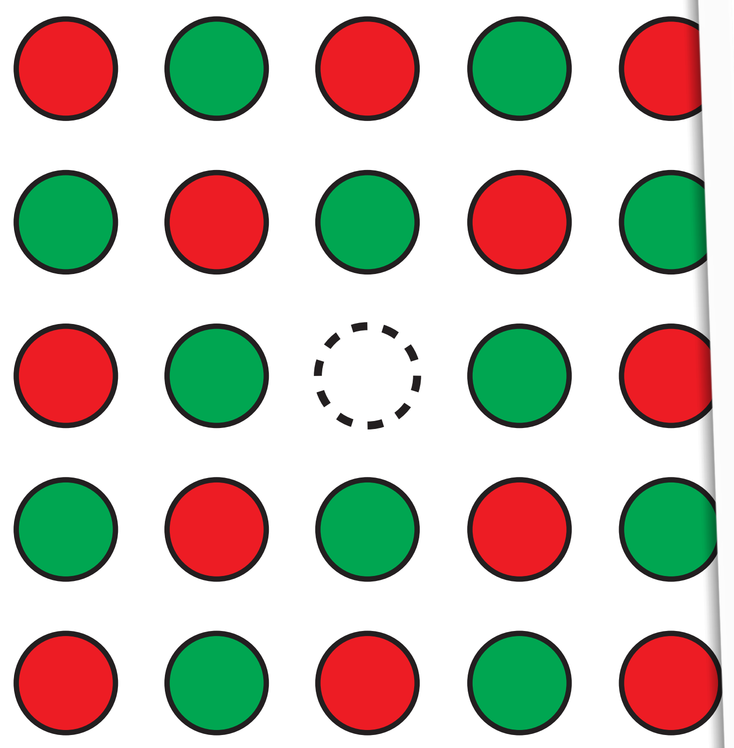
Department of Applied Physics, Yale University, New Haven, Connecticut 06520

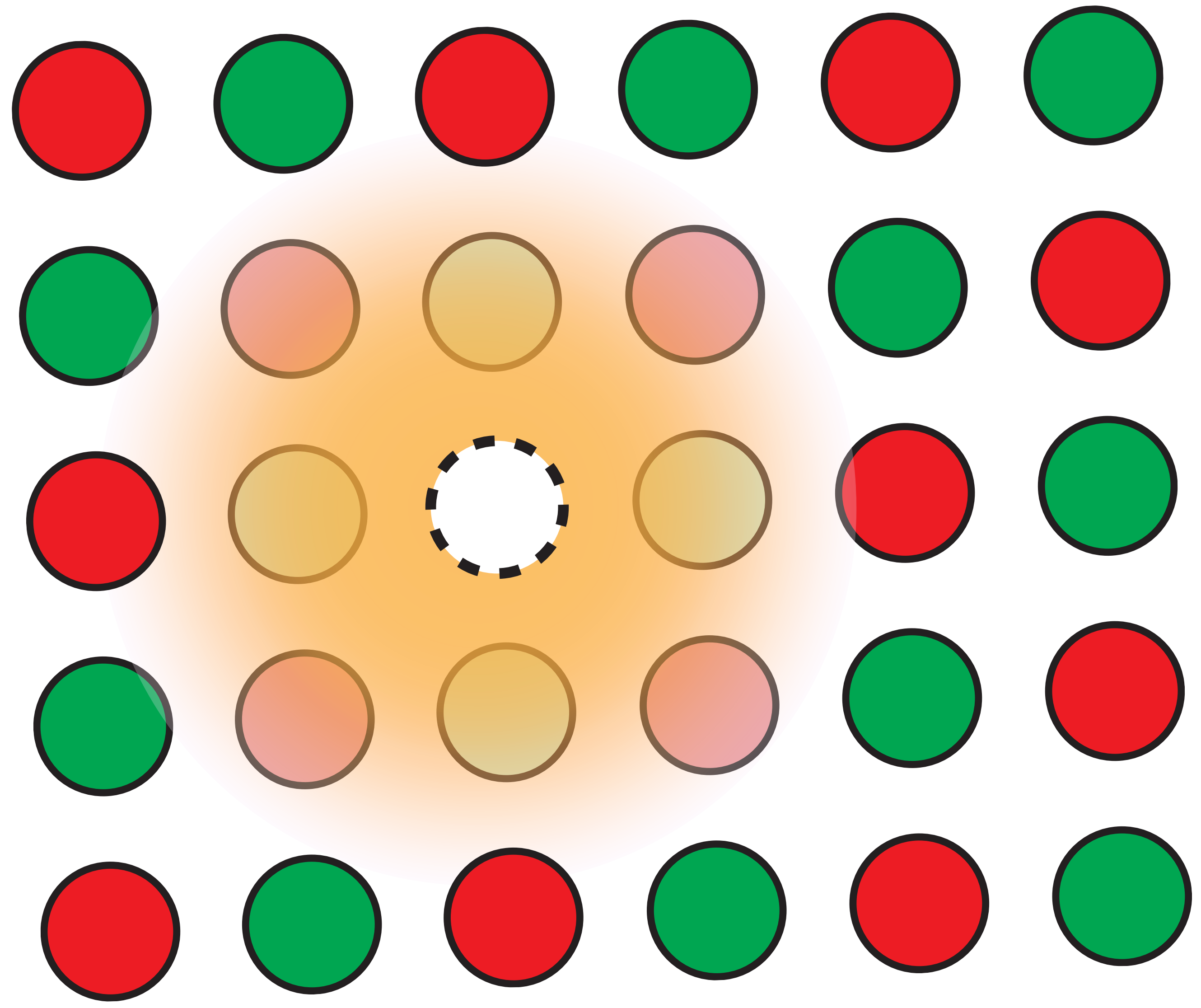
(Received 18 July 1988)

- S. A. Trugman, Phys. Rev. B **37** (1988)
- S. Schmitt-Rink, C. M. Varma & A. E. Ruckenstein PRL **60**, 2793 (1988)
- J. R. Schrieffer, X. G. Wen & S. C. Zhang, Phys. Rev. B **39** (1989)
- C. Kane, P. Lee & N. Read Phys. Rev. B **39** (1989)
- B.D. Simons & J.M.F. Gunn, Phys. Rev. B **41** (1990)
- P. Lee *et al.,* Rev. Mod. Phys. **78** (2006)

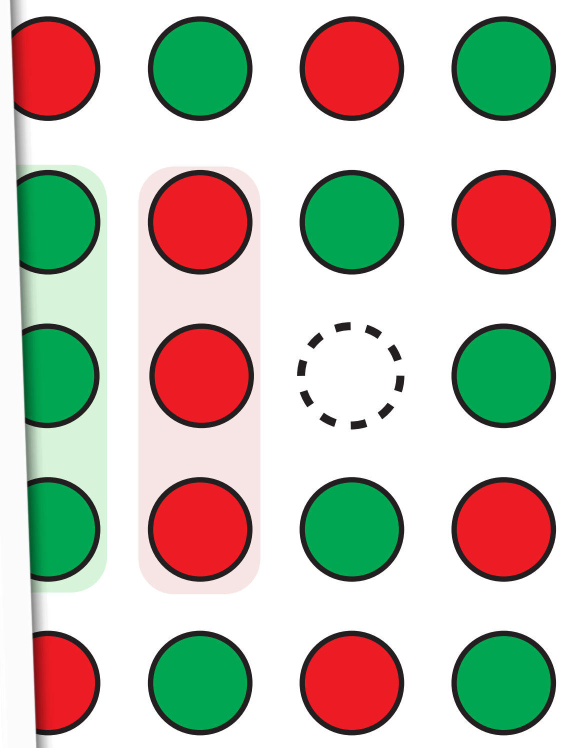
Polaron Formation in 2D

Holes cannot move freely in 2D - holon and spinon are bound





Magnetic polaron is formed!



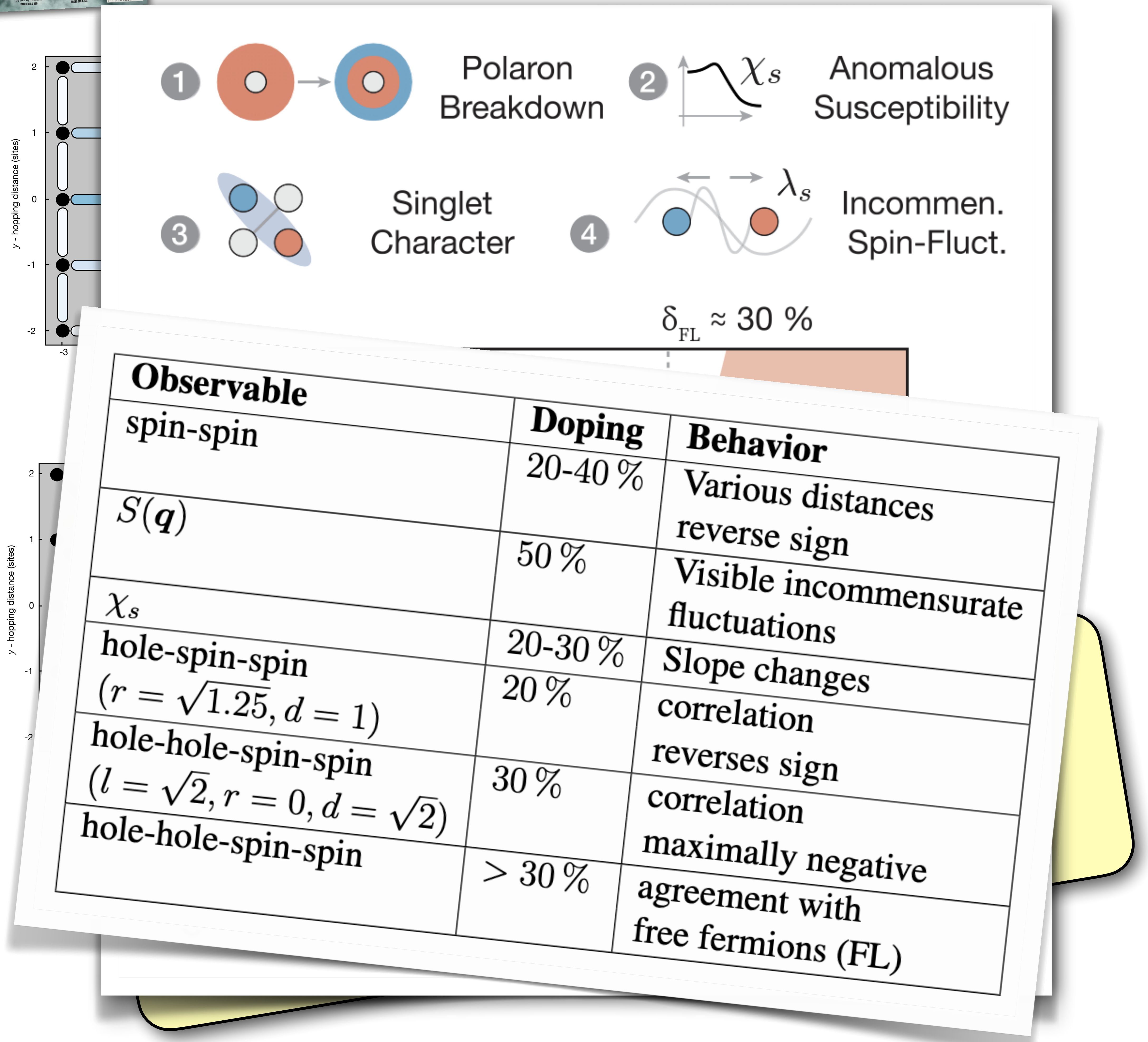
aves frustrated bonds
CONFINEMENT!
lolon form “Parton”

s Phys. Rev. B **42**, 4370 (1990)
F. Grusdt *et al.*, PRX **8**, 011046 (2018)

S. A. Trugman, Phys. Rev. 8 37, 1579 (1988)
J. R. Schrieffer, X. G. Wen & S. C. Zhang, Phys. Rev. B **39** (1989)
C. Kane, P. Lee & N. Read Phys. Rev. **B** 39, (1989)
B. D. Simons and J. M. F. Gunn, Phys. Rev. B 41, 7019 (1990)
P. Lee *et al.,* Rev. Mod. Phys. **78** (2006)

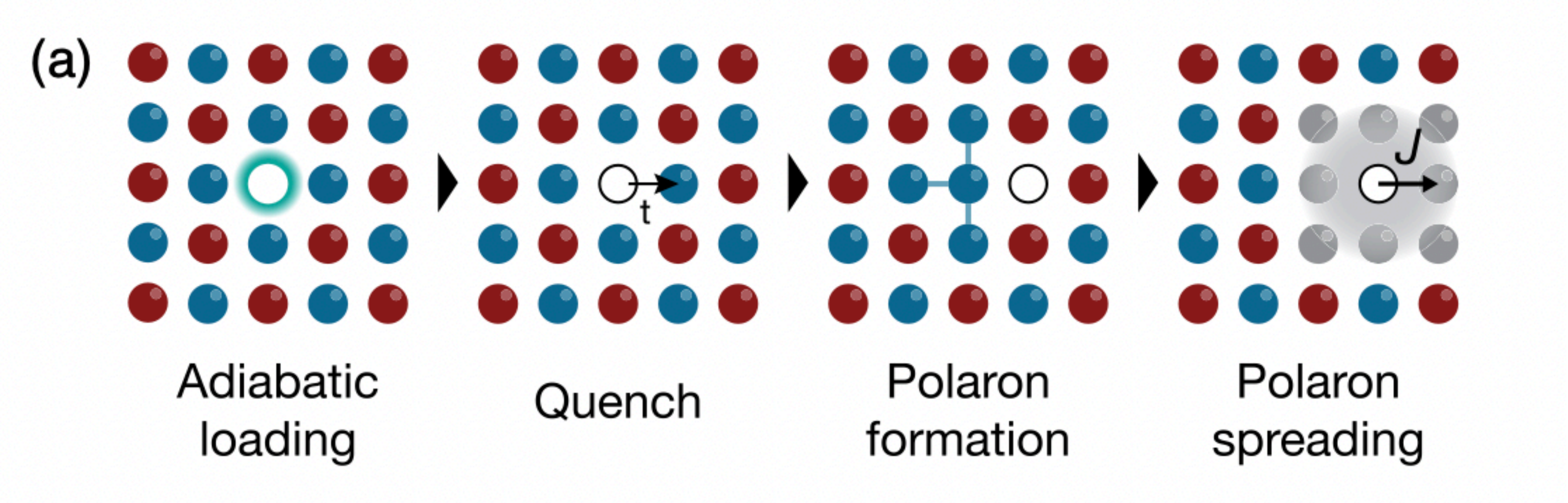
Microscopic Polaron Image & Motion

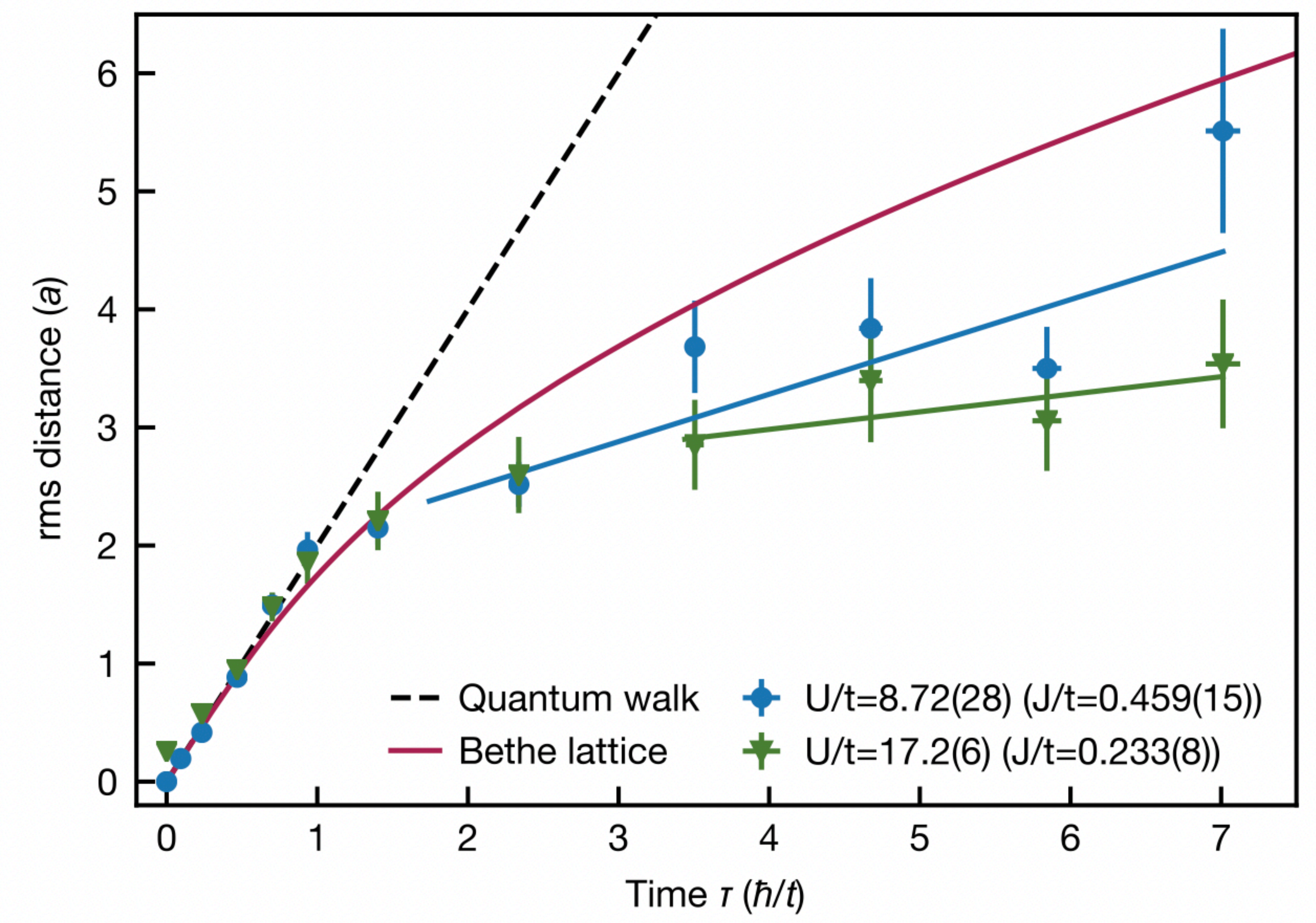
Static (Microscopic Picture)



Observable	Doping	Behavior
spin-spin	20-40 %	Various distances reverse sign
$S(\boldsymbol{q})$	50 %	Visible incommensurate fluctuations
χ_s	20-30 %	Slope changes
hole-spin-spin ($r = \sqrt{1.25}, d = 1$)	20 %	correlation reverses sign
hole-hole-spin-spin ($l = \sqrt{2}, r = 0, d = \sqrt{2}$)	30 %	correlation maximally negative
hole-hole-spin-spin	> 30 %	agreement with free fermions (FL)

Dynamical evolution





J. Koepsell *et al.* Nature **572**, 358 (2019) & J. Koepsell *et al.* **374**, 82 (2021)

See also recent results on frustrated systems: Princeton & Harvard

related: detection of string patterns Ch. Chiu *et al.* Nature (2019)

attractive U: T. Hartke *et al.* arXiv:2208.05948

G. Ji et al. PRX **11,** 021022 (2021)

High Temperature Hole Pairing in Fermi Hubbard Ladders

A. Bohrdt, L. Homeier, I.B., E. Demler & F. Grusdt *Nat. Phys.* **18**, 651 (2022)
S. Hirthe *et al. Nature* **613**, 463–467 (2023)

How to bind with purely repulsive interactions?

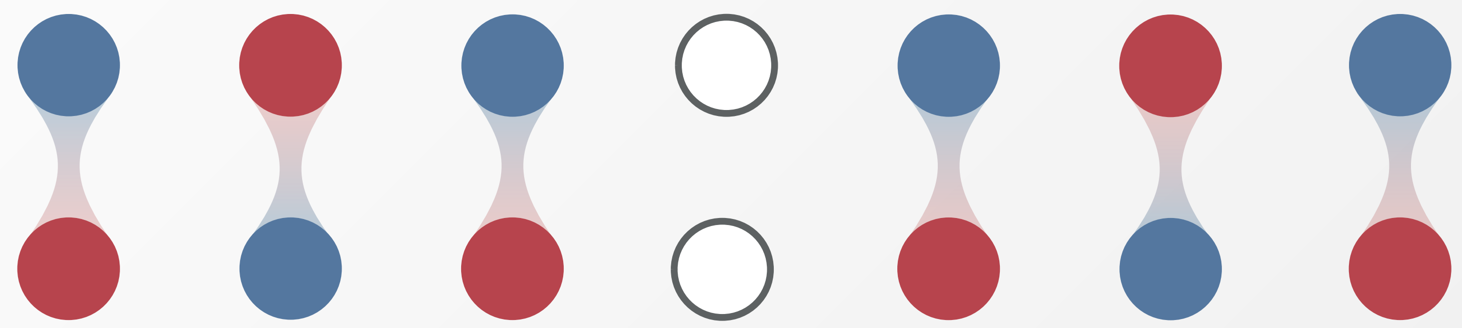
(non standard binding mechanism)

↓

via magnetic correlations!

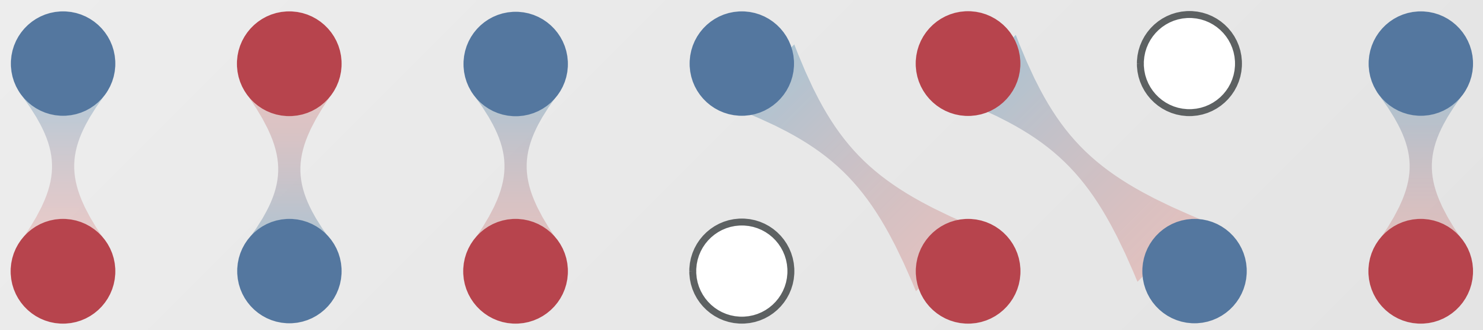
Binding Mechanism

1



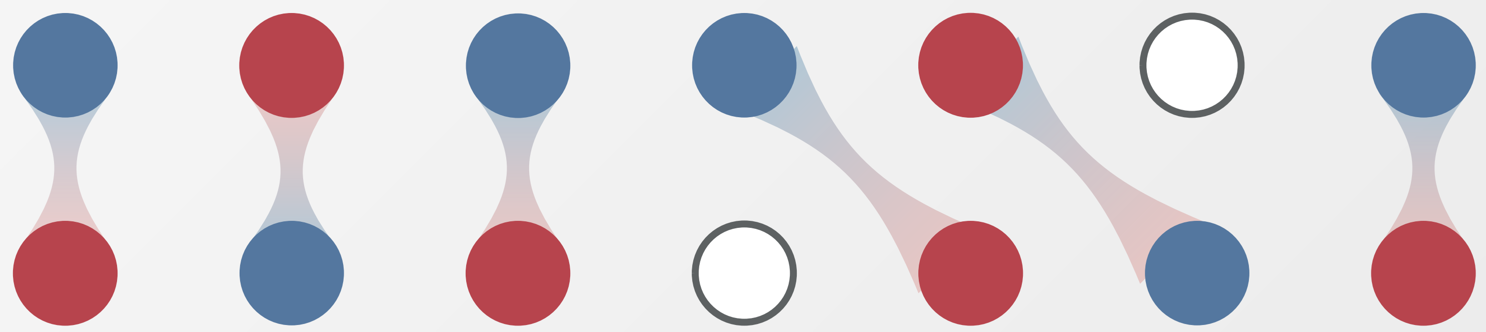
Single holon moves

2



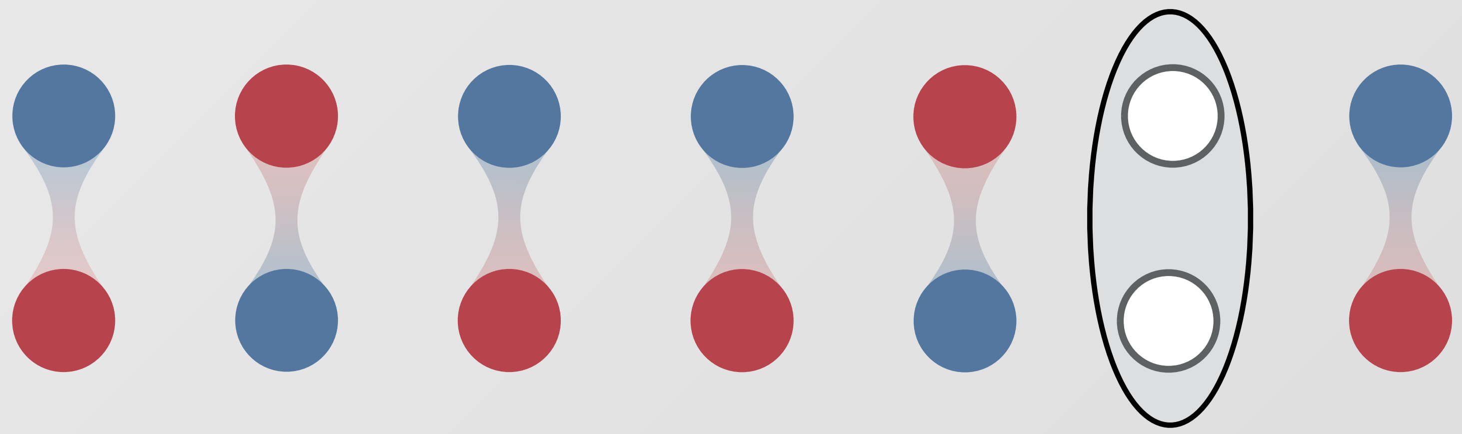
Energy penalty $\approx 2J_{\perp}$

3



Second holon undoes effect of first hole

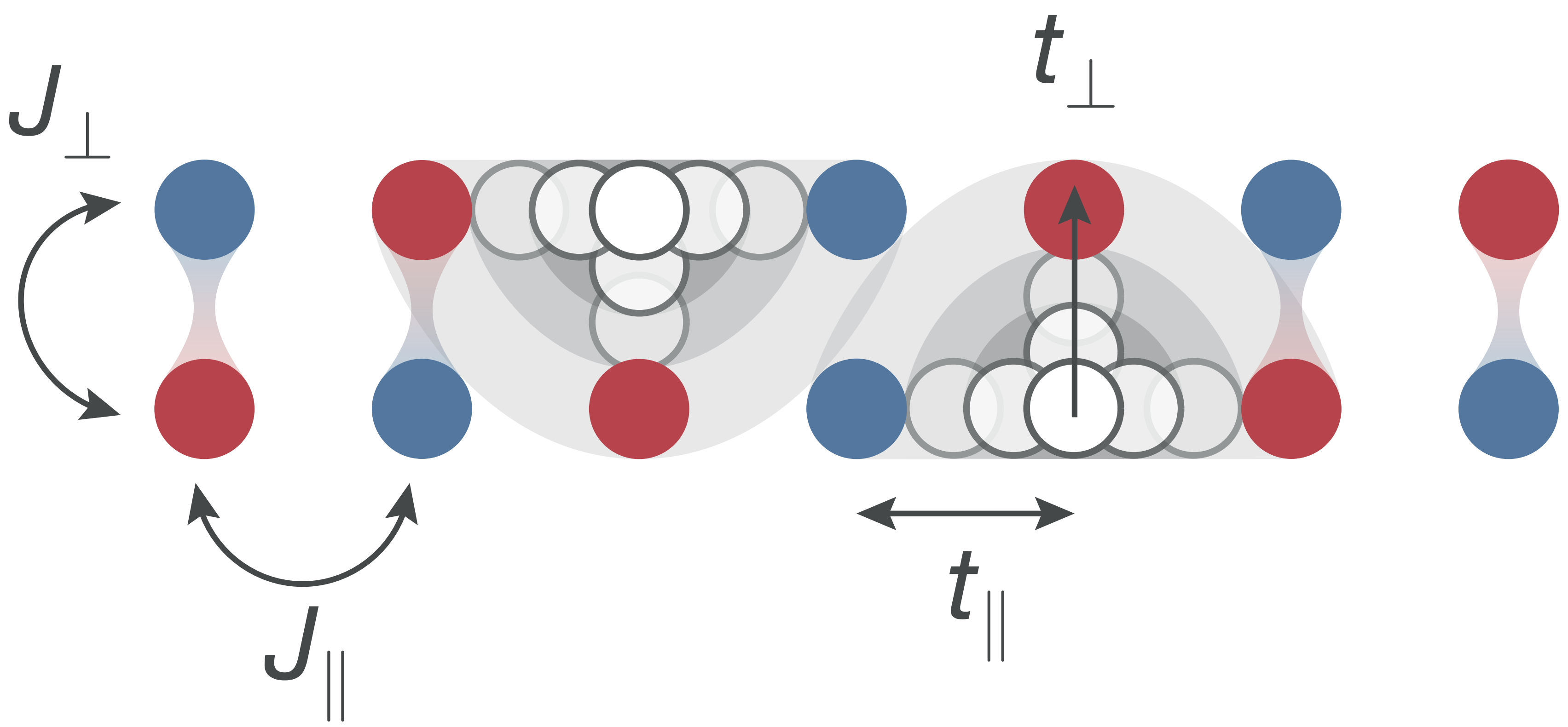
4



Pairing effect!

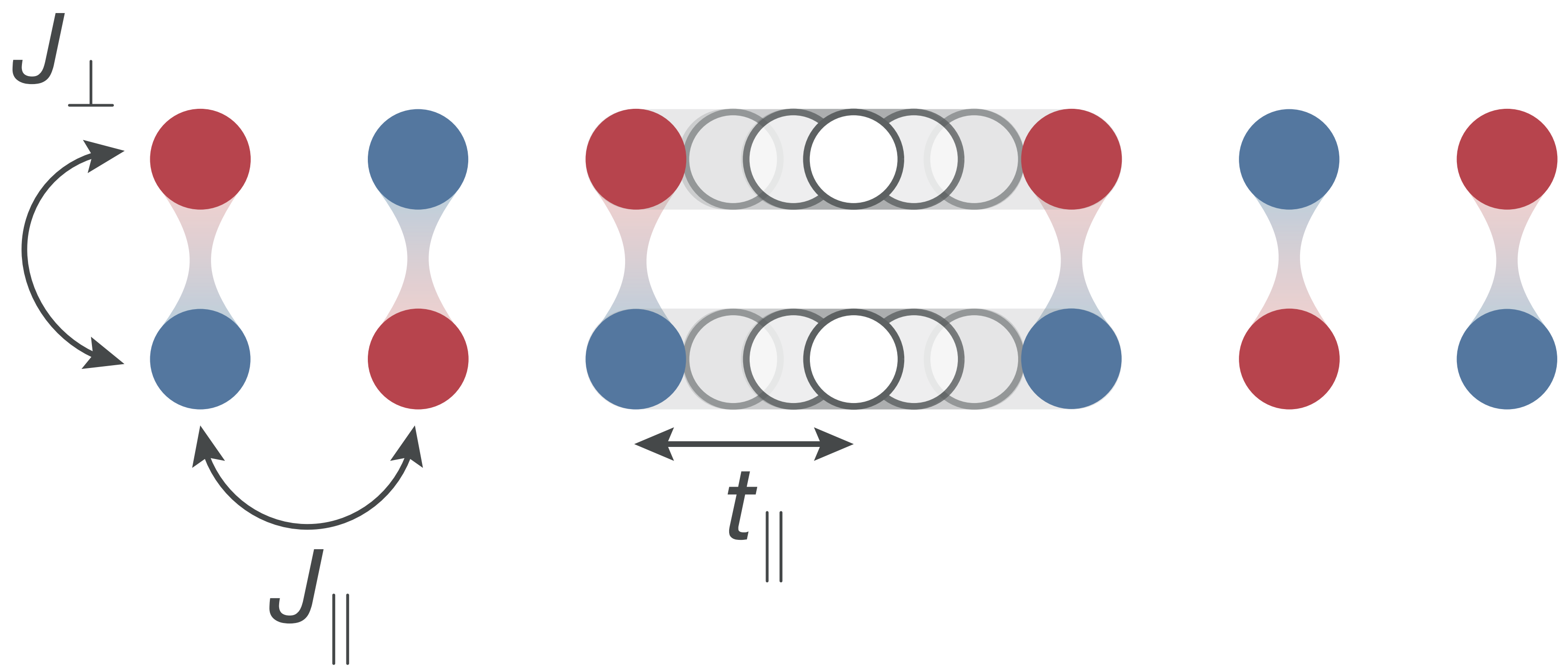
Why Mixed-D Helps

2D



- **Binding reduced** by "Fermi hole-hole repulsion"
- **Weak binding energy** (fraction of $J_\perp$) and **large pairs**
- **low-T required**

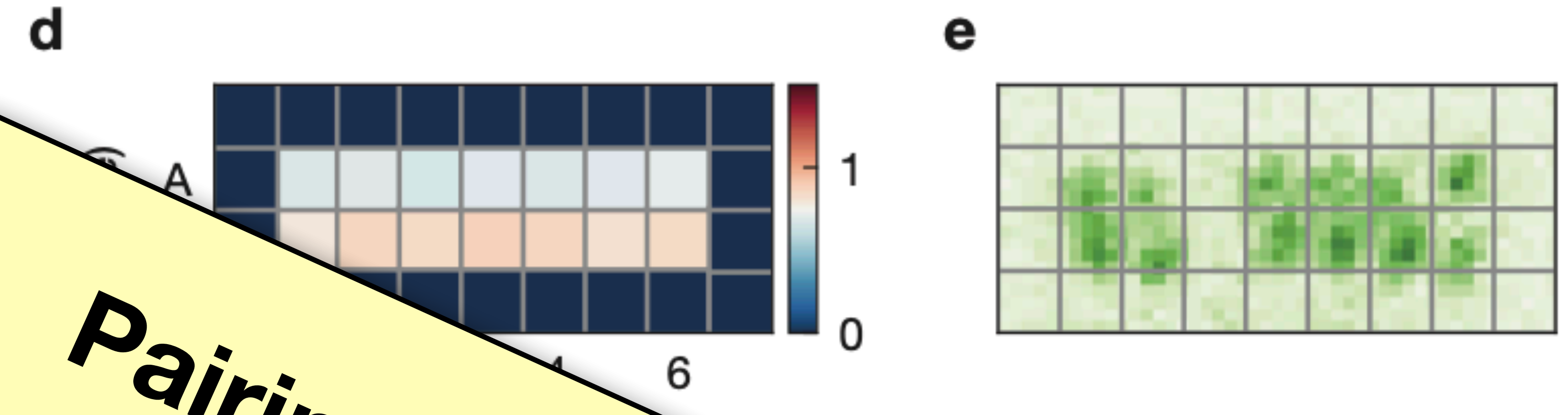
Mixed Dimensions - Charge 1D; Spin 2D



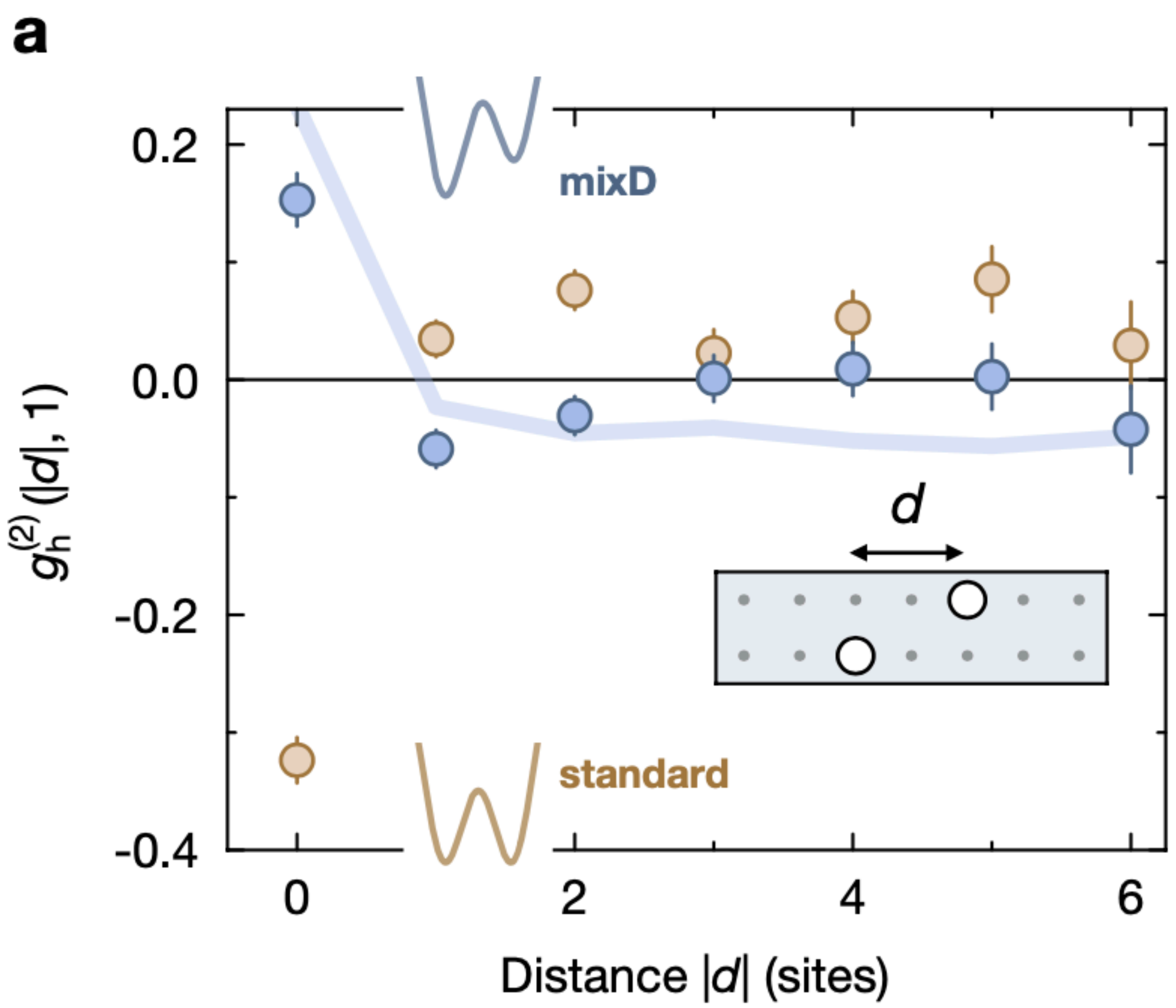
- **Fermi hole-hole repulsion removed**
- **Strong binding energy** (as large as t) and **rung pairs**
- **Much higher-T enabled**

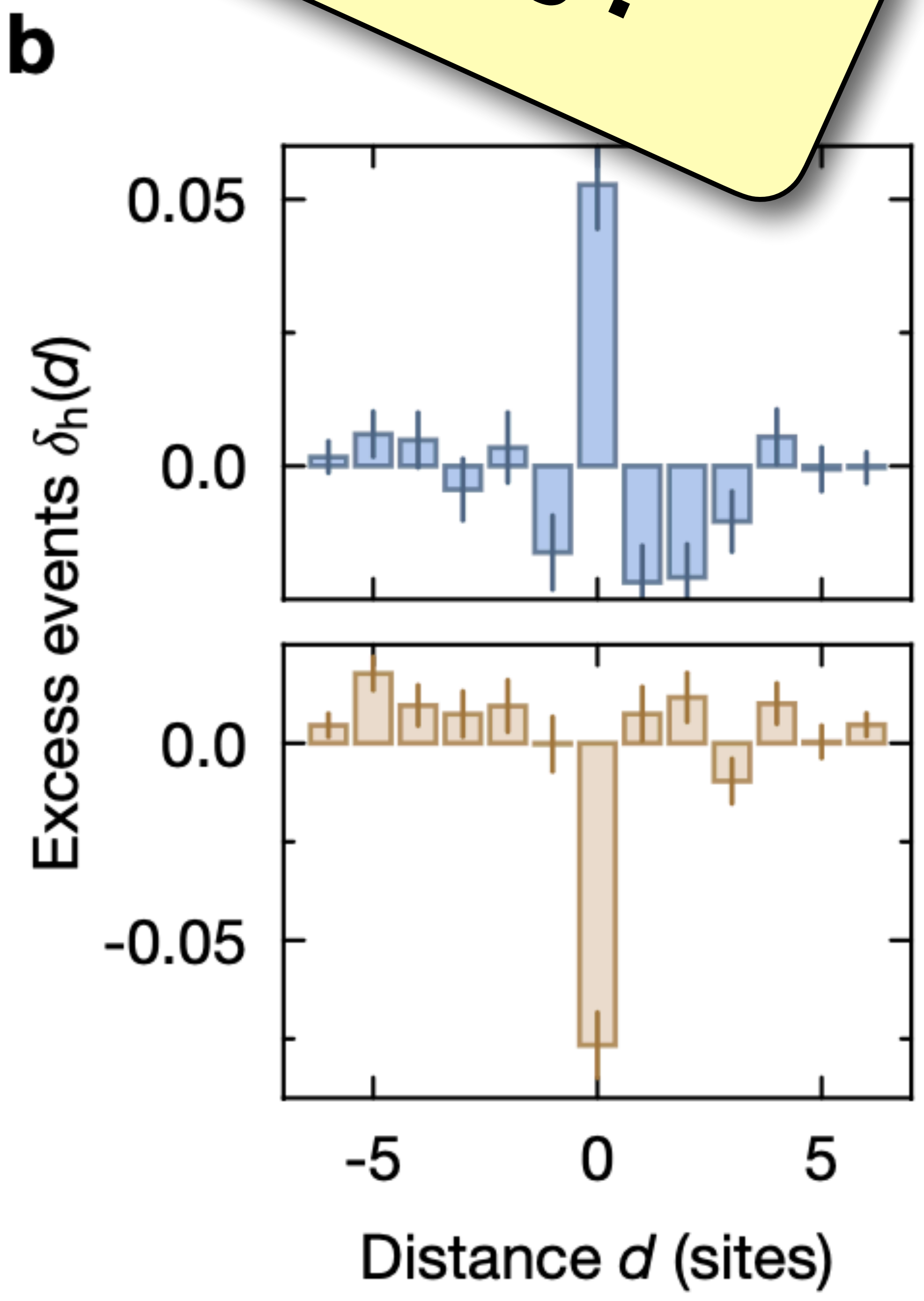
Hole-Hole Correlations

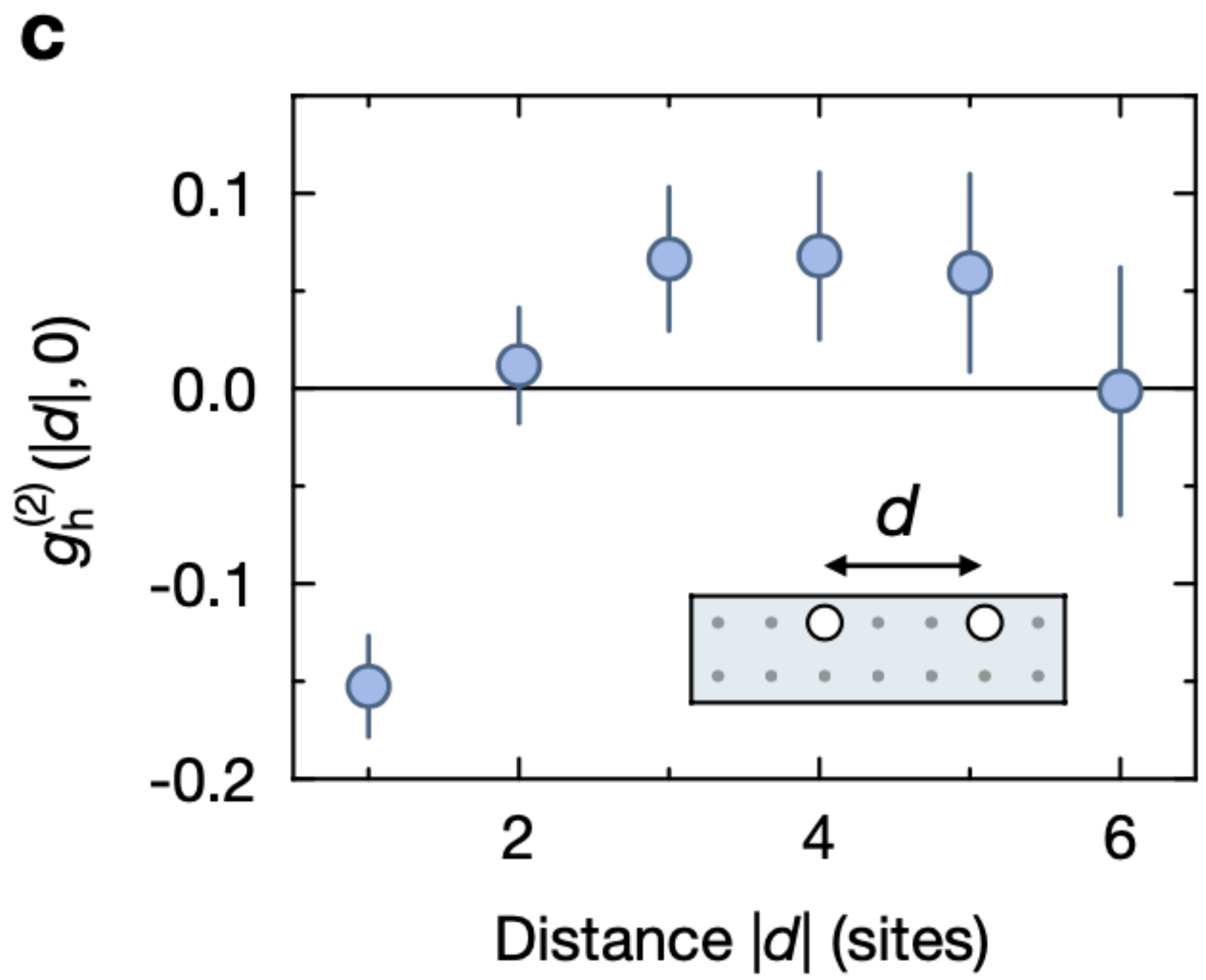
$$g_{\mathrm{h}}^{(2)}(\boldsymbol{r}) = \frac{1}{\mathcal{N}_{\boldsymbol{r}}} \sum_{\substack{i,j \\ \boldsymbol{r}_i - \boldsymbol{r}_j = \boldsymbol{r}}} \left(\frac{\langle \hat{n}_i^{\mathrm{h}} \hat{n}_j^{\mathrm{h}} \rangle}{\langle \hat{n}_i^{\mathrm{h}} \rangle \langle \hat{n}_j^{\mathrm{h}} \rangle} - 1 \right)$$



Pairing !



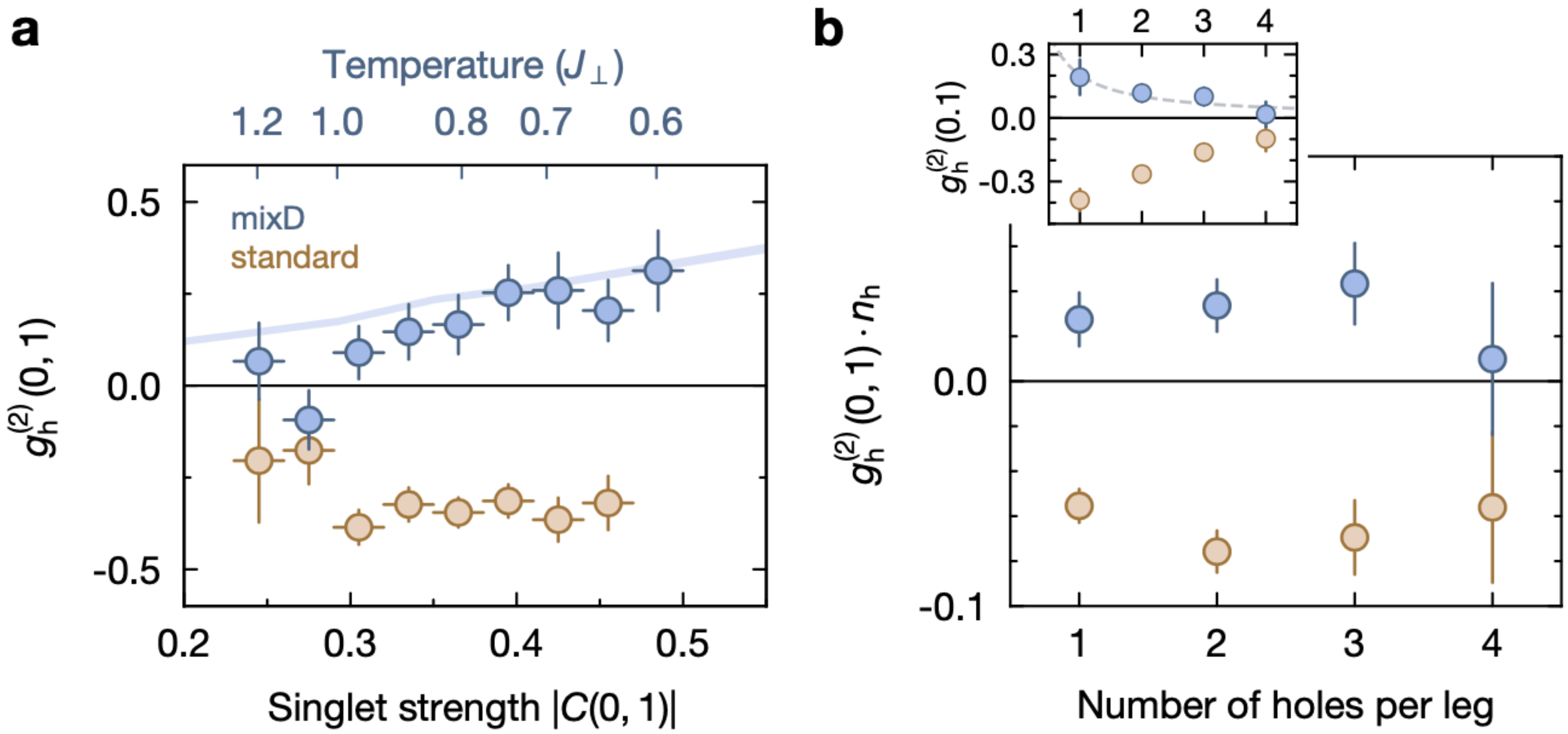




Across Rung

Within Leg

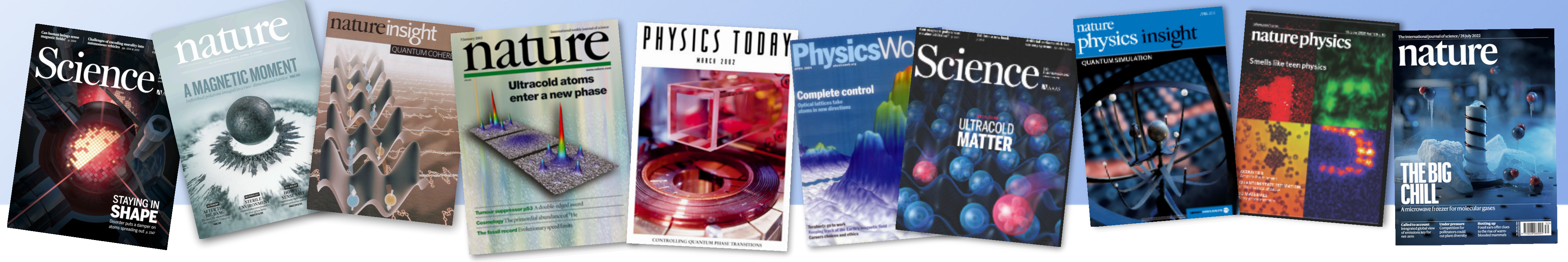
Stability of Pairing Signal



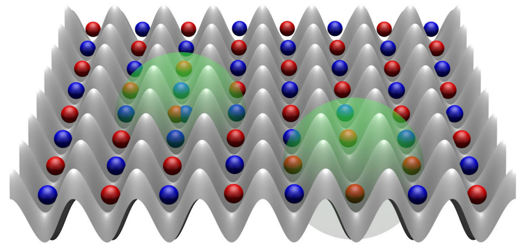
Temperature

Hole Doping

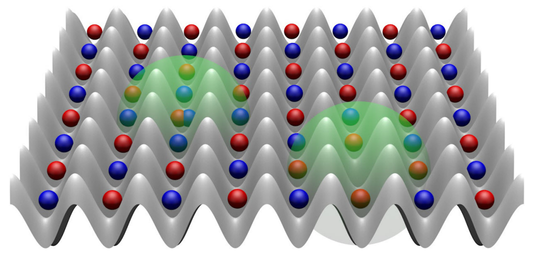
Pair binding energy independent of doping



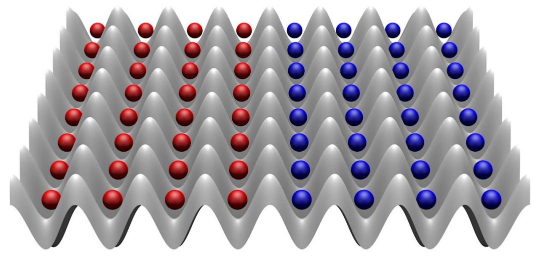
Applications in Many-Body Physics



0 **State Preparation/Engineering and Detection**



1 **Strongly Correlated Electron Physics**
Spin-Charge Separation, Hidden Order, Incommensurate Magnetism, Polarons, Strange Metals, Hole Pairing,…



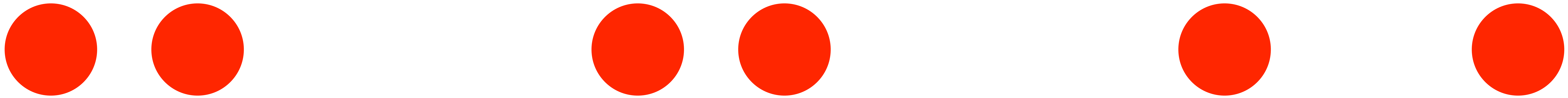
2 **Nonequilibrium Dynamics - Counting Atoms one-by-one**
Fluctuation Hydrodynamics

Promising: Double Quantum Advantage (in Space & Time)

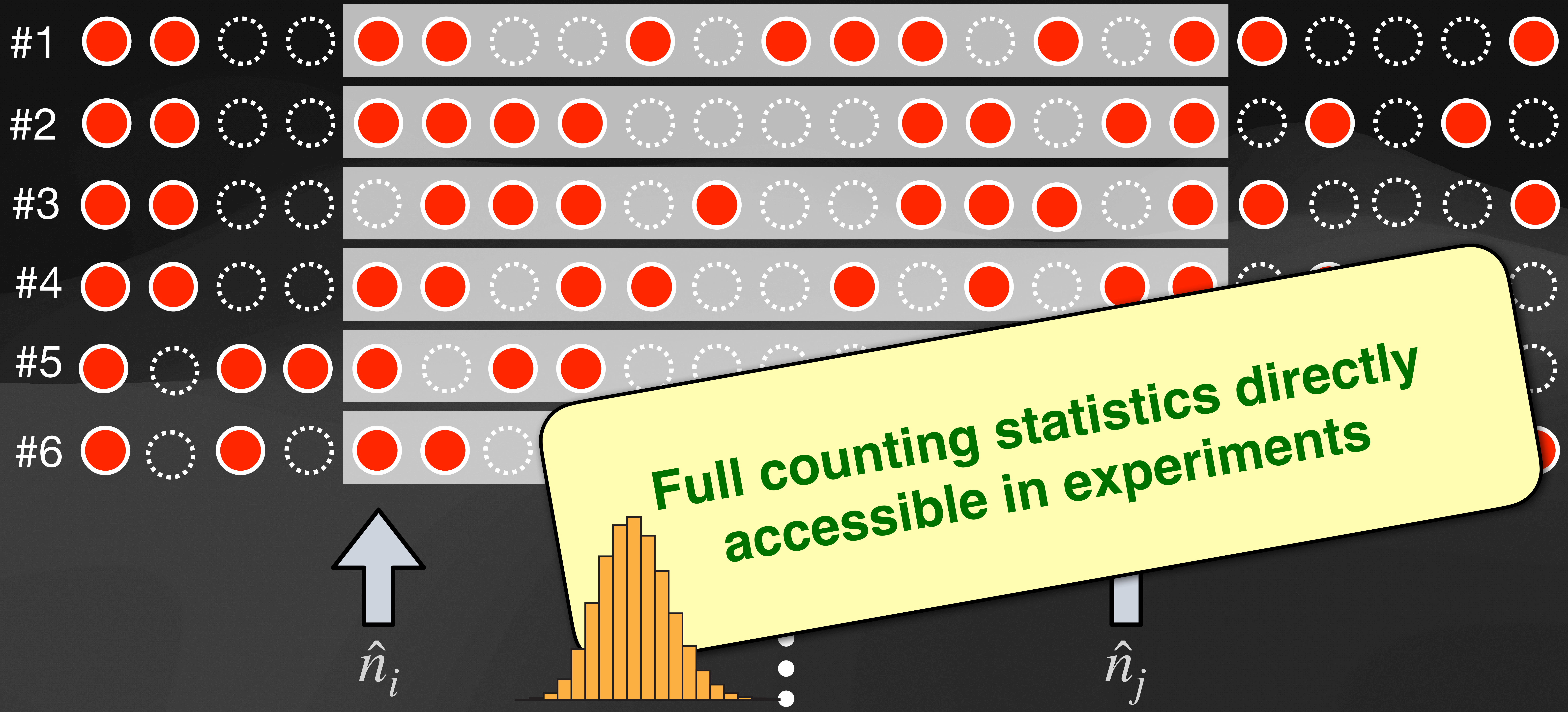
But also challenging for experiments!

- Large System Sizes
- Homogeneous Systems
- Long Time Evolutions

$e^{-i\hat{H}t/\hbar}$ …Time evolution…



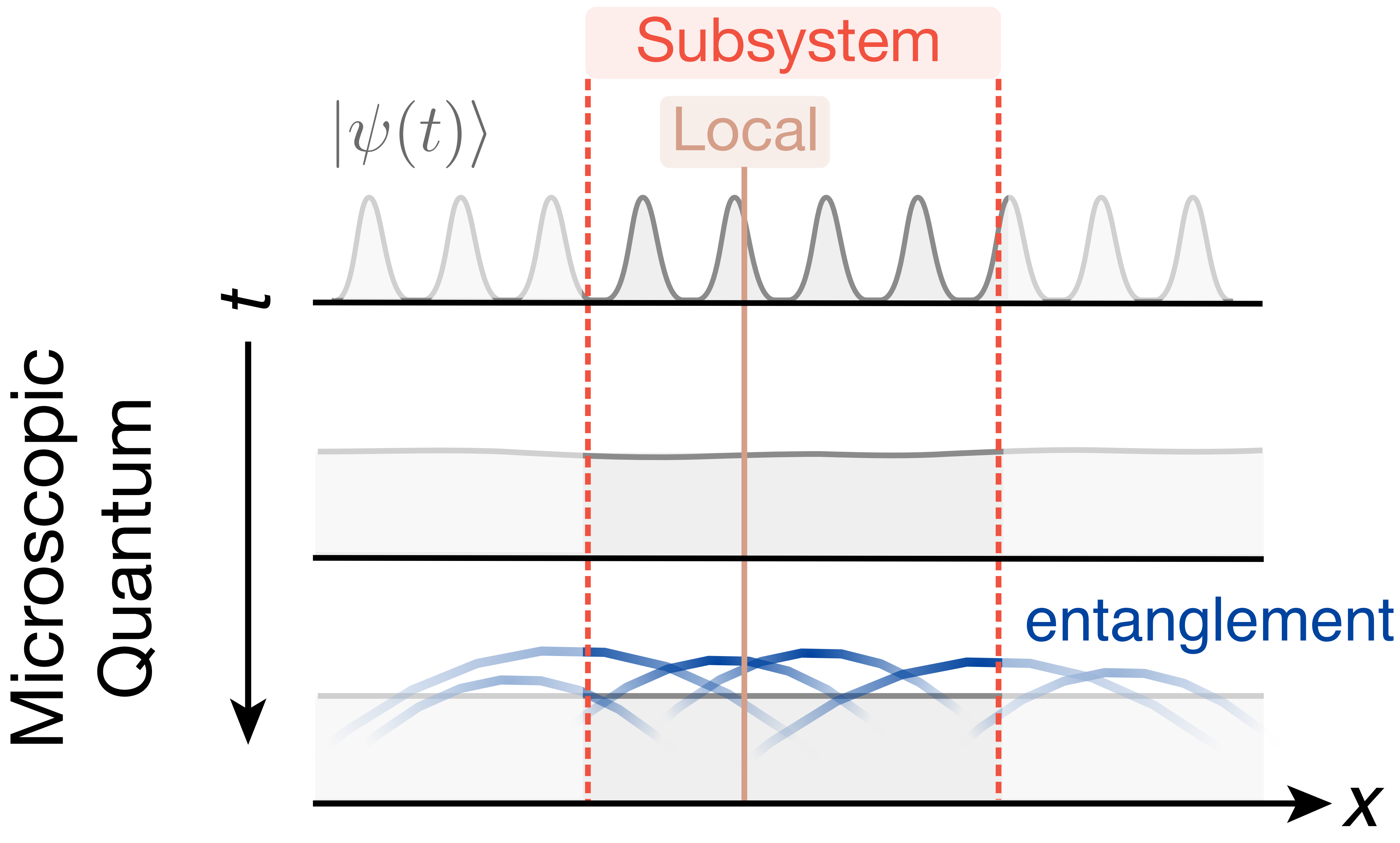
Quantum Transport - Atom-by-Atom

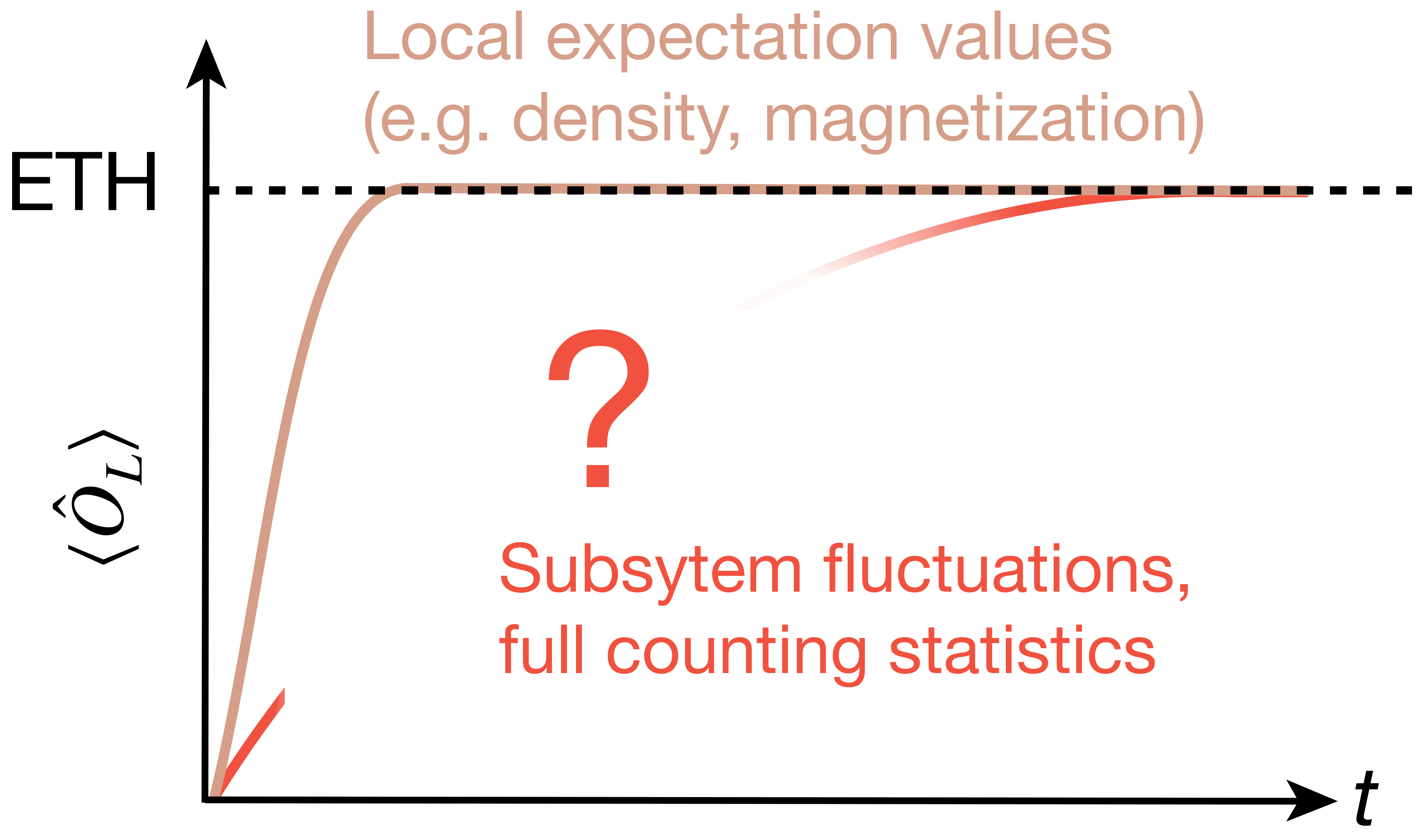


See also: (Endres group) J. Choi *et al.*, Nature **613**, 468 (2023)

Dynamics for Thermalisation

Stages of Dynamics





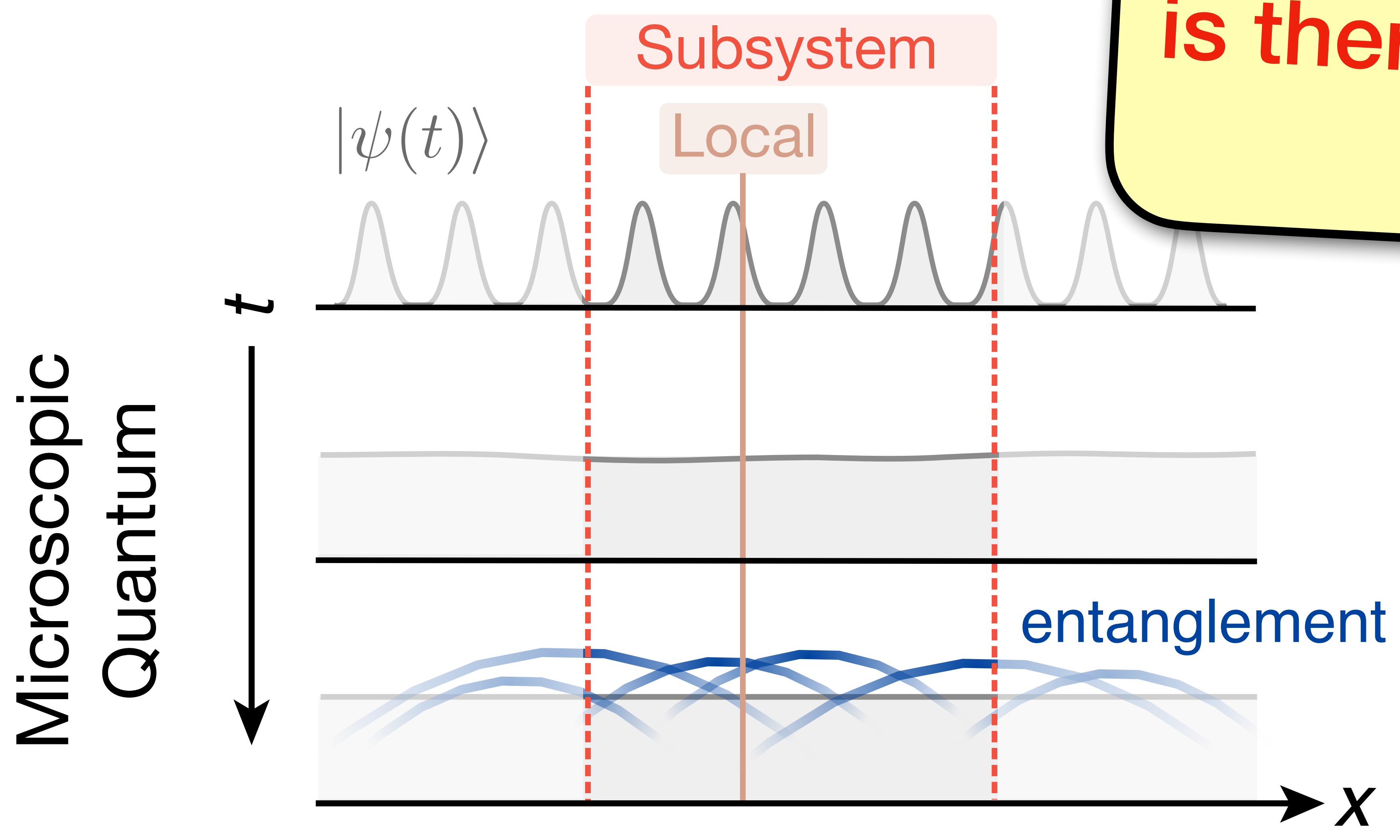
Eigenstate Thermalisation Hypothesis
J. M. Deutsch, Phys. Rev. A **43**, 2046 (1991).
M. Srednicki, Phys. Rev. E **50**, 888 (1994).
M. Rigol, V. Dunjko, and M. Olshanii, Nature **452**, 854 (2008).
R. Nandkishore, Phys. Rev. B **92**, 245141 (2015).
L. D'Alessio, Y. Kafri, A. Polkovnikov, and M. Rigol Adv. Phys. **65**, 239 (2016).

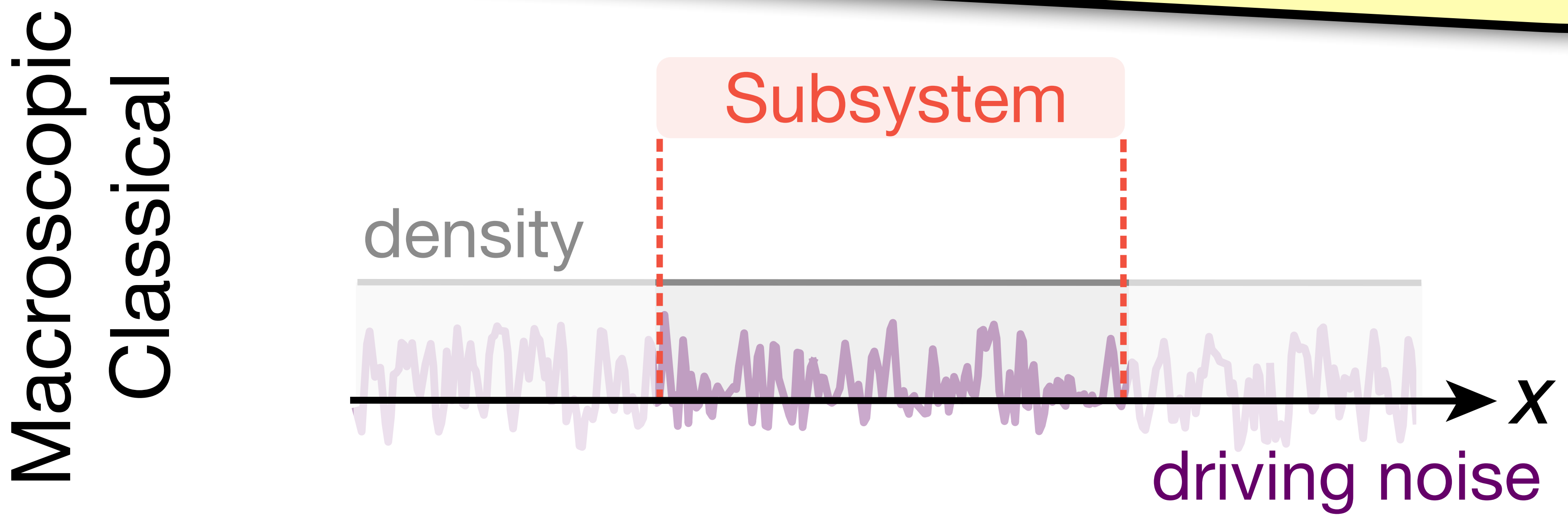
Experiments Thermalisation
A. M. Kaufman et al., Science **353**,794 (2016).

Dynamics for Thermalisation

Can this work for quantum systems as well ???
is there an emergent hydrodynamics of fluctuations

Stages of Dynamics



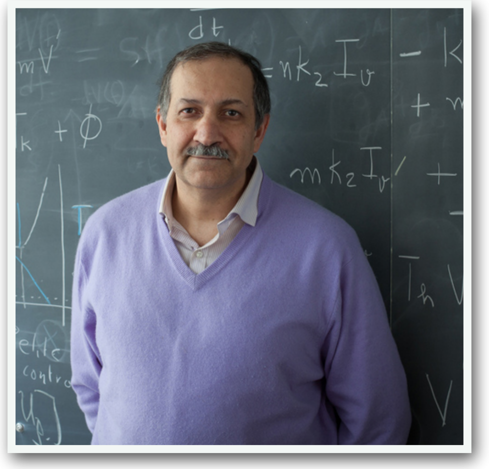


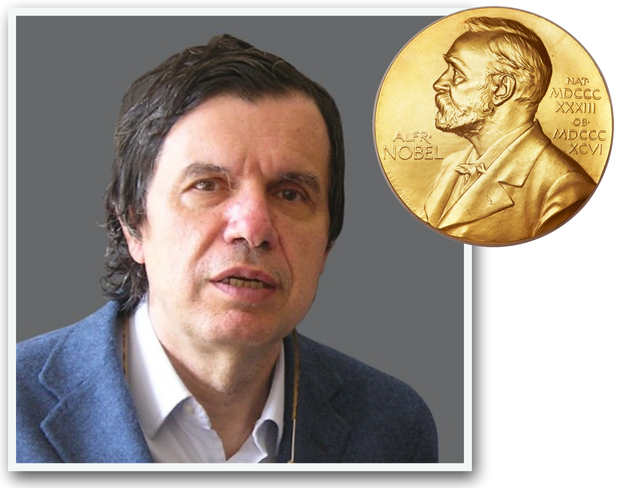
Classical Systems: Macroscopic Fluctuation Theory

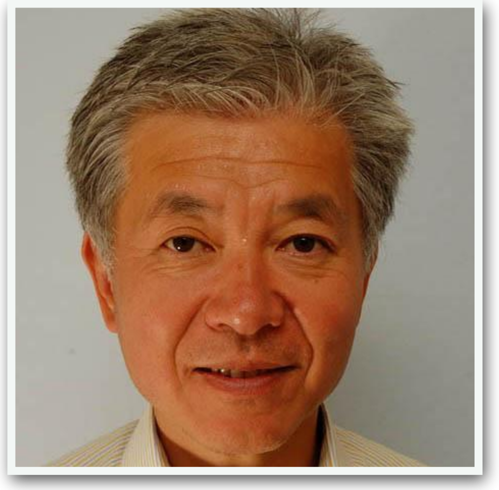
$$\partial_t n + \partial_x j = 0, \quad j = -D(n)\,\partial_x n + \sqrt{2\,D(n)\,\chi(n)}\,\xi$$

Nonlinear stochastic PDE

Bertini, L., De Sole, A., Gabrielli, D., Jona-Lasinio, G. & Landim, C.
Rev. Mod. Phys. **87**, 593–636 (2015).





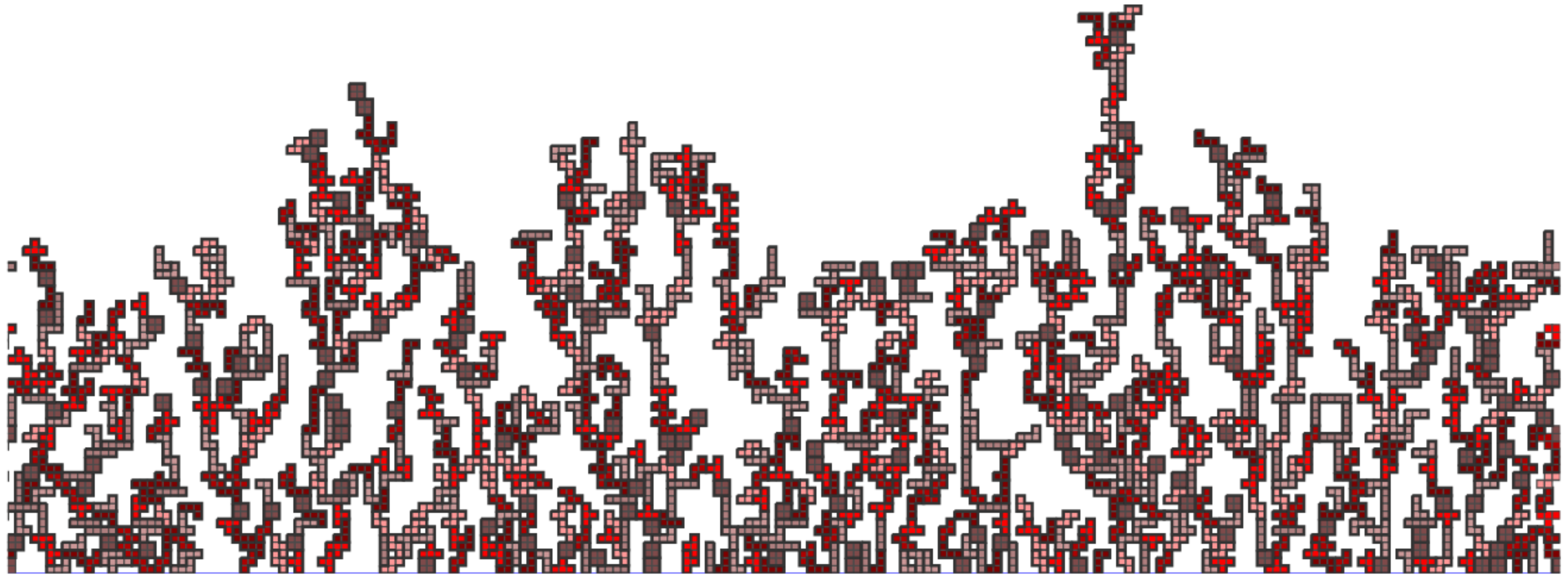


Kardar-Parisi-Zhang Equation

$$\frac{\partial h(\vec{x},t)}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \eta(\vec{x},t)$$

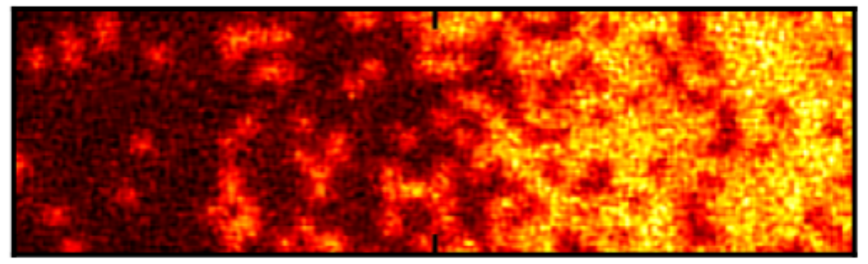
Martin Hairer
Fields Medal
(2014)

Non-linear stochastic differential equation describing **temporal change of height field**



M. Kardar, G. Parisi & Y.-C. Zhang PRL **56,** 889 (1986)
C.A. Tracy & H. Widom Comm. Math. Phys. **159,** 151 (1994)
C.A. Tracy & H. Widom Comm. Math. Phys. **177,** 727 (1994)
M. Prähofer & H. Spohn PRL **84**, 4882 (2000)

$$\hat{S}^z(x,t) \sim \partial_x h(x,t)$$



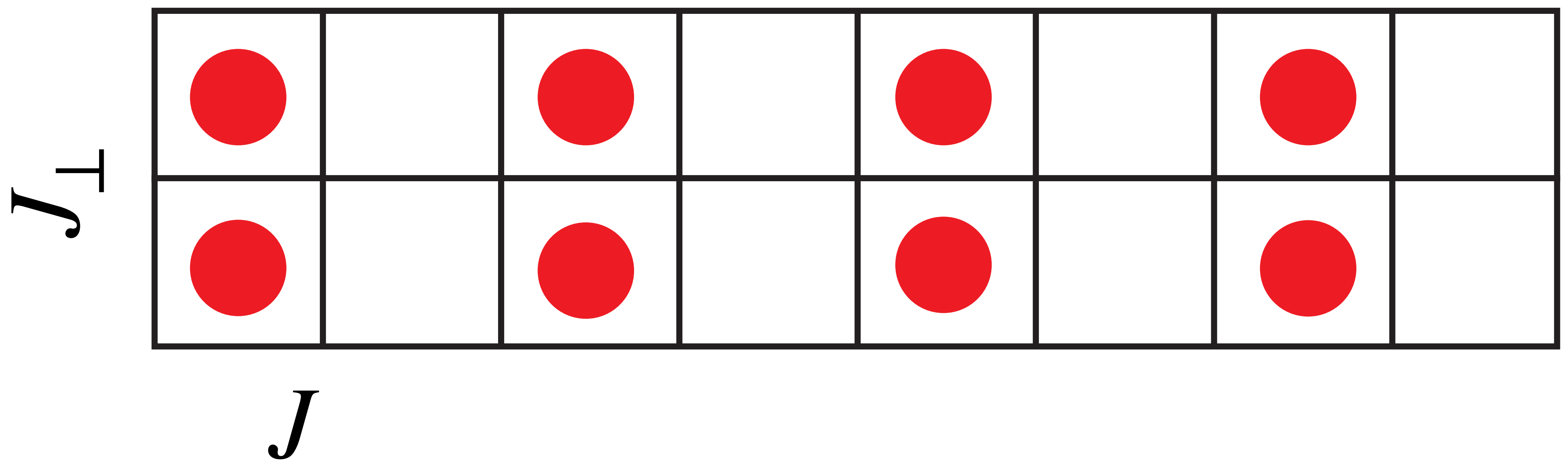
M. Ljubotina *et al.,* Nature Comm. (2017)
M. Ljubotina *et al.*, Phys. Rev. Lett. (2019)
S. Gopalakrishnan and R. Vasseur, Phys. Rev. Lett. (2019)
J. De Nardis, Phys. Rev. Lett. (2019)
S. Gopalakirshnan, R. Vasseur, and B. Ware, PNAS (2019)
V. B. Bulchandani, Phys. Rev. B (2020)

Quantum Exp: D. Wei *et al.* Science **376**, 716 (2022)
see also: A. Scheie *et al.* Nature Phys. **17**, 726 (2021)
Polariton condensate: Q. Fontaine *et al.* Nature **608**, 687 (2022)

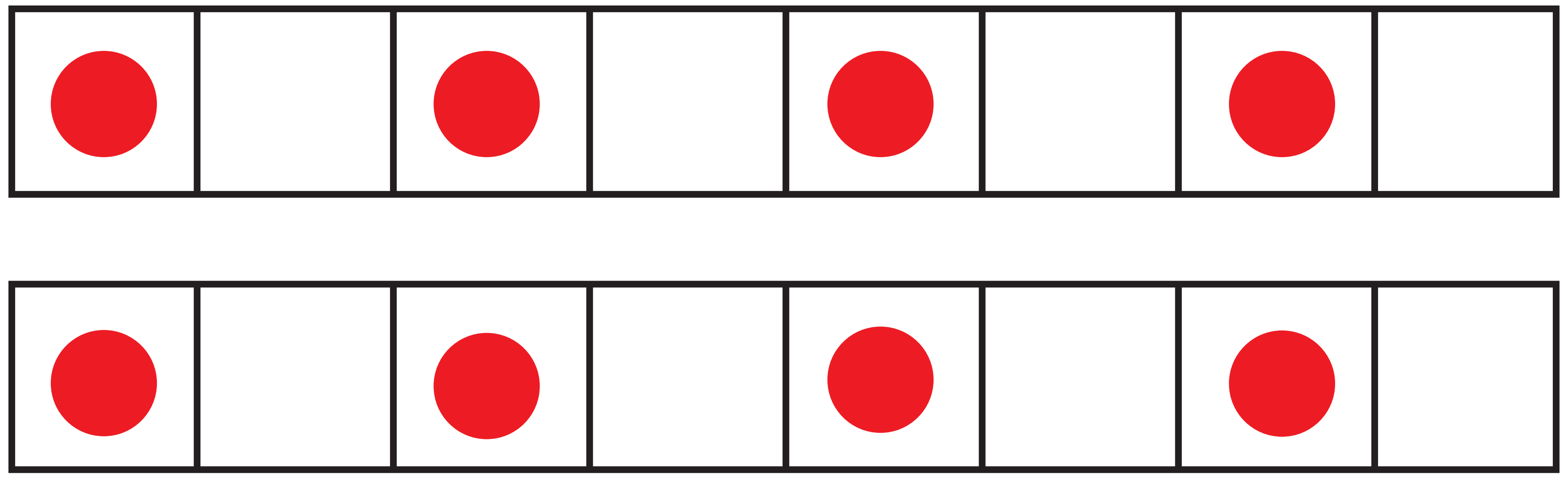
Hardcore bosons

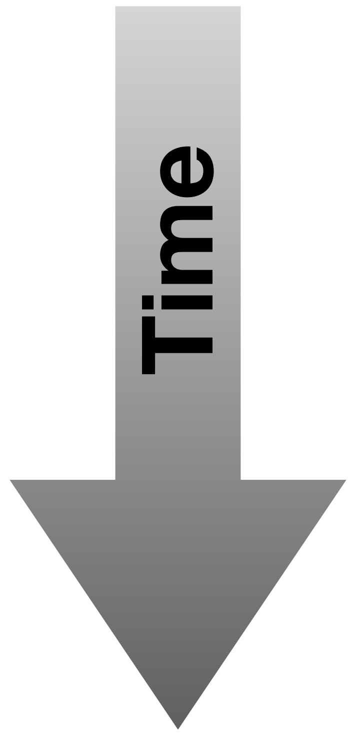
(on a ladder (2x50 sites) with tunable coupling)

Coupled ladders (interacting)

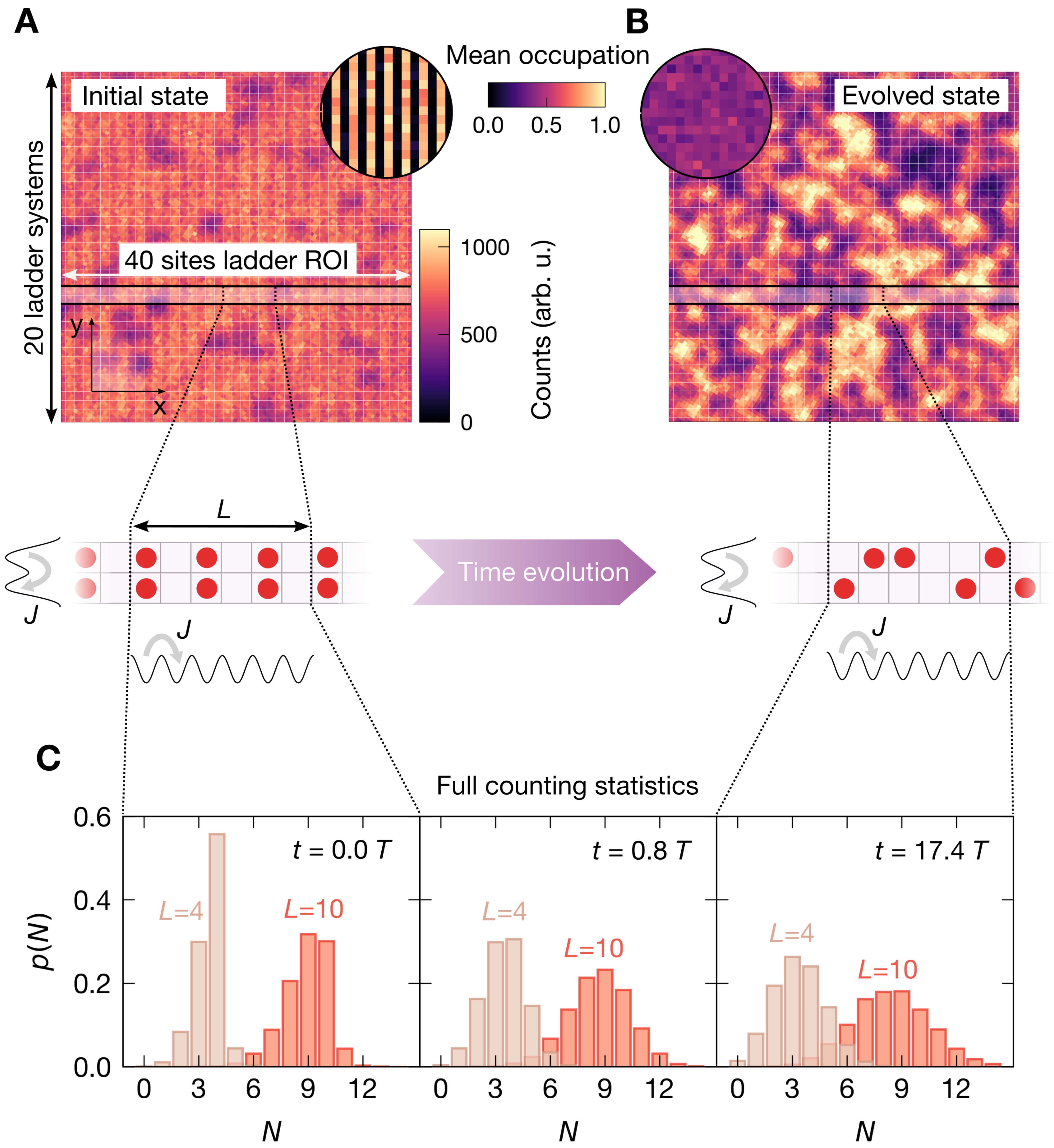


1d chains (free fermions)





Thermalisation: Integrable vs. Chaotic

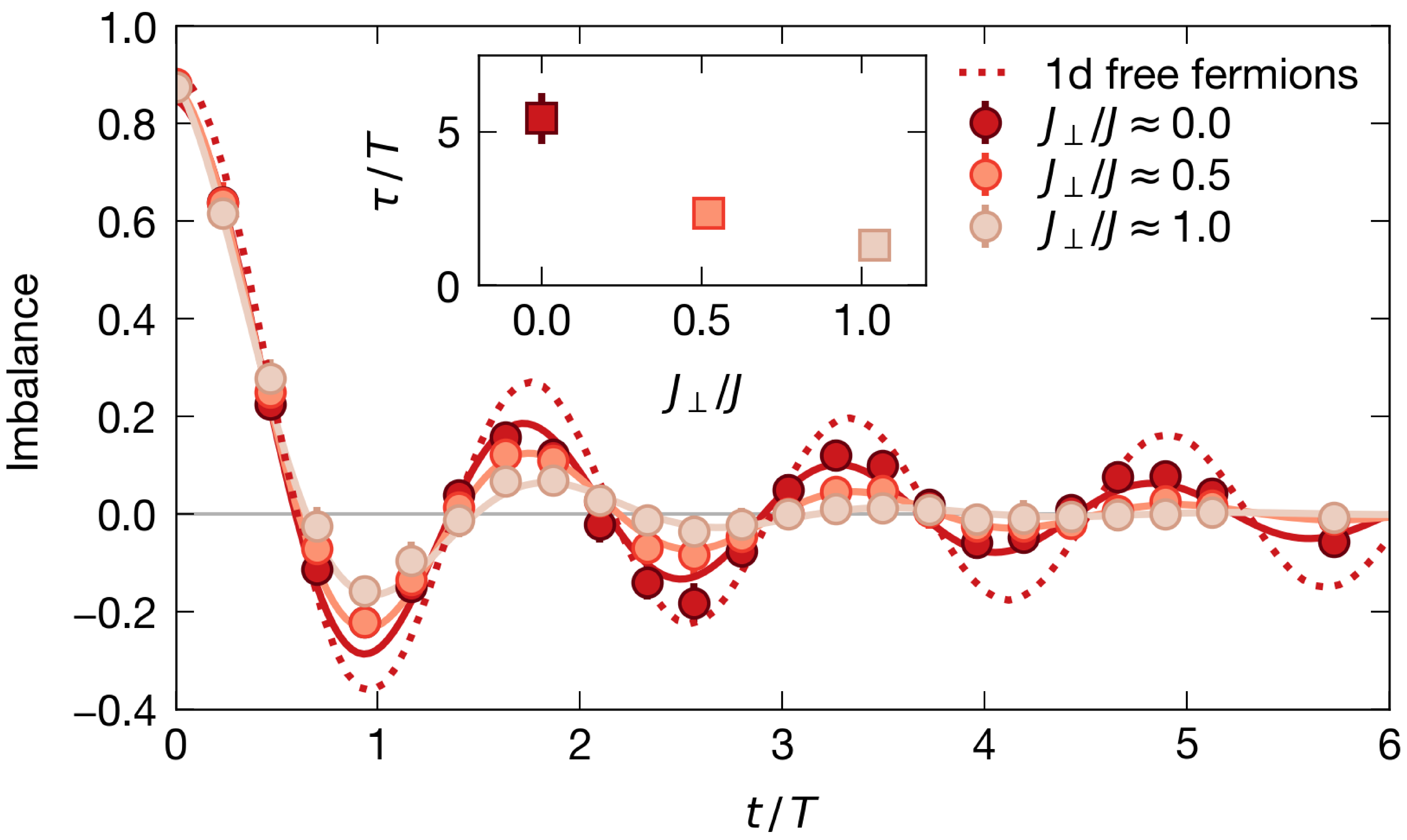


Hardcore bosons (tuneable ladder coupling)

Full counting statistics on different subsystem sizes

Related:
A.M. Kaufman *et al.*
Science **353**, 794 (2016)
and thermalisation
experiments by D. Weiss (PennState)

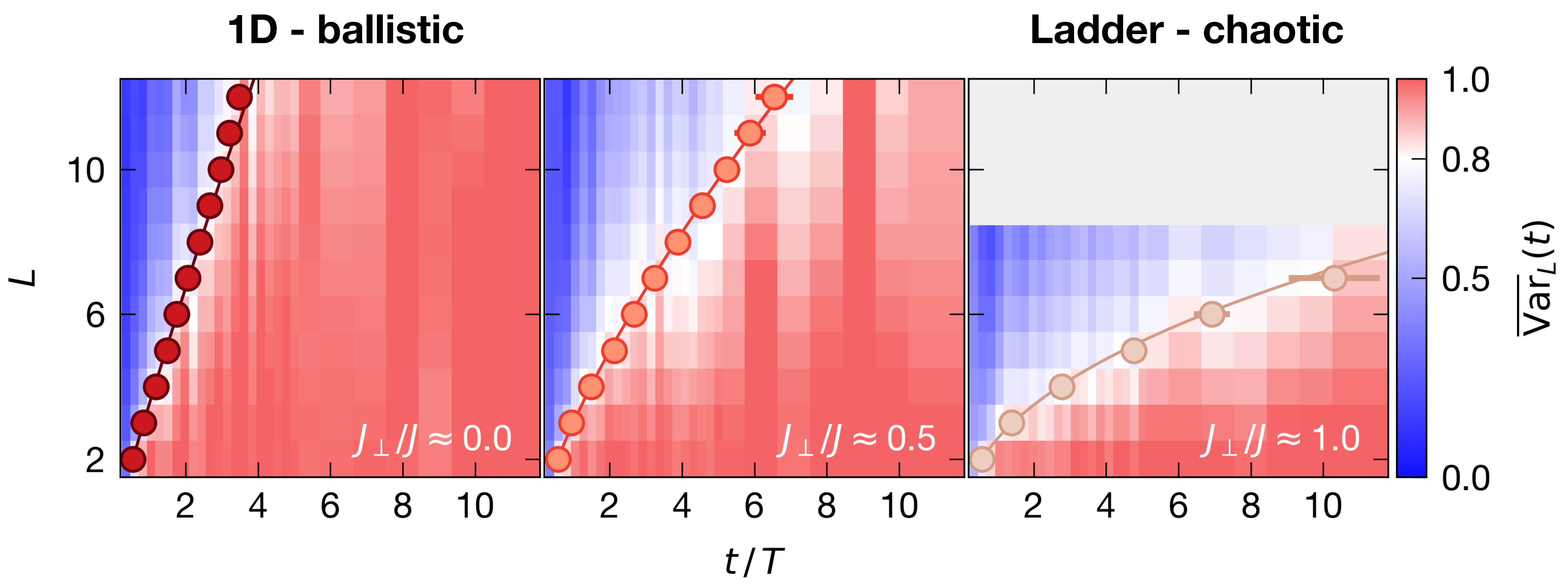
Local equilibration

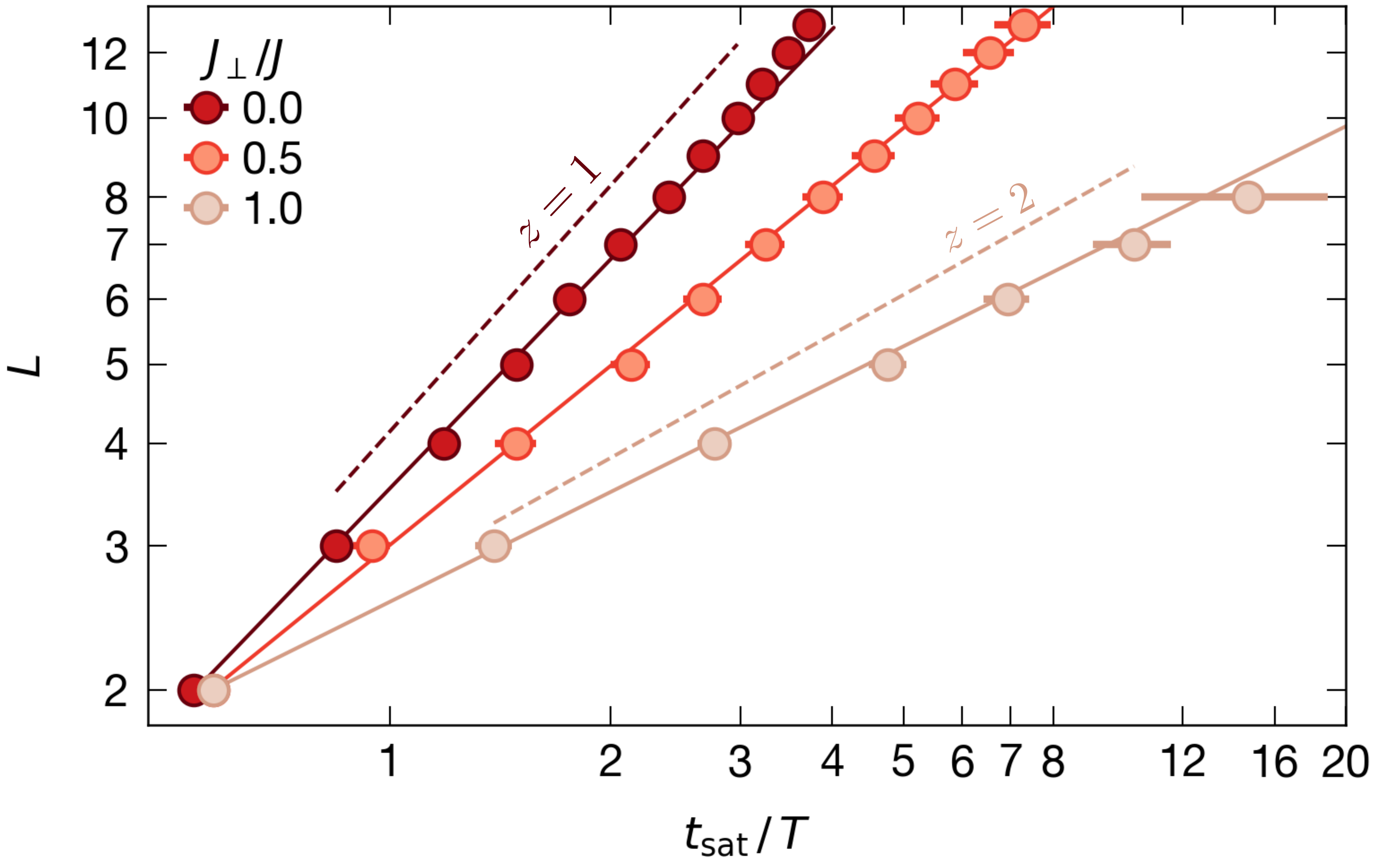


See also: S. Trotzky *et al.* Nat. Phys. **8**, 325 (2012)

Subsystem Fluctuations

Subsystem fluctuations





$$\mathrm{Var}_L(t) \approx \sqrt{\frac{2Dt}{\pi a^2}}$$

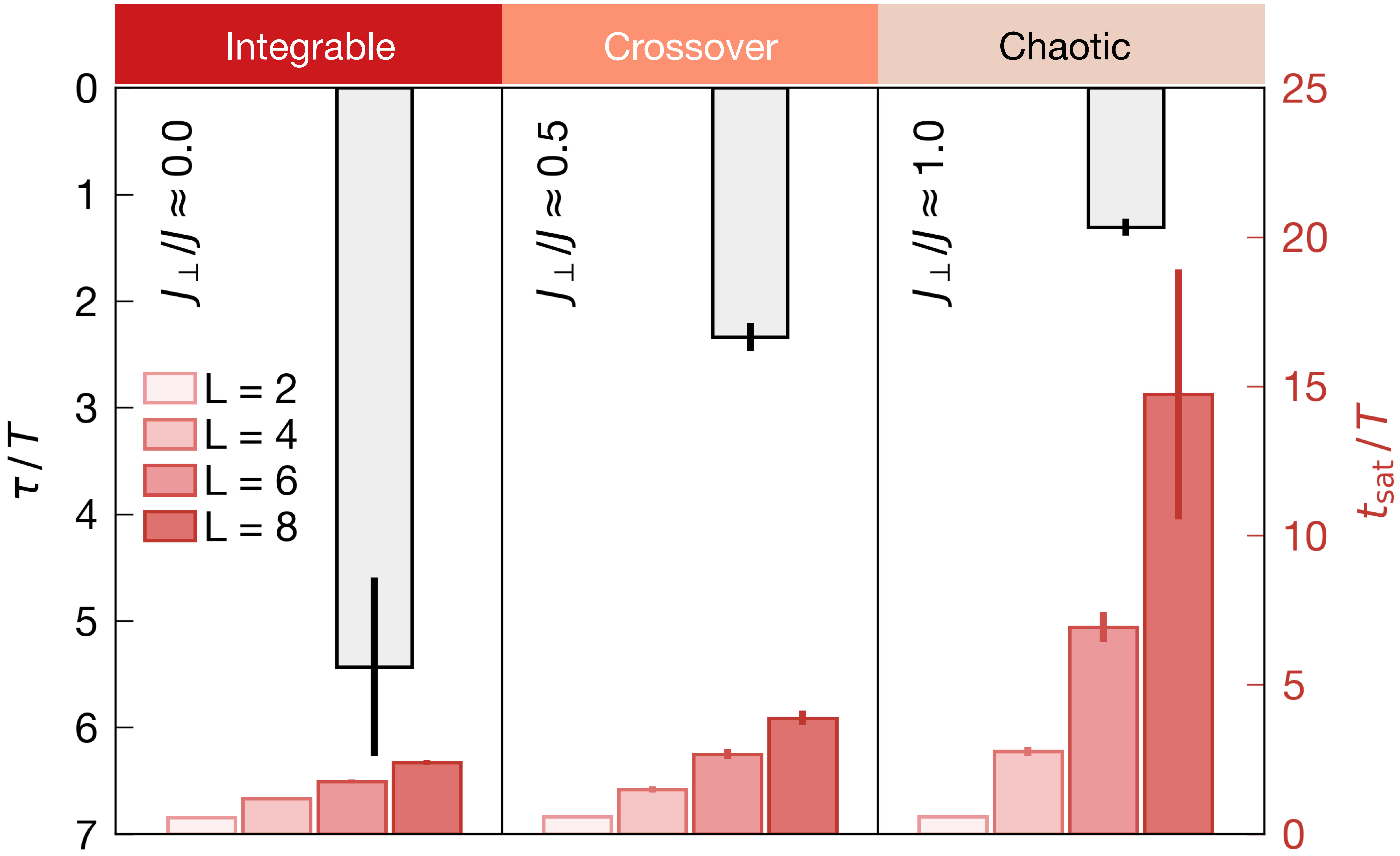
MFT prediction

- Enables quantitative determination of **dynamical exponent z** and **diff… constant**
- **Global equilibrations** takes longer and longer for larger subsystem

$$L \propto t_{sat}^{1/z}$$

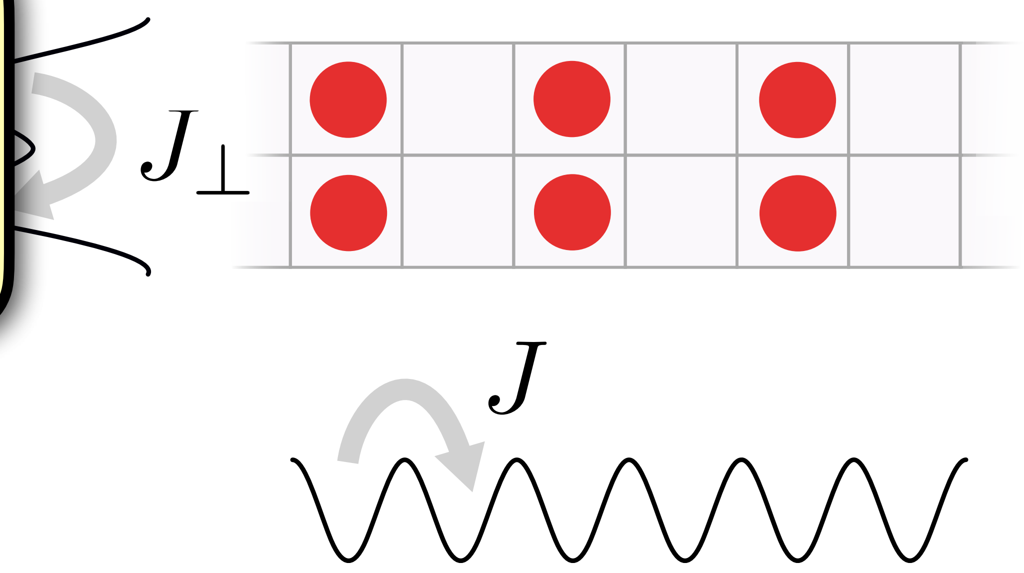
Determination of dynamical exponent

Timescales - Integrable to Chaotic



Density-Density Correlations

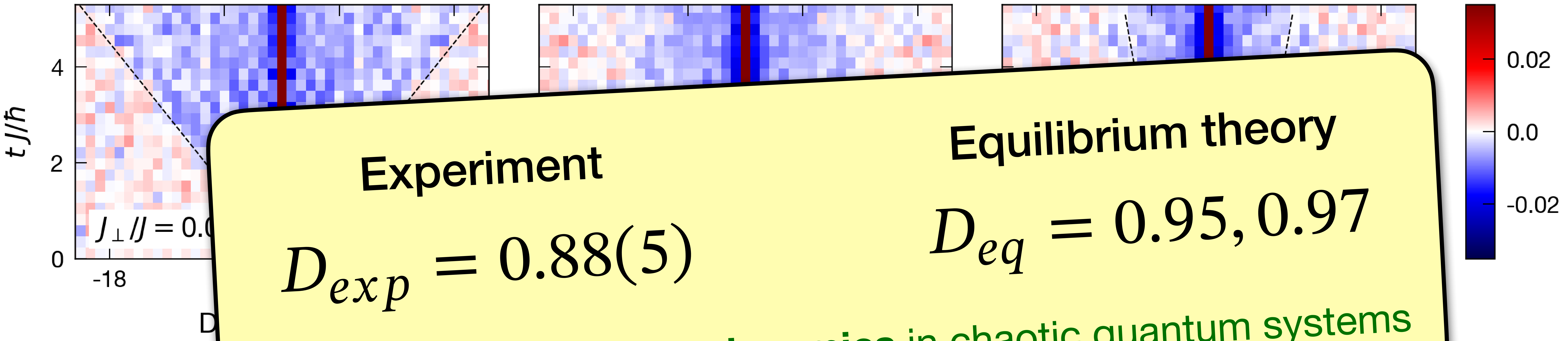
$$C(i-j) = \langle \hat{N}_i \hat{N}_j \rangle - \langle \hat{N}_i \rangle \langle \hat{N}_j \rangle$$



Hardcore bosons
(tuneable ladder coupling)

Integrable (free fermions)

Chaotic



Experiment

$$D_{exp} = 0.88(5)$$

Equilibrium theory

$$D_{eq} = 0.95, 0.97$$

Test for **fluctuating hydrodynamics** in chaotic quantum systems

Rung Density-Density Correlations

See also:
M. Cheneau, ..., I. Bloch, S. Kuhr, Nature (2012)
Y.-G. Zheng, ..., Z.-S. Yuan, J.-W. Pan, arXiv:2210.08556

Equilibrium transport theory:
R. Steinigeweg et al., Phys. Rev. B (2014)
T. Rakovszky, C. W. von Keyserlingk & F. Pollmann Phys. Rev. B (2022)

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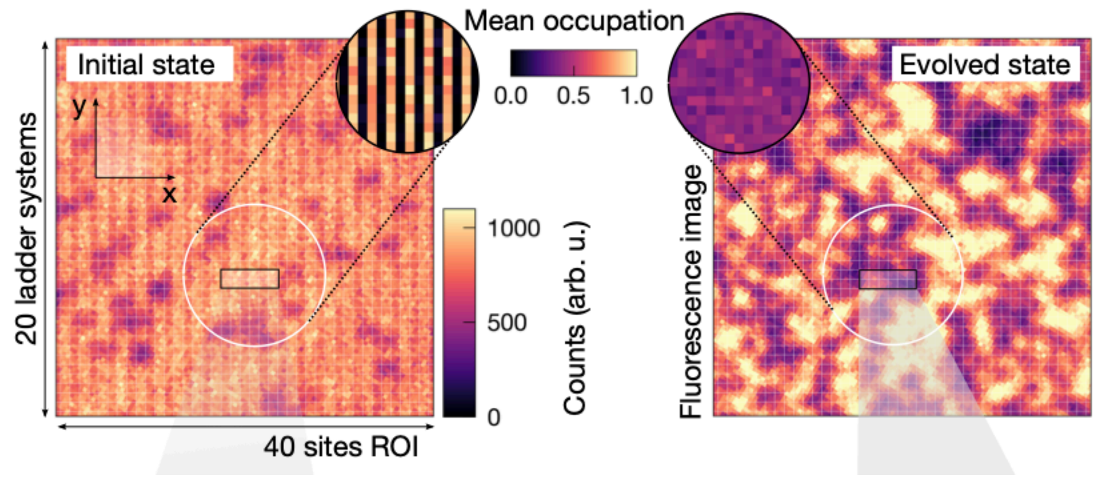
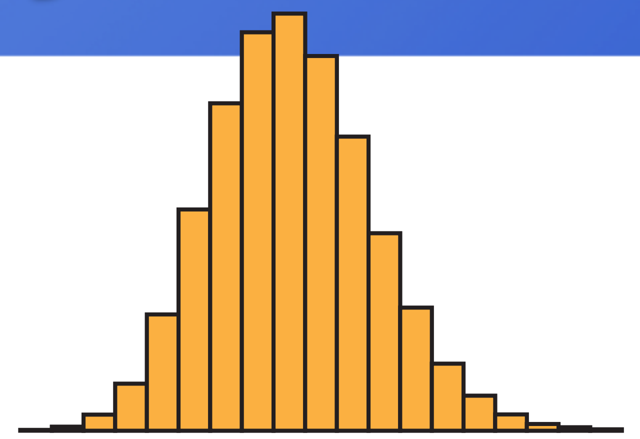
Summary Slide - Fluctuation Hydrodynamics





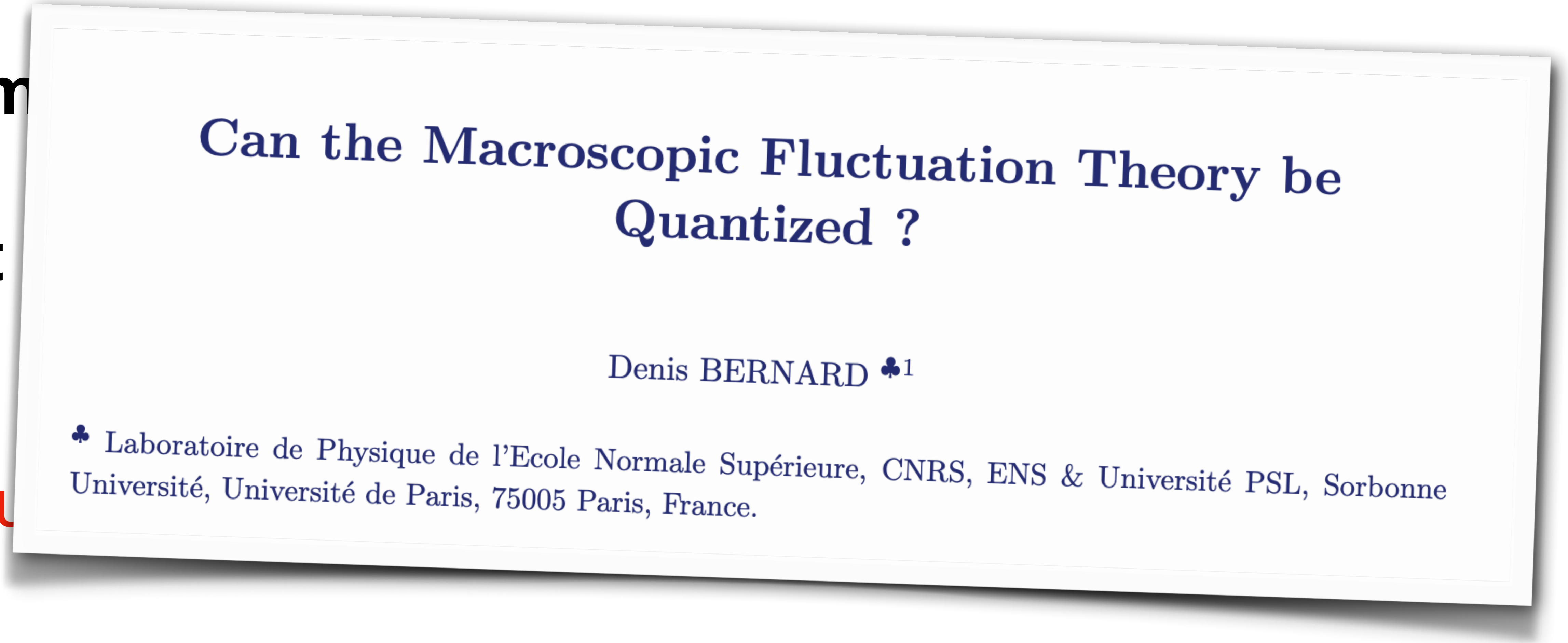
- **Full counting statistic / fluctuations** powerful new observables for quantum transport

- **(Nonlinear) Noisy classical dynamics** can efficiently describe **charge fluctuation dynamics in chaotic quantum many-body systems** via MFT

- Test of **fluctuation-dissipation theorem**

- Determination of **equilibrium transport** through out-of-equilibrium dynamics

- When does MFT fail? Higher order cumu



Can the Macroscopic Fluctuation Theory be Quantized ?

Denis BERNARD ♣1

♣ Laboratoire de Physique de l'Ecole Normale Supérieure, CNRS, ENS & Université PSL, Sorbonne Université, Université de Paris, 75005 Paris, France.

Random Unitary Circuits: E. McCulloch, J. De Nardis, S. Gopalakrishnan & R. Vasseur Full arXiv:2302.01355
Quantising MFT: D. Bernard J. Phys. A: Math. Theor. 54, 433001 (2021).

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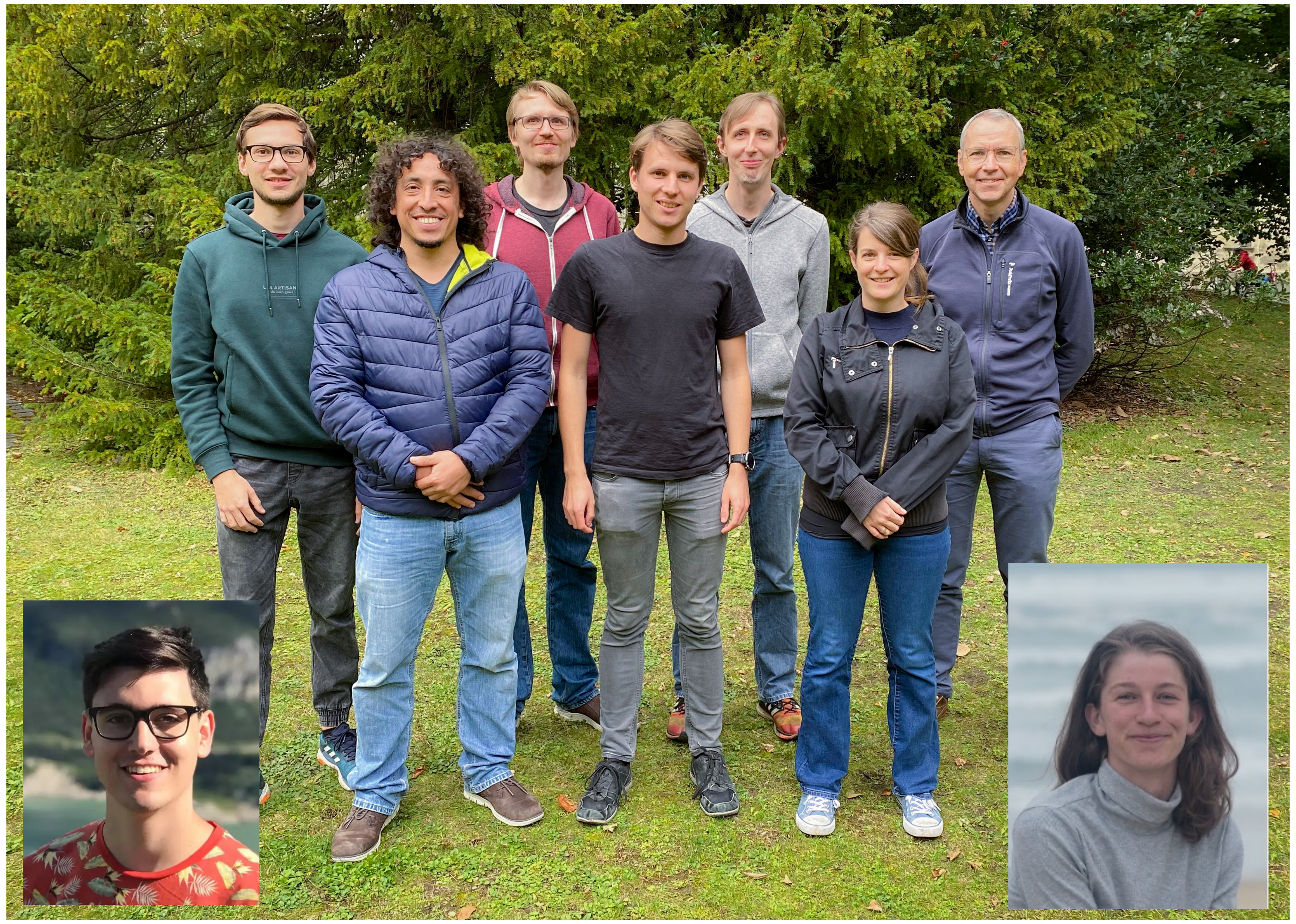

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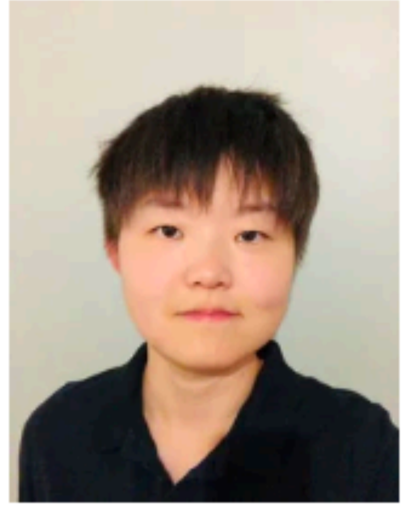

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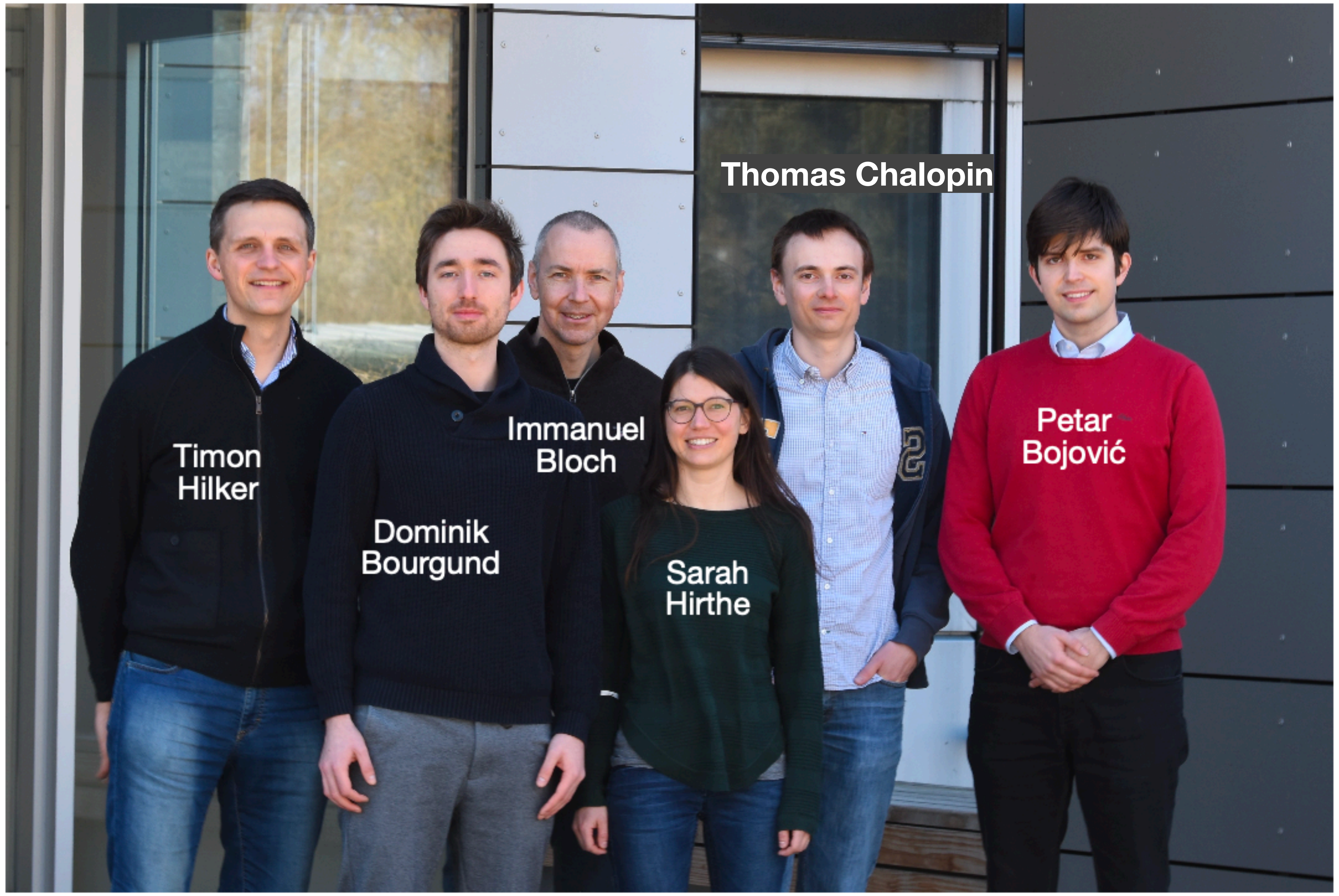

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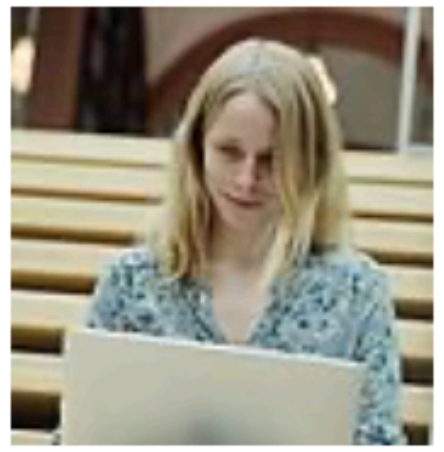

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